

Development of an Optimal Inventory Policy for Deteriorating Items with Stock Level and Selling Price Dependent Demand under Trade Credit

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Abstract

This paper reflect the effect of trade credit on the inventory model to depict the real life situation. Here, model is developed for price and stock dependent demand with constant deterioration rate and trade credit feature. Holding cost is taken as parabolic function of time. With the allowed shortages and partial backlogging concept, this model is formulated for the maximization of total inventory profit. Numerical problems are involved to give the practical result of model and optimal solution. The effect of different parameters involved are shown by the sensitive analysis. This model can be applied for the different business enterprises in order to get maximum profit.

Keywords: EOQ, deterioration, Stock depended demand, Parabolic holding cost, Trade credit

INTRODUCTION

Deterioration is natural phenomenon whose impact or effect cannot be ignored even in daily routine life. Deterioration actually signifies a stage in every item's lifetime after that the item will lose its utility or usefulness or original attribute. Like, fashionable goods, mobile phone, chips etc. in the beginning they all been at the highest level of popularity. Deterioration create a significant effect on the inventory model. Demand is the seed of inventory. Demand can be classified differently according to the need. Demand depends on many factors like time, season, and availability of items. It can be constant and stochastic. When demand depend on the availability and price of the items, it is called as stock dependent demand.

The flexibility given by the supplier to retailer that he needs not to clear all his dues before the delivery is termed as trade credit. In this a specific time is provided by the supplier that he will not ask for any interest after that a certain interest will be applied on the amount to be settle. As in modern era trade credit becomes a common practice, so many of the researchers are taking trade credit as an important parameter while formulating any model for optimal inventory policy.

During the past few years, the researchers have started the focusing on this concept. Gupta and Vrat (1986) were the first who developed models for stock dependent consumption rate. Baker and Urban (1988) established an economic order quantity model for a power form inventory-level-dependent demand pattern. Mandal and Phaujdar (1989) introduced an

economic production quantity model for deteriorating items with constant production rate linearly stock-dependent demand. Researchers like Pal et al. (1993), Giri et.al (1996), Ray et al. (1998), Uthaya Kumar and Parvathi (2006), Roy and Choudhuri (2008), Choudhury et al. (2013) and many others worked on it. Soni and Shah (2008) introduced the optimal ordering policy for an inventory model with stock dependent demand. Wu et. Al (2006) was the first who developed an inventory model for non-instantaneous deteriorating items with stock-dependent demand. Chang et al. (2010) established an optimal replenishment policy for non-instantaneous deteriorating items with stock- dependent demand. Sana (2010) established an EOQ model for perishable items with stock-dependent demand; Gupta et al. (2013) introduced optimal ordering policy for stock-dependent demand inventory model with non-instantaneous deteriorating items. Vipin Kumar et al. (2011) was developed an Inventory Model For Deteriorating Items With Permissible Delay In Payment Under Two-Stage Interest Payable Criterion And Quadratic Demand" Mishra and Tripathy (2012) gave an idea on an inventory model for time dependent Weibull deterioration with partial backlogging, Vipin Kumar et al. (2013) derived a Deterministic Inventory Model for Deteriorating Items with Selling Price Dependent Demand and Parabolic Time Varying Holding Cost under Trade Credit" Palanivel and Uthayakumar (2014) established model for non-instantaneous deteriorating products with time dependent two variable Weibull deterioration rate, where demand rate is power function of time and permitting partial backlogging. Vipin Kumar, Anupama Sharma, C.B.Gupta (2014) established an EOQ Model For Time Dependent Demand and Parabolic Holding Cost With Preservation Technology Under Partial Backlogging For Deteriorating Items. Farughi et al. (2014) modeled pricing and inventory control policy for non-instantaneous deteriorating items with price and time dependent demand permitting shortages with partial backlogging. Vipin Kumar et al. (2015) worked on two-Warehouse Partial Backlogging Inventory Model For Deteriorating Items With Ramp Type Demand". While, Zhang et al. (2015) developed pricing model for non-instantaneous deteriorating item by considering constant deterioration rate and stock sensitive demand. Further, Vipin Kumar, Anupama Sharma, C.B.Gupta (2015) "A Deterministic Inventory Model For Weibull Deteriorating Items with Selling Price Dependent Demand And Parabolic Time Varying Holding Cost Gopal Pathak, Vipin Kumar, C.B.Gupta (2017) A Cost Minimization Inventory Model for Deteriorating Products and Partial

Backlogging under Inflationary Environment . Aditi Khanna, Aakanksha Kishore and Chandra K. Jaggi (2017) Strategic production modeling for defective items with imperfect inspection process, rework, and sales return under two-level trade credit Gopal Pathak, Vipin Kumar, C.B.Gupta (2017) developed An Inventory Model for Deterioration Items with Imperfect Production and Price Sensitive Demand under Partial Backlogging , Mashud et al. (2018) worked on non-instantaneous deteriorating item having different demand rates allowing partial backlogging.

In current chapter, with considered effect of trade credit a mathematical model is developed with the scheme of profit maximization. In this model selling price demand is taken for the formulation of the problem. By considering the stock level of the inventory, the deterioration rate is kept constant and holding cost is dependent on parabolic time parameter. With allowed shortages & partial backlogging concept, delay in payment is taken at two different level. Numerical example, tables and sensitive analysis shows the effect of different parameters on model description.

ASSUMPTIONS

The assumptions used in this model are as follows:

- i. Demand is function of selling price and stock. Defined as: $D\{I(t), s\} = \begin{cases} \frac{a}{s^c} + bI(t) & , I(t) \geq 0 \\ \frac{a}{s^c} & , I(t) < 0 \end{cases}$.where $a (> 0)$ and $b (0 \leq b < 1)$ are initial and stock-dependent consumption rate parameters and $c > 0$.
- ii. Shortages with partial backlogging concept are allowed. The backlogging rate is $B(T-t) = \frac{1}{1 + \delta(T-t)}$, here waiting time is $(T-t)$ is the and δ is backlogging parameter with $0 < \delta < 1$.
- iii. The model is developed for a single product.
- iv. Infinite Time horizon is take with zero lead time
- v. Deteriorated units can't be repaired.
- vi. Under trade credit practice M, is take a grace period provided by the supplier to settle the amount of purchase. Means, no interest will be charged for the interval $[0, M]$ if $T \geq M$. I_c is the interest charged for the interval $[M, T]$. But if $T \leq M$, no interest will be charged.
- vii. I_e defined the interest earn by the retailer for the time $t = 0$ to $t = M$ under trade credit policy.
- viii. Parabolic Holding cost which is increasing function time and taken as $h(t) = h_1 + h_2 t^2$ where $h_1, h_2 \geq 0$

NOTATIONS

The following are the notations used in this model:

- i. $I(t)$: Inventory units at any time t
- ii. θ : Deterioration parameter, where $0 \leq \theta \ll 1$
- iii. O : Per order ordering cost (₹ /Order)
- iv. p : Per unit purchasing cost (₹ /Unit)
- v. S : Per unit Selling price (₹ /Unit)
- vi. α : Per unit Shortage cost (₹ /Unit)
- vii. l : Per unit Lost sale cost (₹ /Unit)
- viii. d : Per unit deterioration cost per unit (₹ /Unit)
- ix. Q_1 : Initial inventory level (Unit)
- x. Q_2 : Maximum backordered quantity (Unit)
- xi. Q : Order quantity (Unit)
- xii. t_1 : No inventory level at this time (Time Unit)
- xiii. T : Cycle time (Time Unit)
- xiv. M : Allowable trade credit period (Time Unit)
- xv. I_e : Interest earns by the rate (%)
- xvi. I_p : Interest charged by the rate (%)
- xvii. U : Unpaid amount at the time of payment (₹)
- xviii. U.T.P: Unit time profit (₹ .)

THE MATHEMATICAL MODEL AND ANALYSIS

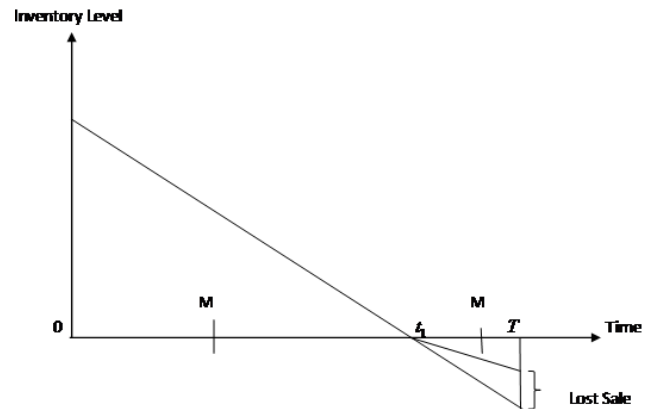


Figure 1. The graphical representation of behavior of inventory level over time

Let $I(t)$ be inventory level at time $t (0 \leq t \leq T)$.The graphical representation of the problem is shown in Fig.3.1. As shown in figure it is clear that during the interval $[0, t_1]$ the inventory starts decreasing due to joint effects of deterioration and demand and then drops to zero at time t_1 . After, during the interval $[t_1, T]$ shortages occur which are partially backlogged.

Hence, the mathematical representation of rate of change of inventory at any time t can be given by following differential equations:

$$\frac{dI(t)}{dt} = -\theta I(t) - D\{I(t), s\}, \quad 0 \leq t \leq t_1 \quad (1)$$

$$\frac{dI(t)}{dt} = -B(T-t)D\{I(t), s\}, \quad t_1 \leq t \leq T \quad (2)$$

With the boundary condition $I(t_1) = 0$

The solutions of the above differential equations are given by

$$I(t) = as^{-c} \left\{ (t_1 - t) + \frac{(\theta + b)}{2} (t_1 - t)^2 \right\}, \quad 0 \leq t \leq t_1 \quad (3)$$

$$\text{And } I(t) = \frac{as^{-c}}{\delta} \ln \left[\frac{1 + \delta(T-t)}{1 + \delta(T-t_1)} \right], \quad t_1 \leq t \leq T \quad (4)$$

With the help of equation (3) and (4) the maximum inventory level and the maximum amount of demand backlogged during the first replenishment, we get

The maximum positive inventory

$$= Q_1 = I(0) = as^{-c} \left\{ t_1 + \frac{(\theta + b)}{2} t_1^2 \right\} \quad (5)$$

The maximum backordered quantity

$$= Q_2 = -I(T) = \frac{as^{-c}}{\delta} \ln [1 + \delta(T - t_1)] \quad (6)$$

Hence, the order quantity per cycle is given by

$$Q = Q_1 + Q_2 = as^{-c} \left\{ T + \frac{(\theta + b)}{2} t_1^2 + \frac{1}{\delta} \ln [1 + \delta(T - t_1)] \right\} \quad (7)$$

$$\begin{aligned} \text{T.R.P} &= \frac{I}{T} [\text{Sales Revenue} - \text{Ordering Cost} - \text{Holding} \\ &\text{Cost} - \text{Deterioration Cost} - \text{Shortage Cost} - \text{Lost Sales Cost} \\ &- \text{Interest Charged} + \text{Interest Earned}] \end{aligned} \quad (8)$$

a) Sales Revenue: The Sales Revenue per cycle is

$$SR = (s - p)(Q_1 + Q_2) = (s - p)Q$$

$$SR = (s - p)as^{-c} \left\{ T + \frac{(\theta + b)}{2} t_1^2 + \frac{1}{\delta} \ln [1 + \delta(T - t_1)] \right\} \quad (9)$$

b) Ordering Cost: Per cycle ordering cost

$$OC = O \quad (10)$$

c) Holding Cost: Per cycle Inventory holding cost is

$$\begin{aligned} HC &= \int_0^{t_1} (h_1 + h_2 t^2) I(t) dt \\ HC &= as^{-c} \left\{ h_1 \left(\frac{t_1^2}{2} + \frac{(\theta + b)}{6} t_1^3 \right) + h_2 \left(\frac{t_1^4}{12} + \frac{(\theta + b)}{30} t_1^5 \right) \right\} \end{aligned} \quad (11)$$

d) Deterioration Cost: The cost associated with the deteriorated units is calculated as

$$\begin{aligned} DC &= d \left\{ Q_1 - \int_0^{t_1} (a + bI(t) - cs) dt \right\} \\ DC &= das^{-c} \left\{ \frac{\theta}{2} t_1^2 - \frac{b(\theta + b)}{6} t_1^3 \right\} \end{aligned} \quad (12)$$

e) Shortage Cost: Per cycle the shortage is

$$\begin{aligned} SC &= \alpha \int_{t_1}^T \{-I(t)\} dt \\ SC &= \alpha as^{-c} \left\{ T - t_1 - \frac{1}{\delta} \ln [1 + \delta(T - t_1)] \right\} \end{aligned} \quad (13)$$

f) Lost Sale Cost: The opportunity cost due to the lost sale during the interval $[t_1, T]$ is

$$\begin{aligned} LSC &= l \int_{t_1}^T as^{-c} \{1 - B(T - t)\} dt \\ LSC &= las^{-c} \left[T - t_1 + \frac{1}{\delta} \ln [1 + \delta(T - t_1)] \right] \end{aligned} \quad (14)$$

g) Interest Payable and Interest Earned : To calculate interest payable and interest earned the following two cases arises:

- i. When allowed trade credit period M is greater than the time t_1 i.e. ($M \geq t_1$)
- ii. When allowed trade credit period M is less than the time t_1 i.e. ($M < t_1$)

i. Case.1: When $M \geq t_1$

In this case, the allowed trade credit period is greater than the time when the retailer sold all the stock. So, at the time of payment, retailer will have enough money to pay all the dues. Hence, the interest charged in this case will be zero.

Inventory level

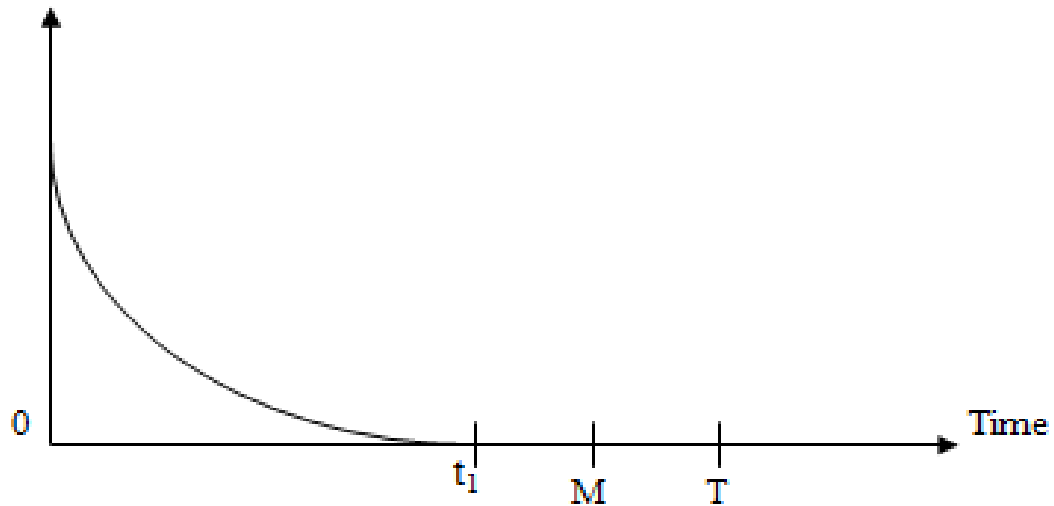


Figure 2. when allowed trade credit period M is greater than the time t_1 ($M \geq t_1$)

$$I.C_1 = 0 \quad (15)$$

$$I.E_1 = sI_e \left[\int_0^{t_1} D\{I(t),s\} t dt + (M - t_1) \int_0^{t_1} D\{I(t),s\} dt \right]$$

$$I.E_1 = sI_e a s^{-c} \left[M \left\{ t_1 + b \left\{ \frac{t_1^2}{2} + \frac{(\theta + b)}{6} t_1^3 \right\} \right\} - \frac{t_1^2}{2} - b \left\{ \frac{t_1^3}{3} + \frac{(\theta + b)}{8} t_1^4 \right\} \right] \quad (16)$$

In this case, the Total Retailer's unit time Profit of the system will be

$$U.T.P(s, t_1, T) = \frac{1}{T} [SR - OC - HC - DC - SC - LSC - I.C_1 + I.E_1]$$

$$U.T.P(s, T) = \frac{O}{T} + \frac{as^{-c}}{T} \left[\begin{aligned} & (s-p) \left\{ T + \frac{(\theta+b)}{2} t_1^2 + \frac{1}{\delta} \ln[1 + \delta(T - t_1)] \right\} \\ & - \left\{ h_1 \left(\frac{t_1^2}{2} + \frac{(\theta+b)}{6} t_1^3 \right) + h_2 \left(\frac{t_1^4}{12} + \frac{(\theta+b)}{30} t_1^5 \right) \right\} \\ & - d \left\{ \frac{\theta}{2} t_1^2 - \frac{b(\theta+b)}{6} t_1^3 \right\} - \alpha \left\{ T - t_1 - \frac{1}{\delta} \ln[1 + \delta(T - t_1)] \right\} \\ & - I \left[T - t_1 + \frac{1}{\delta} \ln[1 + \delta(T - t_1)] \right] + sI_e \left\{ \begin{aligned} & M \left(t_1 + b \left(\frac{t_1^2}{2} + \frac{(\theta+b)}{6} t_1^3 \right) \right) \\ & - \frac{t_1^2}{2} - b \left(\frac{t_1^3}{3} + \frac{(\theta+b)}{8} t_1^4 \right) \end{aligned} \right\} \end{aligned} \right] \quad (17)$$

ii Case.2: When $M < t_1$

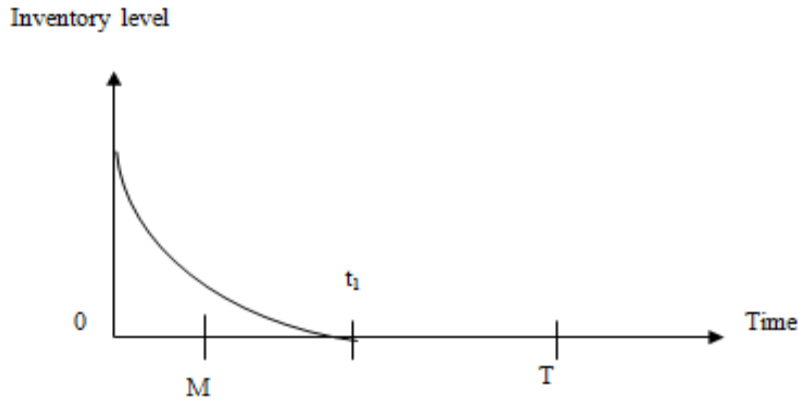


Figure 3. When allowed trade credit period M is less then the time ($M < t_1$)

On the bases of the total available capital at the time of payment we considered the following two sub-cases arise:

2.1. When $sD[0, M] + I.E_2[0, M] \geq pI(0)$

2.2. When $sD[0, M] + I.E_2[0, M] < pI(0)$

Case.2.1. When $sD[0, M] + I.E_2[0, M] \geq pI(0)$

This is case when the retailer is able to clear all his dues at $t = M$. In this case the Interest charged will be zero.

$$I.C_{2,1} = 0 \tag{18}$$

$$I.E_{2,1} = sI_e \left[\int_0^M D\{I(t), s\} t dt + \int_M^{t_1} D\{I(t), s\} t dt \right]$$

$$I.E_{2,1} = sI_e a s^{-c} \left\{ \frac{t_1^2}{2} + b \left(\frac{t_1^3}{6} + \frac{(\theta + b)}{24} t_1^4 \right) \right\} \tag{19}$$

$$I.E_2[0, M] = sI_e a s^{-c} \left\{ \frac{M^2}{2} + b \left(\frac{M^3}{6} + \frac{(\theta + b)}{24} M^4 \right) \right\} \tag{20}$$

In this case, the Total Retailer's Profit per unit time of the system will be

$$U.T.P_{2,1}(s, T) = \frac{1}{T} [SR - OC - HC - DC - SC - LSC - I.C_{2,1} + I.E_{2,1}]$$

$$U.T.P_{2,1}(s, t_1, T) = \frac{O}{T} + \frac{as^{-c}}{T} \left[\begin{aligned} & (s-p) \left\{ T + \frac{(\theta + b)}{2} t_1^2 + \frac{1}{\delta} \ln [1 + \delta(T - t_1)] \right\} \\ & - \left\{ h_1 \left(\frac{t_1^2}{2} + \frac{(\theta + b)}{6} t_1^3 \right) + h_2 \left(\frac{t_1^4}{12} + \frac{(\theta + b)}{30} t_1^5 \right) \right\} \\ & - d \left\{ \frac{\theta}{2} t_1^2 - \frac{b(\theta + b)}{6} t_1^3 \right\} - \alpha \left\{ T - t_1 - \frac{1}{\delta} \ln [1 + \delta(T - t_1)] \right\} \\ & - l \left[T - t_1 + \frac{1}{\delta} \ln [1 + \delta(T - t_1)] \right] + sI_e \left\{ \frac{t_1^2}{2} + b \left(\frac{t_1^3}{6} + \frac{(\theta + b)}{24} t_1^4 \right) \right\} \end{aligned} \right] \tag{21}$$

Case.2.2. When $sD[0, M] + I.E_2[0, M] < pI(\theta)$

This case depicts the situation when retailer is not able to clear his dues within the grace period, so he has to pay interest. Therefore, interest will be charged on unpaid amount. We have

$$D[0, M] = \int_0^M D\{I(t), s\} dt$$

$$D[0, M] = as^{-c} \left\{ M + b \left(\frac{M^2}{2} + \frac{(\theta + b)}{6} M^3 \right) \right\} \quad (22)$$

Unpaid amount = $U = pI(\theta) - (sD[0, M] + I.E_2[0, M])$

$$U = as^{-c} \left[\begin{array}{l} p \left\{ t_1 + \frac{(\theta + b)}{2} t_1^2 \right\} - s \left\{ M + b \left(\frac{M^2}{2} + \frac{(\theta + b)}{6} M^3 \right) \right\} \\ -sI_e \left\{ \frac{M^2}{2} + b \left(\frac{M^3}{6} + \frac{(\theta + b)}{24} M^4 \right) \right\} \end{array} \right] \quad (23)$$

Interest charged on this unpaid amount will be

$$I.C_{2.2} = (t_1 - M) I_c U$$

$$I.C_{2.2} = I_c (t_1 - M) as^{-c} \left[\begin{array}{l} p \left\{ t_1 + \frac{(\theta + b)}{2} t_1^2 \right\} - s \left\{ M + b \left(\frac{M^2}{2} + \frac{(\theta + b)}{6} M^3 \right) \right\} \\ -s \left\{ \frac{M^2}{2} + b \left(\frac{M^3}{6} + \frac{(\theta + b)}{24} M^4 \right) \right\} \end{array} \right] \quad (24)$$

and interest earned is given by

$$I.E_{2.2} = I.E_{2.1} = sI_e as^{-c} \left\{ \frac{t_1^2}{2} + b \left(\frac{t_1^3}{6} + \frac{(\theta + b)}{24} t_1^4 \right) \right\} \quad (25)$$

In this case, the Total Retailer's Profit per unit time of the system will be

$$U.T.P_{2.2}(s, T) = \frac{1}{T} [SR - OC - HC - DC - SC - LSC - I.C_{2.2} + I.E_{2.2}]$$

$$U.T.P_{2.2}(s, t_1, T) = \frac{O}{T} + \frac{as^{-c}}{T} \left[\begin{array}{l} (s-p) \left\{ T + \frac{(\theta + b)}{2} t_1^2 + \frac{1}{\delta} \ln[1 + \delta(T - t_1)] \right\} \\ - \left\{ h_1 \left(\frac{t_1^2}{2} + \frac{(\theta + b)}{6} t_1^3 \right) + h_2 \left(\frac{t_1^4}{12} + \frac{(\theta + b)}{30} t_1^5 \right) \right\} \\ - d \left\{ \frac{\theta}{2} t_1^2 - \frac{b(\theta + b)}{6} t_1^3 \right\} - \alpha \left\{ T - t_1 - \frac{1}{\delta} \ln[1 + \delta(T - t_1)] \right\} \\ - l \left[T - t_1 + \frac{1}{\delta} \ln[1 + \delta(T - t_1)] \right] \\ - I_c (t_1 - M) \left\{ \begin{array}{l} p \left(t_1 + \frac{(\theta + b)}{2} t_1^2 \right) - s \left\{ M + b \left(\frac{M^2}{2} + \frac{(\theta + b)}{6} M^3 \right) \right\} \\ -sI_e \left\{ \frac{M^2}{2} + b \left(\frac{M^3}{6} + \frac{(\theta + b)}{24} M^4 \right) \right\} \end{array} \right\} \\ + sI_e \left\{ \frac{t_1^2}{2} + b \left(\frac{t_1^3}{6} + \frac{(\theta + b)}{24} t_1^4 \right) \right\} \end{array} \right] \quad (26)$$

The goal of the present model is to maximize the total retailer's profit per unit time.

$$\max : U.T.P(s, t_1, T)$$

$$= \begin{cases} U.T.P_1(s, t_1, T) & M \geq t_1 \\ U.T.P_{2,1}(s, t_1, T), \quad sD[0, M] + I.E_2[0, M] \geq pI(0) \\ U.T.P_{2,2}(s, t_1, T), \quad sD[0, M] + I.E_2[0, M] \leq pI(0) \end{cases} \quad M \leq t_1$$

The non-linearity of the objective functions in the equations (17), (21), and (26) does not allow us to obtain the closed form solution. Therefore, we analyze the model with numerical values for the inventory parameters.

NUMERICAL EXAMPLES

Case 1: When $M \geq t_1$: The following input data of different parameter are considered for the numerical illustration.

$$[T, a, c, p, b, \alpha, \theta, I_e, M, d, l, O, h_1, h_2] = [1, 1000, 0.02, 15, 0.05, 10, 0.001, 0.04, 0.9, 16, 12, 500, 0.02]$$

Corresponding to these data, the following optimal value of t_1 , $I(0)$ and unit time profit exists:

$$t_1 = 0.810656 \text{ days, } s = 34.29 \text{ ₹ /unit, } I(0) = 998.133 \text{ unit, } U.T.P. = ₹ 4491.74 \text{ respectively.}$$

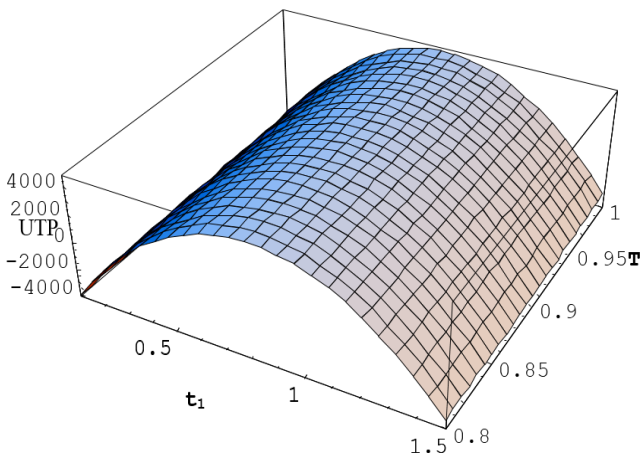


Figure 4: Concavity of the U.T.P. function (case 1)

Case 2.1. When $M \leq t_1$ and

$sD[0, M] + I.E_2[0, M] \geq pI(0)$ The following input data of different parameter are considered for the numerical illustration.

$$[T, a, c, p, b, \alpha, \theta, I_e, M, d, l, O, h_1, h_2] = [1, 1000, 0.02, 15, 0.05, 10, 0.001, 0.03, 0.9, 16, 12, 500, 1, 0.02]$$

Corresponding to these data, the following optimal value of t_1 , $I(0)$ and unit time profit exists:

$$t_1 = 0.844698 \text{ days, } s = 35.79 \text{ ₹ / unit, } I(0) = 1005.43 \text{ unit, } U.T.P. = ₹ 4405.81 \text{ respectively.}$$

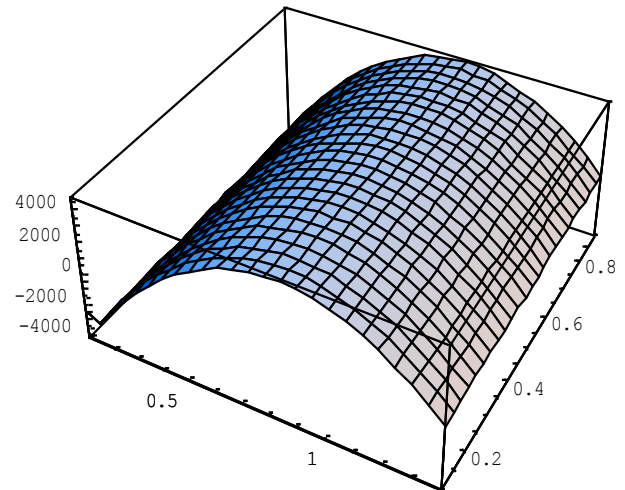


Figure 5: Concavity of the U.T.P. function (case 2.1)

Case 2.2. When $M < t_1$ and

$sD[0, M] + I.E_2[0, M] < pI(0)$: The following input data of different parameter are considered for the numerical illustration.

$$[T, a, c, p, b, \alpha, \theta, I_e, M, d, l, O, h_1, h_2] = [1, 1000, 0.02, 15, 0.05, 10, 0.001, 0.03, 0.9, 16, 12, 500, 1, 0.02]$$

Corresponding to these data, the following optimal value of t_1 , $I(0)$ and unit time profit exists:

$$t_1 = 0.832929 \text{ days, } s = 36.12 \text{ ₹ per Unit, } I(0) = 1003.13 \text{ unit, } U.T.P. = ₹ 4403.11 \text{ respectively.}$$

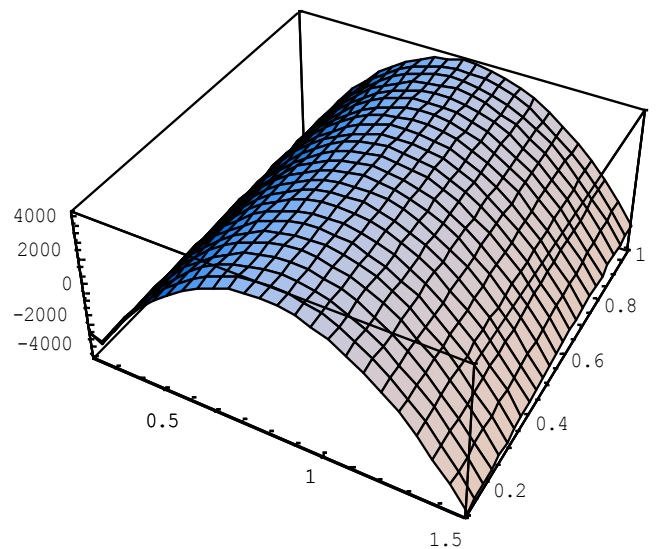


Figure 6: Concavity of the U.T.P. function (case 2.1 and case 2.2)

SENSITIVITY ANALYSIS: To reduce the length of the paper here we discussed only one case

Case 1. When $M \geq t_1$:

Table 1

Parameters	%	Values	t_1	I(0)	U.T.P.
A	-20%	800	0.81066	798.366	3492.69
	-15%	850	0.81066	848.308	3742.45
	-10%	900	0.81066	898.25	3992.22
	-5%	950	0.81066	948.192	4241.98
	0%	1000	0.81066	998.134	4491.74
	5%	1050	0.81066	1048.08	4741.5
	10%	1100	0.81066	1098.02	4991.26
	15%	1150	0.81066	1147.96	5241.03
	20%	1200	0.81066	1197.9	5490.79
B	-20%	0.04	0.80975	994.648	4477.95
	-15%	0.0425	0.80998	995.518	4481.39
	-10%	0.045	0.8102	996.388	4484.84
	-5%	0.0475	0.81043	997.26	4488.29
	0%	0.05	0.81066	998.133	4491.74
	5%	0.0525	0.81088	999.006	4495.19
	10%	0.055	0.81111	999.881	4498.65
	15%	0.0575	0.81134	1000.76	4502.1
	20%	0.06	0.81156	1001.63	4505.56
C	-20%	0.016	0.81066	998.273	4492.44
	-15%	0.017	0.81066	998.238	4492.26
	-10%	0.018	0.81066	998.203	4492.09
	-5%	0.019	0.81066	998.168	4491.92
	0%	0.02	0.81066	998.133	4491.74
	5%	0.021	0.81066	998.098	4491.57
	10%	0.022	0.81066	998.063	4491.39
	15%	0.023	0.81066	998.028	4491.22
	20%	0.024	0.81066	997.993	4491.04
	-20%	0.0008	0.81056	998.044	4490.42
	-15%	0.00085	0.81058	998.061	4490.75
	-10%	0.0009	0.81061	998.077	4491.08
	-5%	0.00095	0.81063	998.094	4491.41
	0%	0.001	0.81066	998.133	4491.74
	5%	0.00105	0.81068	998.155	4492.07
	10%	0.0011	0.81071	998.177	4492.4
	15%	0.00115	0.81073	998.199	4492.73
	20%	0.0012	0.81075	998.221	4493.06

OBSERVATIONS

The model is solved for three different conditions depending on allowable trade credit period and available money at that time. For different conditions the optimal value of ' t_1 ', ' S ', ' $I(0)$ ' and unit time profit has been calculated. Observing these results we arrive at the following conclusion.

1. We have changed all the parameters by -20%, -10%, 0%, 10%, 20%. It has been observed that as compared to different parameters a, b, c, d, θ the critical time t_1 is quite stable.
2. With the variation in parameters 'a' and 'b' the value of initial stock and unit time profit mildly increases while an increase in demand parameter 'c' shows the reverse effect on initial amount of stock level and unit time profit.
3. An increase in deterioration parameter ' θ ' also results and increase in initial stock level as well as unit time profit of the system.
4. It is also observed that with allowed extension in payment than critical time, the profit get extremized.

CONCLUSION

In this chapter, a mathematical model is formulated with selling price dependent demand for obsolete items by taking parabolic time dependent demand. To keep the model more related to real base condition shortages are allowed and partially backlogged. The delay in payment of the stock is kept at two different level. Depending on this delay time three different cases are taken for modelling. The model is developed for maximization of profit. The different graphs, table, and sensitive analysis shows that how by considerable effect of different parameters, a unique optimal solution of the problem exists.

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