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Mathematical Modeling of Blood Flow through an Inclined Axially Non-Symmetric Stenosed Catheterized Artery with Body Acceleration

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Abstract:

In this paper, a mathematical model has been developed to study the pulsatile flow of blood through an axially non-symmetric but radially symmetric stenosed inclined catheterized artery with periodic body acceleration and slip at wall. In our study, blood is assumed to be Newtonian fluid, since in general stenosis is developed in large arteries. Here, perturbation as well as analytical methods are used to finding the solutions of non-linear partial differential equations of various flow variables. In this work, various interesting results are obtained regarding velocity field, volumetric flow rate, wall shear stress and effective viscosity of blood during catheterization in inclined arteries. The variation of flow variables with different parameters are obtained graphically and discussed elaborately. Here, it has been observed that velocity increases with increase of inclination, time, body acceleration to a certain limit and then decreases. Also, wall shear stress increases with increase of inclination but decrease with increase of body acceleration and slip velocity. Volumetric flow rate increases with mild increase of body acceleration and Effective viscosity also increase with increase of stenosis shape parameter.

Keywords: Newtonian Fluid, Pulsatile flow, Inclined, Stenosis, Body acceleration, Slip velocity.

Mathematical Subject Classification: 76Z05, 74G10.

INTRODUCTION

The one of the most leading cause of death in the world is heart diseases. The most common types of disease are ischemia, atherosclerosis and angina pectoris (Mekheimer and El Kot [1]). Ischemia is the temporary deficiency of oxygen in a part of body due to constriction or obstruction of blood in the blood vessels. Atherosclerosis is a type of arteriosclerosis, which comes from Greek word athero (meaning paste) and sclerosis (hardness). This is because of deposit of fatty substances , cholesterol, cellular waste products, calcium, fibrin etc. This abnormal growth in the lumen of an artery is called stenosis (atherosclerosis). Such sever growth on the artery wall results in serious circulatory disorders as proposed by Young [2], Biswas [3], Bali and Awasthi [4], Biswas and Chakraborty [5] etc. These circulatory disorders may be included as narrowing in artery leading to the reduction and impediment to blood flow in the constricted artery regions, the blockage of artery in making the flow irregular, causing abnormality in blood flow.

With the development of medical technology, there has been considerable increase in use of catheter of various sizes for coronary Angiography, balloon Angioplasty, Renal Angiography etc. In medical science, a catheter is a thin tube made from medical grade materials inserted in the body to treat diseases or perform surgical procedure. The catheter has immense importance and standard tool for diagnosis and treatment of cardiovascular diseases.

In general, blood exhibits Non-Newtonian character at low shear rates [6], but in larger arteries with diameters about and above 1mm , blood behaves like Newtonian fluids [7]. In general, stenosis normally generated and developed in large diameter arteries in the range of 500 μm to 2000 μm, so we may consider the behaviour of blood as Newtonian fluid.

A good number of researchers viz. Haldar [8], Srivastava [9], Srivastava [10], Chakraborty and Mandal [11], Mandal [12], EI-Shahed [13], Jung et al.[14], Liu et al.[15], Sankar and Lee [16, 17], Srivastava and Srivastava[18] etc. have studied the flow of blood through stenosed arteries.

In recent years, considerable amount of researchers have studied the flow of blood through catheterized artery. The researchers have analysed the flow of blood in arteries by considering the catheter and the artery as rigid co-axial cylinders , assuming blood as either a Newtonian or a Non-Newtonian fluid.

Back [19], investigated the influence and size of the catheter on measurement of mean pressure drop and resistance to flow of blood through coronary vessels. Mc Donald [20], investigated the theoretical corrections of pressure gradient for pulsatile flow of blood through catheterized artery. Kerahalios [21] and Jayaraman and Tiwari [22] studied the effect of catheterization on various flow characteristics in a curved artery. Back and Denton [23], investigated and estimated the wall shear stress and its clinical importance in coronary Angioplasty. Daripa and Dash[24], studied the

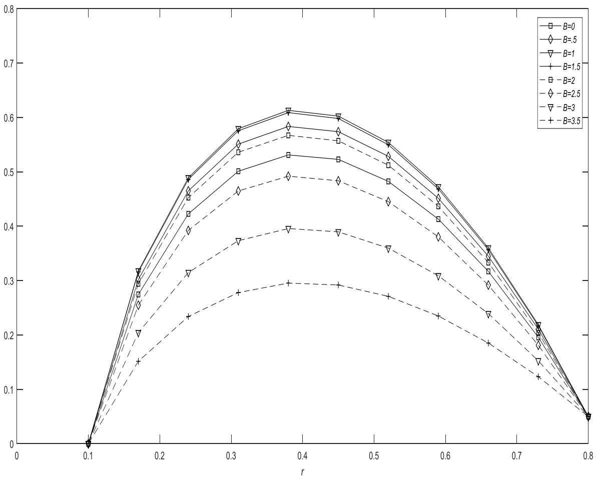

Fig5: Axial velocity u versus radial distance r for various values of Body acceleration parameter B.

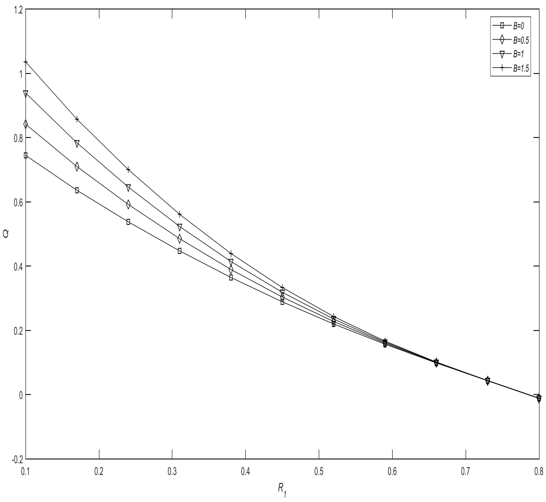

Fig.6: Variation of volumetric flow rate Q versus catheter radius R_1 for various values of Body acceleration parameter B.

Fig.6, represents the variation volumetric flow rate Q with variation of catheter radius R_1 for different values of body acceleration parameter B with fixed values of R = 0.8, t = π/3, β=π/4, u_s = 0.05. From this figure, it is observed that volumetric flow rate increases with increase of body acceleration parameter from B = 0 to B = 1.5, but along with increase of catheter radius R_1 towards R, the flow rate Q become decreases continuously towards zero.

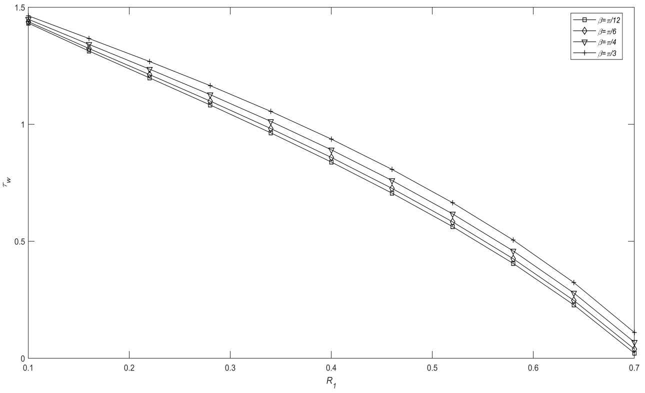

Fig7: Variation of Shear Stress τ_w versus catheter radius R_1 for various values of inclination β.

Fig.7, represents the variation of wall shear stress τ_w with the variation of catheter radius R_1 for various values of inclination β (= π/12, π/6, π/4, π/3) and with fixed values of R = 0.8, t = π/3, B = 0.5, u_s =0.05. From this figure, it is observed that the wall shear stress increases with increase of inclination from β = $\frac{\pi}{12}$ to $\frac{\pi}{3}$. Further, for narrow the catheter, the effect of inclination on wall shear stress observed to be very low.

Fig.8, shows the variation of wall shear stress τ_w with variation of catheter radius R_1 for various values of body acceleration parameter B . From this figure, we observed that wall shear stress decreases with increase of body acceleration parameter. Hence, from Fig.5, Fig.6 and Fig.8, we observed that a mild periodic body acceleration is advisable during catheterization for diagnosis or treatment of cardiovascular diseases in inclined artery to keep the cardiac condition of the patient stable .

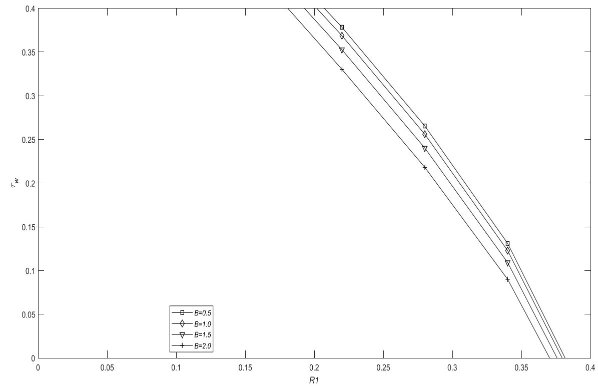

Fig.8: Variation of Shear stress τ_w versus catheter radius R_1 for various values of Body acceleration parameter B.

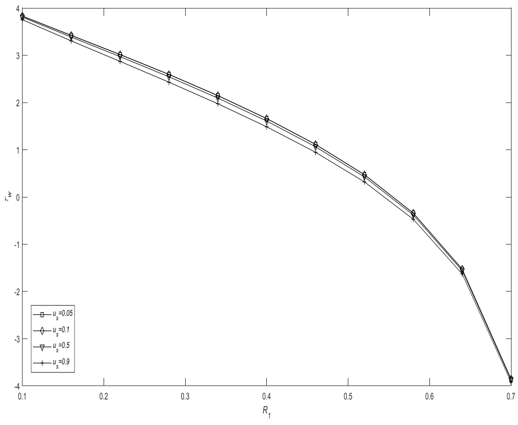

Fig.9: Shear Stress τ_w versus Catheter radius R_1 for various values of slip velocity u_s

Fig.9, represents the variation of wall shear stress τ_w with the variation of catheter radius R_1 for various values of slip velocity u_s and with fixed values of R = 0.8, t = π/2, β = π/4, B=0.5. From the figure, it is observed that the wall shear stress decreases with increase of slip velocity.

Fig.10, depicts the variation of wall shear stress τ_w with the variation of time t for various values of slip velocity u_s and with fixed values of R = 0.8, B = 0.5, β = π/4, R_1 = 0.1. Here also, as like in Fig.9, wall shear stress decreases with increase of slip velocity.

Hence, from Fig.9 and Fig10, we observed that if slip velocity increases then wall shear stress decreases during catheterization. So, drugs that have the capacity to increase slip velocity at artery wall may be beneficial for patient during the catheterization process in inclined artery.

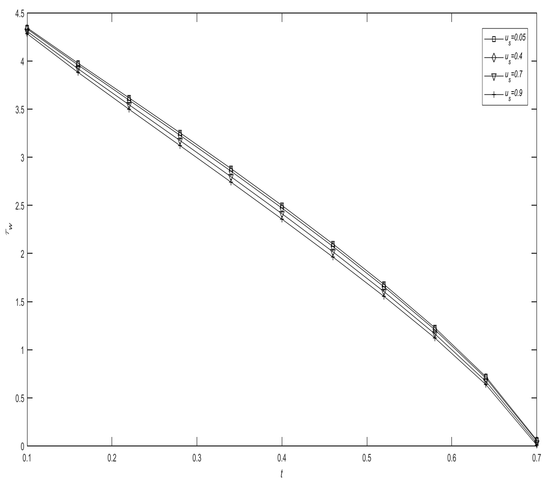

Fig.10: Shear Stress τ_w versus time t for various values of slip velocity u_s

Fig.11 shows the variation of effective viscosity μ_e with the variation catheter radius R_1 for various values of stenosis shape parameter n (= 2,3,4,5) and with fixed values of t = π/4, β=π/3, $u_s = 0.05,\ L_0 = 1,\ z = 8,\ d = 7.5,\ \delta = 0.2$. From the figure, it is observed that effective viscosity increases with increase of stenosis shape parameter. Further, it is also observed that with increase of catheter radius, the effective viscosity increases proportionately and after a certain limit of R_1 = 0.3, the effective viscosity shows abnormal behaviour.

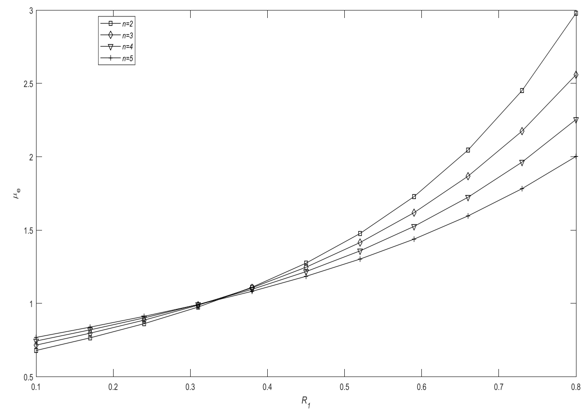

Fig11: Variation of effective viscosity μ_e versus catheter radius R_1 for various values of stenosis shape parameter of n.

CONCLUSION

Our theoretical model study may be summarised to the following conclusions:

I. Velocity of blood increases with increase of inclination, but catheter radius should be as fine as possible, otherwise flow velocity will not reached to the level of expectations. Hence for treatment of cardiovascular disease in inclined artery by catheterization, the catheter should be as fine as possible for the safety of patient.

II. For diagnosis and treatment of coronary artery disease by means of angiography, angioplasty etc., a mild periodic body acceleration is advisable during catheterization process. We may also strongly recommend to use catheter of radius as fine as possible to keep the volumetric flow of blood in standard level.

III. Wall shear stress increases with increase of inclination, but with decrease of catheter radius, the effect of inclination on wall shear stress become negligible. Further catheterization may not be advisable for arteries with higher inclination to keep control on wall shear stress.

IV. A mild periodic body acceleration is advisable to minimize wall shear stress during angiography, angioplasty etc.

V. The drugs that may increase the wall slip velocity are advisable during catheterization to minimize the wall shear stress.

VI. As effective viscosity increases with increase of stenosis shape parameter, for the safety of the patient catheterization is not suggestive in case of inclined artery with stenosis of high shape parameter.

On the basis of above discussions, we may conclude that this model may be considered as reference model for the safety of patient with coronary artery diseases during diagnosis or treatment by means of angiography, angioplasty etc. in an inclined artery. Further, more effective models may be studied for radially non-symmetric stenosis of higher shape parameter as well as accurately calculated limiting value of inclination and body acceleration.

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