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An Inventory Model with Time-Varying Demand for Non- Instantaneous Deteriorating Items with Maximum Life Time

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Abstract

We develop the inventory model for non-instantaneous deteriorating Items. Here demand is taken to be time varying. Generally, most of the proposed models researcher considered the constant deterioration rate but in actual situations, maximum items deteriorate due to finish of their maximum life time. The main idea integrated for the modeling of the proposed inventory model is: time-varying demand, time varying sales revenue, and sensitivity analysis of parameters with profit function. To find the optimal total profit and the optimal order quantity, a numerical example is provided to demonstrate the practical usage of the mathematical result and sensitivity analysis of the optimal total profit function with respect to constraints is carried out.

Keywords: Inventory Model, Time-varying demand, Maximum life time and time-varying Sales revenue cost.

INTRODUCTION

In the recent years, the problem of deterioration in the inventory models has received considerable attention. Harris [1] developed the first EOQ (Economic Order Quantity) inventory model, which was generalized by Wilson [2] who invented a formula to find economic order quantity. Whitin [3] discussed the inventory model with the fashionable products deteriorating at the end of the storage period. Study of deteriorating inventory model began with Ghare and Schrader [4] who developed an inventory model for an exponentially decaying inventory and also established the EOQ inventory model with fix deterioration rate and without shortages. Covert and Philip [5] extended Ghare and Schrader's inventory model and presented an EOQ model with variable rate of deterioration by considering a two-parameter Weibull distribution. This inventory model was further modified by Philip [6] by assuming a three-parameter Weibull distribution. Misra [7] considered an EPQ inventory model with deteriorating items and assuming the two type of deterioration rate constant and variable form. After that the different types of order-level inventory models for deteriorating items at a constant deteriorate rate were discussed [8-11]

Dave [12] presented inventory ordering policies for deteriorating items. Kang and Kim [13] studied on the price and production inventory model with deteriorating items. Aggarwal and Jaggi [14] presented an inventory model with ordering policies for deteriorating items. Raafat [15] gave a survey of the existing literature on constantly deteriorating inventory model. Some of the recent works in this field have been done by Chung and Ting [16], Goyal and Ganasekaran [17].

Chang and Dye [20] proposed a model with partial backlogging and time-varying demand. In the recent research work, the relationship between time and deteriorating rate is a common phenomenon assumed in inventory models. This type of situation, we have several scenarios; including linear deteriorating rate Bhunia and Maiti [18], Mukhopadhyay et al. [24] and deteriorating rate is consider other time function Abad [23]. Wee [19] discussed a deteriorating inventory model for quantity discount and pricing under the partial backordering. Shah et al. [21], Goyal and Giri [22] gave recent trends of modelling in deteriorating item inventory. In reality, not all kinds of inventory items deteriorated as soon as they obtained by the retailer. During the fresh time of product/item, the original quality is maintained because of no deterioration in product. Ouyang et al. [26] called this occurrence as "non-instantaneous deterioration", and they developed a non-instantaneous inventory model for deteriorating items under the permissible delay in payments. Manna and Chaudhuri [25] presented an EOQ inventory model having demand rate of ramp type for time dependent deterioration rate and unit production cost under the shortages. Zhou and Gu [27] gave an optimal production-inventory policy for deteriorating items.

Liao [28] gave an economic order quantity (EOQ) inventory model for non-instantaneous receipt with exponential deteriorating item under the two level trade credits. Li et al. [29] suggested a review on deteriorating inventory study. In this study they compared with the extant reviews of Raafat [15] and Goyal [22]. This research paper proposed some new key parameters which should be assumed in the deteriorating inventory studies. Singh and Malik [31] considered an Optimal ordering policy inventory model with linear deterioration and exponential demand under the two storage capacities. Sana [30] examined optimal selling price and lot size inventory model with time varying deterioration under the partial backlogging. Singh and Malik [32] developed an stock dependent demand inventory model with two storages capacity for non-instantaneous deteriorating items. Sarkar [34] proposed an EOQ inventory model with permissible delay in payments and time dependent deterioration rate. In this paper he dealt with an EOQ model for finite replenishment rate in which demand and deterioration rate both are time-dependent. Sett et al. [33] investigated an inventory model of two-warehouse considering quadratically increasing demand and time varying deterioration.

Gupta et al. [35] discussed an optimal ordering policy with stock-dependent demand inventory model for non-instantaneous deteriorating items. Sarkar and Sarkar [36] discussed an improved inventory model for time varying deterioration with stock-dependent demand under the partial backlogging. Sarkar and Sarkar [37] developed an economic quantity model with probabilistic deterioration rate in the production system. Ghoreishi et al. [38] considered an optimal pricing and ordering policy for non-instantaneous deteriorating items under the inflation. Sheng et al. [39] developed an time varying deteriorating items having inventory model under the trade credit. Sarkar et.al. [40] presented an inventory model with quality improvement and backorder price discount under controllable lead time. Malik et. al. [41] proposed an inventory model with non-instantaneous and time-varying deteriorating Items.

Most researchers on inventory models do not consider simultaneously the phenomena with non-instantaneous and time varying deterioration, because these phenomena are not uncommon in real life situations, so we incorporate them in our model. A numerical example, graphical images with sensitivity analysis are provided to the study of the effect of changes in the associated parameters on the optimal solution.

NOTATIONS AND ASSUMPTIONS

For developing the inventory model, we use the following notations and assumptions:

Notations

d_1+d_2t : the demand rate,

Co : the ordering cost per order,

C_h : the inventory holding cost per unit time per unit item,

Cp : the purchasing cost per unit item,

Cd : the deteriorating cost per unit item,

S_1-S_2t : the sales revenue cost per unit item,

$\theta(t)$: the time varying deterioration rate at time t, $\theta(t) = \dfrac{1}{1+R-t}$, where $0 \le \theta(t) \le 1$,

L : maximum inventory level

R : maximum life time of the deteriorating item,

$I_1(t)$: the inventory level at the time $[0, t_1]$ with no deterioration in the product.

$I_2(t)$: the inventory level at the time $[t_1, T]$ with the deterioration in the product starts.

t_1 : the time interval in which the product has no deterioration (*i.e.,* fresh product time),

T : the ordering cycle

TP : the total profit per unit time of the inventory system.

Assumptions

- Demand is the linear increasing function of time.
- It is assumed that in this inventory model the time duration $[0, t_1]$, the product has no deterioration (*i.e.,* fresh product time). After the interval $[0, t_1]$, on-hand inventory deteriorates and there is no repair or replacement of the deteriorated units. The deterioration function depends upon time, where R is the maximum life time of the item.
- Shortages are not allowed. The Lead time is zero or negligible.

MATHEMATICAL MODEL

In the time-interval $[0, t_1]$, the inventory level (I_1) decreases due to demand rate, and no deterioration during this period of time. After the time t_1, the demand and deterioration start simultaneous and stop whenever the inventory reduces to zero level. The inventory levels at any time t during the time interval [0, T] can be represented by the differential equations given below:

$$\frac{dI_1(t)}{dt} = -(d_1 + d_2 t) \qquad \text{…. (1)}$$

$$\frac{dI_2(t)}{dt} + \theta(t) I_2(t) = -(d_1 + d_2 t) \qquad \text{…. (2)}$$

With the boundary conditions, respectively. By solving these differential equations, we get the inventory level as follows:

$$I_1(t) = L - d_1 t - d_2 \frac{t^2}{2} \qquad \text{…. (3)}$$

$$I_2(t) = d_1(1+R-t)\log\left(\frac{1+R-t}{1+R-T}\right) + d_2(1+R-t)\left[\begin{array}{l} -T+t \\ +(1+R)\log\left(\dfrac{1+R-t}{1+R-T}\right) \end{array}\right] \qquad \text{…. (4)}$$

Assuming that continuity of I (t) at $t=t_1$, it follows from equations (3) and (4) that

$$L = d_1\left[t_1 + (1+R-t_1)\log\left(\frac{1+R-t_1}{1+R-T}\right)\right] + d_2\left[\frac{t_1^2}{2} + (1+R-t_1)\left[-T+t_1 + (1+R)\log\left(\frac{1+R-t_1}{1+R-T}\right)\right]\right] \quad \ldots\ldots (5)$$

The optimum profit per cycle contains the following values:

The ordering cost per cycle is OC= C_0 …. (6)

The deterioration cost per cycle is cycle is given by

$$DC = C_d\int_{t_1}^{T}\theta(t)I_2(t)dt$$

$$= d_1C_d\left[-(T-t_1)+(1+R-t_1)\log\left(\frac{1+R-t_1}{1+R-T}\right)\right] + d_2C_d\left\{\begin{array}{l}\left(\frac{-T^2}{2}-\frac{t_1^2}{2}+Tt_1\right)\\ -(1+R)(T-t_1)[\log(1+R-T)+1]\\ +(1+R)^2\log\left(\frac{1+R-t_1}{1+R-T}\right)\\ +T(1+R)\log(1+R-T)\\ -t_1(1+R)\log(1+R-t_1)\end{array}\right\} \quad \ldots. (7)$$

The purchasing cost per cycle is given by

$$PC = C_P * L$$

$$= C_P\left\{\begin{array}{l} d_1\left[t_1 + (1+R-t_1)\log\left(\frac{1+R-t_1}{1+R-T}\right)\right]\\ + d_2\left[\frac{t_1^2}{2} + (1+R-t_1)\left[\begin{array}{l}-T+t_1\\ +(1+R)\log\left(\frac{1+R-t_1}{1+R-T}\right)\end{array}\right]\right]\end{array}\right\} \quad \ldots. (8)$$

The sales revenue cost per cycle is given by

$$SRC = C_S\int_0^T (d_1+d_2t)dt$$

$$= \left(s_1d_1T + \frac{T^2}{2}(s_1d_2 - s_2d_1) - \frac{T^3}{3}(s_2d_2)\right) \quad \ldots. (9)$$

The holding cost per cycle is given by

$$HC = C_h\left(\int_0^{t_1} I_1(t)dt + \int_{t_1}^{T} I_2(t)dt\right)$$

$$= C_h\left\{\begin{array}{l} Lt_1 - d_1\frac{t_1^2}{2} - d_2\frac{t_1^3}{6} + \left(\begin{array}{l}-T(1+R)(T-t_1)\\ +\frac{(1+R+T)}{2}\left(T^2-t_1^2\right)\\ -\frac{(T^3-t_1^3)}{3}\end{array}\right)\\ +\left\{\begin{array}{l}d_1\\ +d_2(1+R)\end{array}\right\}\left(\begin{array}{l}(1+R)\left[\begin{array}{l}-(T-t_1)\\ +\left(\begin{array}{l}1+R\\ -t_1\end{array}\right)\log\left(\frac{1+R-t_1}{1+R-T}\right)\end{array}\right]\\ -\frac{\left(T^2-t_1^2\right)}{4} - \frac{1}{2}(1+R)(T-t_1)\\ +\left(\frac{(1+R)^2+t_1^2}{2}\right)\log\left(\frac{1+R-t_1}{1+R-T}\right)\end{array}\right)\end{array}\right\} \quad \ldots. (10)$$

Thus the optimum profit (TP) per cycle per unit time is given by

$$TP = \frac{1}{T}[SRC - OC - HC - DC - PC] \quad \ldots.(11)$$

To obtain the optimal cycle time T* we have to maximize the total present value of profit. To maximize the TP, we have to differentiate with respect to T and equate it to zero

$$\frac{dTP}{dT} = 0 \quad \ldots. (12)$$

and
$$\frac{d^2TP}{dT^2} < 0 \quad \ldots. (13)$$

The profit function is highly non-linear; therefore it cannot inventory system is maximized by a developed solution procedure. Also a numerical example, graphical design with the sensitivity analysis is utilized to illustrate this developed inventory model.

SOLUTION PROCEDURE

Step 1. Input the value of the parameters C_o, C_h, C_p, C_s, C_d, R, D, t_1 in equation (12);

Step 2. Now using the equation (12) obtain T and from the relation (11) obtain TP;

Step 3. Substituting the value of T in the equation (13) and check the optimal profit TP. If satisfied then go to stop otherwise repeat the process from step 1 to 3.

NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

To demonstrate the above solution procedure, we take the following example: C_o=1500, S_1=220, S_2=1.5, C_p=120, C_h=0.10, C_d=0.08, R=30, t_1=0.5, d_1=800 and d_2=50. Using optimization techniques we obtain the optimal ordering cycle T*=4.4007 and corresponding TP*=80352.1294 and L*=4265.79 are evaluated and tabulated in Table 1 for varying model's parameters:

Change in		t_2	L^*	TP*	$\frac{d^2TP}{d^2t}$
d_1	1000	3.2590	4325.06	98365.5185	-998.4189
	800	3.9007	4265.79	80352.1294	-711.3266
	600	4.8637	4280.68	62792.9517	-522.4506
d_2	62.5	4.6199	5315.25	82895.5609	-687.5045
	50	3.9007	4265.79	80352.1294	-711.3266
	37.5	3.1315	3307.53	78215.0523	-835.3825
R	37.5	5.4445	6074.70	82408.4605	-495.2960
	30	3.9007	4265.79	80352.1294	-711.3266
	22.5	2.4709	2727.34	78783.2769	-1239.0863
S_1	275	6.3378	7436.92	131982.3169	-499.7438
	220	3.9007	4265.79	80352.1294	-711.3266
	165	2.5201	2744.41	31366.0723	-1476.4368
S_2	1.875	3.6304	3953.44	79599.1311	-797.0746
	1.5	3.9007	4265.79	80352.1294	-711.3266
	1.125	4.2084	4630.62	81164.3268	-636.4948
C_p	150	2.5891	2816.19	52261.2185	-1468.3896
	120	3.9007	4265.79	80352.1294	-711.3266
	90	6.7683	8069.03	111246.8092	-409.0152
C_h	0.125	4.1307	4537.53	80350.2451	-737.5057
	0.10	3.9007	4265.79	80352.1294	-711.3266
	0.075	3.6301	3952.97	80391.2233	-680.8128
C_o	1875	3.9276	4297.42	80267.1764	-713.6959
	1500	3.9007	4265.79	80352.1294	-711.3266
	1125	3.8733	4233.74	80437.6096	-708.9079
C_d	0.10	3.9001	4265.11	80350.9429	-711.5090
	0.08	3.9007	4265.79	80352.1294	-711.3266
	0.06	3.9012	4266.47	80353.3162	-711.1441

From the above table we represent the following results:

1. When the demand rate D increases, then the total profit increases.
2. When the maximum life time R and sales revenue cost S_1 increase, then the total profit increases.
3. When the purchasing cost C_p, holding cost C_h, ordering cost C_o are increase and deteriorating cost Cd are increases, then total profit decreases.

The following graphs (Fig.1 and Fig.2) show the relation between the total profit TP and the time duration t_1 and T.

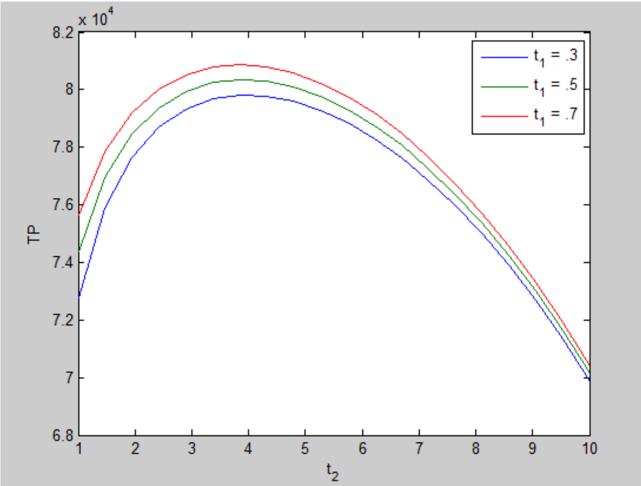

Figure 1: Total Profit w.r.t. T and t_2

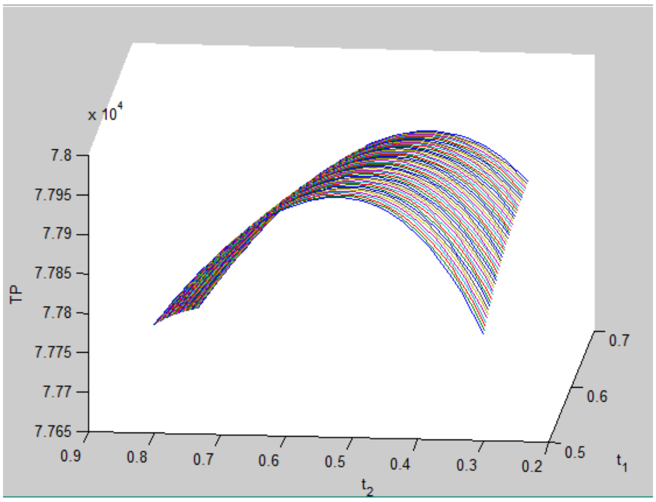

Figure 2: Graph representation of Total Profit w.r.t. T, t_1 and t_2

CONCLUSIONS

In most of the developed inventory models researcher have discussed with the constant deterioration rate. But in actual conditions, maximum products/items deteriorate due to expiration of their maximum life time. According to author's, such form of view for time varying deterioration function of the time with assuming non-instantaneous items, has not yet been proposed. A numerical example and sensitivity analysis are presented to the study of the effect of changes in the related parameters on the optimum function. Further, a future research direction, in the study for inventory model with variable demand, stock dependent demand, price and multi valued demand, variable holding cost, inflation, probabilistic demand and deterioration, partial backlogging, two storages, reliability and trade credit.

REFERENCES

[1] Harris F. W. Operations and Cost. A. W. Shaw Company, Chicago, 48 -54 (1915).

[2] Wilson R. H. A Scientific Routine for Stock Control. Harvard Business Review 13, 116-128 (1934).

[3] Whitin T.M., Theory of Inventory Management. Princeton University Press, Princeton, NJ (1957).

[4] Ghare P.M. and Schrader G.F., An inventory model for exponentially deteriorating items. Journal of Industrial Engineering 14, 238-243 (1963).

[5] Covert R. P. and Philip G.C., 1973. An EOQ model for items with Weibull distribution deterioration. AIIE Transactions 5, 323-326 (1973).

[6] Philip G.C., A generalized EOQ model for items with Weibull distribution deterioration. AIIE Transactions 6, 159-162 (1974).

[7] Mishra R.B., Optimum production lot-size model for a system with deteriorating inventory. International Journal of Production Research Society 13, 495-505 (1975).

[8] Shah Y. K. and Jaiswal M.C., An order level inventory model for a system with constant rate of deterioration. Opsearch 14, 174-184 (1977).

[9] Aggarwal S. P., A note on an order-level inventory model for a system with constant rate of deterioration. Opsearch 15, 184–187 (1978).

[10] Dave, U., and Patel, L.K., (T, Si) policy inventory model for deteriorating items with time proportional demand. Journal of Operational Research Society 32, 137- 142 (1981).

[11] Roy chowdhury M. and Chaudhuri K.S., An order level inventory model for deteriorating items with finite rate of replenishment. Opsearch 20, 99-106 (1983).

[12] Dave U., An order level inventory model for deteriorating items with variable instantaneous demand and discrete opportunities for replenishment. Opsearch 23, 244-249 (1986).

[13] Kang, S., and Kim, A., A study on the price and production level of the deteriorating inventory system. International Journal of Production Research 21, 899-908 (1983).

[14] Aggarwal, S.P. and Jaggi, C.K., Ordering policy for decaying inventory. International Journal of System Science 20, 151-155 (1989).

[15] Raafat, F., Survey of literature on continuously deteriorating inventory model. Journal of the Operational Research Society 42, 27-37 (1991).

[16] Chung KJ, Ting PS., A heuristic for replenishment for deteriorating items with a linear trend in demand. Journal of Operational Research Society

44, 1235–1241 (1993).

[17] Goyal S. K. and A. Gunasekaran, An integrated production-inventory-marketing model for deteriorating items. Computers & Industrial Engineering 28, 755–762 (1995).

[18] Bhunia A. K. and M. Maiti, Deterministic inventory model for deteriorating items with finite rate of replenishment dependent on inventory level. Computers & Operations Research 25, 997–1006 (1998).

[19] Wee H.-M., Deteriorating inventory model with quantity discount, pricing and partial backordering. International Journal of Production Economics 59, 511–518 (1999).

[20] Chang H J, Dye CY., An EOQ model for deteriorating items with time varying demand and partial backlogging. Journal of Operational Research Society 50, 1176–1182 (1999).

[21] Shah N. H. and Y. K. Shah, Literature survey on inventory model for deteriorating items. Economics Annals 44, 221-237 (2000).

[22] Goyal, S. K. and B. C. Giri., Recent trends in modeling of deteriorating inventory. European Journal of Operational Research 134, 1-16 (2001).

[23] Abad, P. L., Optimal price and order size for a reseller under partial backordering. Computers & Operations Research 28, 53–65 (2001).

[24] Mukhopadhyay S., R. N. Mukherjee, and K. S. Chaudhuri, Joint pricing and ordering policy for a deteriorating inventory. Computers & Industrial Engineering 47, 339–349 (2004).

[25] Manna S. K. and Chaudhuri, K. S., An EOQ model with ramp type demand rate, time dependent deterioration rate, unit production cost and shortages. European Journal of Operational Research 171, 557–566 (2006).

[26] Wu K.S., Ouyang L. Y. and Yang C.Y., An optimal replenishment policy for non-instantaneous deteriorating items with stock dependent demand and partial backlogging. International Journal of Production Engineering 101, 369-384 (2006).

[27] Zhou S.-Y. and Gu, F.-W., An optimal production-inventory policy for deteriorating items. Logistics Technology 26, 43–46 (2007).

[28] Liao J J., An EOQ model with non-instantaneous receipt and exponential deteriorating item under two-level trade credit. International Journal of Production Economics 113, 852–861 (2008).

[29] Li, R., Lan, H. and Mawhinney J. R., A review on deteriorating inventory study. Journal of Service Science and Management 3, 117-129 (2010).

[30] Sana S. S., Optimal selling price and lot size with time varying deterioration and partial backlogging. Applied Mathematics and Computation 217, 185–194 (2010).

[31] Singh, S.R. and Malik, A.K., Optimal ordering policy with linear deterioration, exponential demand and two storage capacity. International Journal of Mathematical Sciences 9, 513–528 (2010).

[32] Singh, S.R. and Malik, A.K., An Inventory Model with Stock-Dependent Demand with Two Storages Capacity for Non-Instantaneous Deteriorating Items, International Journal of Mathematical Sciences and Applications 1, 1255-1259 (2011).

[33] Sett, B. K., Sarkar, B., Goswami A., A two-warehouse inventory model with increasing demand and time varying deterioration. Scientia Iranica, E 19, 1969–1977 (2012).

[34] Sarkar, B., An EOQ model with delay-in-payments and time-varying deterioration rate. Mathematical and Computer Modelling, 55, 367–377 (2012).

[35] Gupta K. K., A. Sharma, P. R. Singh, A. K. Malik, Optimal Ordering Policy for Stock-dependent Demand Inventory Model with Non-Instantaneous Deteriorating Items, International Journal of Soft Computing and Engineering 3, 279-281 (2013).

[36] Sarkar B. and Sarkar, S., An improved inventory model with partial backlogging, time varying deterioration and stock-dependent demand, Economic Modelling, vol. 30, pp. 924–932 (2013).

[37] Sarkar, M. and Sarkar, B., An Economic Manufacturing Quantity Model with Probabilistic Deterioration in a Production System. Economic Modelling, 31, 245-252 (2013).

[38] Ghoreishi M., A. Mirzazadeh and Weber, G. W., Optimal pricing and ordering policy for non-instantaneous deteriorating items under inflation and customer returns, Optimization, 63, 1785-1804 (2014).

[39] Sheng-C. C. Teng, J. T., Inventory and credit decisions for time-varying deteriorating items with up-stream and down-stream trade credit financing by discounted cash flow analysis, European Journal of Operational Research, Vol. 243, Issue 2, pp. 566–575 (2015).

[40] Sarkar, B., Mandal, B. and Sarkar, S., Quality Improvement and Backorder Price Discount under Controllable Lead Time in an Inventory Model, Journal of Manufacturing Systems, 35, 26-36 (2015).

[41] A. K. Malik, Chandra Shekhar, Vikram Vashisth, Anuj Kumar Chaudhary and S R Singh. Sensitivity analysis of an inventory model with non-instantaneous and time-varying deteriorating Items, AIP Conference Proceedings 1715, 020059 (2016); doi: 10.1063/1.4942741.