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Special $D(k^2+1)$ Dio-quadruples Involving k-Jacobsthal and k- Jacobsthal Lucas Numbers

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Abstract

This paper concerns with the study of three distinct integers a, b, c such that product of any two from the set added with k -times their sum and increased by k^2+1 is a perfect square.Also, we show that the triple can be extended to the quadruple with property $D(k^2+1)$

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Mathematics Subject Classification:11D99

INTRODUCTION

The problem of constructing the sets with property that the product of any two of its distinct elements is one less than a square has a very long history and such sets were studied by Diophantus. A set of m positive integers $\{a_1, a_2, a_3 \ldots\ldots\}$ is said to have the property $D(n)$, $n \epsilon z-\{0\} if\ a_i a_j+n$, a perfect square for all $1 \leq i \leq j \leq m$ and such a set is called a Diophantine m-tuples with property $D(n)$. Many mathematician considered the construction of different formulations of Diophantine triples with property $D(n)$ for any arbitrary integer n and also for any linear polynomials in n. In this context, one may refer [1-18] for an extensive review of various problems on Diophantine triples, quadruples.This paper aims using at constructing special dio-quadruple where the product of any two from the set added with k -times their sum and increased by k^2+1 is a perfect square. Also we show that the triple can be extended to the quadruple with property $D(k^2+1)$

METHOD OF ANALYSIS

We see that $(a-1)(a+1)+k(a-1+a+1)+k^2+1$ is a perfect square.

Let $c_s(n)$ be any non-zero integer such that

$$(a-1+k)\, c_s + (a-1)k + k^2 + 1 = \alpha_s^2 \quad (1)$$

$$(a+1+k)c_s + (a+1)k + k^2 + 1 = \beta_s^2 \quad (2)$$

Eliminating c_s from (1) and (2)

$$(a+1+k)\alpha_s^2 - (a-1+k)\beta_s^2 = 2 \quad (3)$$

Introducing the linear transformation

$$\alpha_s = X_s + (a-1+k)\, T_s \quad (4)$$

$$\beta_s = X_s + (a+1+k)\, T_s$$

In (3) we get,

$$X_s^2 = (a-1+k)\,(a+1+k)T_s^2 + 1 \quad (5)$$

Which is pellian equation whose general solution is given by

$$X_s = \frac{1}{2}[\left(\left(a+k+\sqrt{D}\right)^{s+1}\right) + (\left(a+k-\sqrt{D}\right)^{s+1})]$$

$$T_s = \frac{1}{2\sqrt{D}}[\left(\left(a+k+\sqrt{D}\right)^{s+1}\right) + (\left(a+k-\sqrt{D}\right)^{s+1})] \quad (6)$$

Here $X_0 = a+k$, $D = (a+k)^2 - 1$ and $T_0 = 1$

$$c_0(n) = \frac{\alpha_0^2 - k^2 - 1 - (a-1)k}{(a-1+k)}$$

$$c_0(n) = 4a + 3k$$

Note that $(a-1, a+1, c_0(n))$ is the special Diophantine triple with property $D(k^2+1)$

Now substuting s=1 in (6) we have

$$X_1 = (a+k)^2 + D \quad \text{and} \quad T_1 = 2(a+k)$$

$$c_1(n) = \frac{\alpha_1^2 - k^2 - 1 - (a-1)k}{(a-1+k)}$$

$$c_1(n) = 16(a+k)^3 - 4a - 5k$$

Thus we obtain $(a-1, a+1, 4a+3k, 16(a+k)^3 - 4a - 5k)$ as a diophantine quadruple with the property $D(k^2+1)$

Again, taking $s = 2,3$ in (6) and using (1), we get

$$c_2(n) = 64(a+k)^5 - 48(a+k)^3 + 8(a+k) - k$$

$$c_3(n) = 256(a+k)^7 - 320(a+k)^5 + 112(a+k)^3 - 8(a+k) - k$$

It is seen that $[a-1, a+1, c_1(n)\ c_2(n)]$ is the special Diophantine quadruple with property $D(k^2+1)$.Repeating the above process, it is observed that [a,b, $c_{m-1}(n)\ c_m(n)$], m=1,2,3…. Represents special dio-quadruples with property $D(k^2+1)$.

A few illustrations are given below:

(a,k)	$a-1,a+1,c_0(n)\ c_1(n)$	$a-1,a+1,c_1(n)\ c_2(n)$	$a-1,a+1,c_2(n)\ c_3(n)$
(1,1)	(0,2,7,119)	(0,2,119,1674)	(0,2, 1674,23407)
(2,1)	(1,3,11,414)	(1,3,414,14279)	(1,3, 14279,485111)
(3,1)	(2,4,15,1007)	(2,4,1007,62495)	(2,4,62495,4922335)

Remark 1:

When $a = j_{k,n}$ is the k-jacobsthal number, $k \epsilon N$, we get the following dio-quadruple

(n,k)	$j_{k,n}-1,j_{k,n}+1,c_0(n)\ c_1(n)$	$j_{k,n}-1,j_{k,n}+1,c_1(n)\ c_2(n)$	$j_{k,n}-1,j_{k,n}+1,c_2(n)\ c_3(n)$
(1,1)	(0,2,7,119)	(0,2,119,1679)	(0,2, 1679,23407)
(2,1)	(0,2,7,119)	(0,2,119,1679)	(0,2, 1679,23407)
(3,1)	(2,4,15,1007)	(0,2,1007,62495)	(0,2, 62495,3873759)

Remark 2 ; Similarly we have to obtain and verify k-jacobsthal lucas $(\hat{j}_{k,n}-1, \hat{j}_{k,n}+1, 4\hat{j}_{k,n}+3k, 16(\hat{j}_{k,n}+k)^3 - 4\hat{j}_{k,n}-5k)$

$$c_2(n) = 64(\hat{j}_{k,n}+k)^5 - 48(\hat{j}_{k,n}+k)^3 + 8(\hat{j}_{k,n}+k) - k$$

$$c_3(n) = 256(\hat{j}_{k,n}+k)^7 - 320(\hat{j}_{k,n}+k)^5 + 112(\hat{j}_{k,n}+k)^3 - 8(\hat{j}_{k,n}+k) - k$$

A few illustrations are given below: $k \epsilon N$, we get the following dio-quadruple

(n,k)	$\hat{j}_{k,n}-1,\hat{j}_{k,n}+1,c_0(n)\ c_1(n)$	$\hat{j}_{k,n}-1,\hat{j}_{k,n}+1,c_1(n)\ c_2(n)$	$\hat{j}_{k,n}-1,\hat{j}_{k,n}+1,c_2(n)\ c_3(n)$
(1,1)	(0,2,7,119)	(0,2,119,1687)	(0,2,1687,23317)
(2,1)	(4,6,23,3431)	(4,6,3431,487343)	(4,6,487343,69199439)
(3,1)	(6,8,31,8153)	(6,8,8153,134193215)	(6,8,134193215,205487127)

Remark 3: we all observe that $(a-r, a+r, 4a+3k)$ is the special Diophantine triple with property $D(k^2+r^2)$ such that product of any two from the set added with k -times their sum and increased by k^2+r^2 is a perfect square.

Note that $(a-r, a+r, 4a+3k)$ is the special Diophantine triple with property $D(k^2+r^2)$.

A few illustrations are given below:

(a,r,k)	$(a-1,a+1,4a+3k)$
(1,1,1)	(0,2,7)
(2,1,1)	(1,3,11)
(3,1,1)	(2,4,15)

CONCLUSION

In the construction of the special dio-quadruple we have assumed the product ab added with k-times their sum and increased by k^2+1 is a perfect square.One may search for special dio-quadruples consisting of other special numbers with suitable property.

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