k-Power Domination of Crown Graph

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Abstract

The problem of monitoring an electric power system by placing as few phase measurement units (PMUs) in the system as possible is closely related to the well-known domination problem in graphs. The power domination number $\gamma_p(G)$ is the minimum cardinality of a power dominating set of G. In this paper, we investigate the power domination problem in a biclique crown graph.We also find the power domination number of direct product of some classes of graphs.

Keywords: Power Domination, Crown Graphs, Minimum Power Dominating Set

1. INTRODUCTION

The power domination problem arose in the context of monitoring electric power networks. A power network contains a set of nodes and a set of edges connecting the nodes. It also contains a set of generators,which supply power ,and a set of loads, where the power is directed to. In order to monitor a power network we need to measure all the state variables of the network by placing measurement devices. A Phase Measurement Unit (PMU) is a measurement device placed on a node that has the ability to measure the voltage of the node and current phase of the edges connected to the node and to give warnings of system-wide failures. The goal is to install the minimum number of PMUs such that the whole system is monitored. Now, let the graph G=(V,E)represent an electric power system, where a vertex represents an electrical node and an edge represents a transmission line joining two electrical nodes. In the following, we use the notations as in [3].For a vertex v of G, let N (v) denote the open neighborhood of v, and for a subset S of V(G), let N (S) = $U_{v \in s}$N (v) − S. The closed neighborhood N [S]of a subset is the set N [S] =N(S)∪ S.

For a connected graph G and a subset S S⊆ V (G),let M (S) denote the set monitored by S.

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1. Domination step :

M (S) ← S U N (S)

2. Propagation step :

As long as there exists v   ∈   M (S) such that N (v)∩(V (G) − M (S)) = {w }setM (S) ←   M (S) U {w}.

A set S is called a power dominating set of G if M (S) = V (G) and the power domination number of G $\gamma_p(G)$, is the minimum cardinality of a power dominating set of G. For any graph G, $1 \leq \gamma_p(G) \leq \gamma(G)$. The problem of deciding if a graph G has a power dominating set of cardinality k has been shown to be NP-complete even for bipartite graphs, chordal graphs [5] or even split graphs [8]. On the other hand, the problem has efficient solutions on trees [5], as well as on interval graphs [8]. Some upper bounds for $\gamma_p(G)$ are discussed in [11] and [12].

In this paper, we investigate the power domination problem in a biclique crown graph. We also show that the power domination number of generalized a biclique crown graph even path is one. We compute the power domination number of the indirect product of some classes of graphs.

All the graphs considered here are undirected, finite and simple. For all basic concepts and notations not mentioned in this paper, we refer [10]In this paper, we investigate the power domination problem in a biclique crown graph. We also show that the power domination number of generalized a biclique crown graph even path is one. We compute the power domination number of the indirect product of some classes of graphs.

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Definition 1: The crown graph $s_n^0$for n ≥ 3 is the graph with vertex set V=$\{u_1, u_2, u_3, u_{4........u_n}, v_1, v_2, v_3 ....v_n\}$and an edge fromV=$\{u_i v_i; 1 \leq i, j \leq$n;i≠j} Therefore $s_n$coincides with the complete bipartite graphKn, n with horizontal edges removed.

Definition 2: The -**crown graph** for an integer is the **graph** with vertex set. (1) and edge set. (2) It is therefore equivalent to the complete bipartite **graph** with horizontal edges removed.
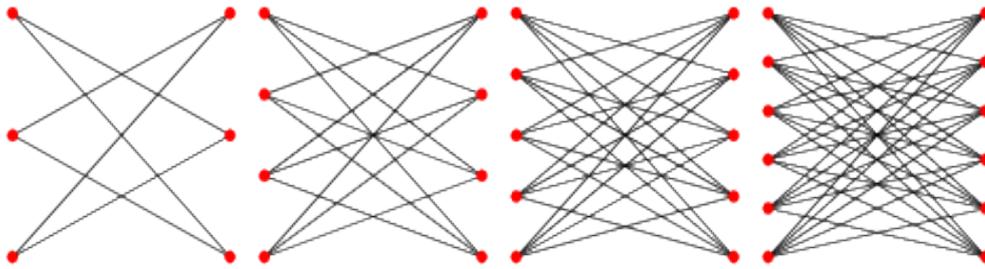
The n-crown graph for an integer $n \geq 3$ is the graph with vertex set

$$\{x_0, x_1, \ldots, x_{n-1}, y_0, y_1, \ldots, y_{n-1}\} \qquad (1)$$

and edge set

$$\{(x_i, y_j) : 0 \leq i, j \leq n-1, i \neq j\}. \qquad (2)$$

It is therefore equivalent to the complete bipartite graph $K_{n,n}$ with horizontal edges removed.

Definition 3 (k-Power Dominating Set)

A set S such that $p_G^\infty(S) = V(G)$ is a k-power dominating set of G. The least cardinality of such a set is called the k-power domination number of G, written $\gamma_{P,K}(G)$. A $\gamma_{P,K}(G)$. set is a minimum k-power dominating set of G. When G is clear from context, we will use $p^i(S)$ to denote $p_G^i(S)$. From this definition, the following observation clearly holds.

Keywords: Power domination, Mycielskian, Generalized Mycielskian, Direct. A crown graph Hm,n is a graph obtained from the complete bipartite graph Km,n by power domination. In this paper we show that for $H_{m,n}$, m,n≥K+2 graphs.

Observation 1.

The class of complete graphs coincides with the class of 1-word-representable graphs. In particular, the complete graph's representation number is 1.For a vertex v in a graph G denote by N(v) the neighbourhood of v

i. e. the set of vertices adjacent to v. Clearly, if a graph is bipartite then the neighbourhood of each vertex induces an independent set.

Observation 2.

For any vertex v ∈ V in a word-representable graph G = (V, E), the set A = N(v) is splittable.

Observation 3.

If n ≥ 5 then in any word w k-representing $H_{n,n}$ the set A = {1, . . . , n} is splittable.

Observation 4.

If n ≥ 5 then the crown graph Hn,n is ⌈n/2⌉-representable.

Theorem.

If $\lambda_1(G)$ is the largest minimum dominating eigen value of $A_D(G)$,then $\lambda_1(G) \geq 2m + k\,n$ where k is the domination number.

MAIN RESULTS:

Theorem 1

Let G be $H_{n,n}$ n ≥K+2,then $\gamma_{p,k}$ ≥n-k.

Proof :

Suppose S is a k- power domination set of $H_{n,n}$ with $|s|$=n-k-1.

Every member of S dominates every other member of the row and columns to which it belongs .Then induces a subgraphs H with x rows and y columns , where x,y≤n-k-1. $H_{n,n}$ / H is $H_{\propto,\beta}$ with ∝≥k+1,β ≥k+1,choose u∈N[s] / s, then u is a adjacent to k+1 vertices in $H_{\propto,\beta}$ a contradiction .Hence $\gamma_{p,k}$ ≥n-k.
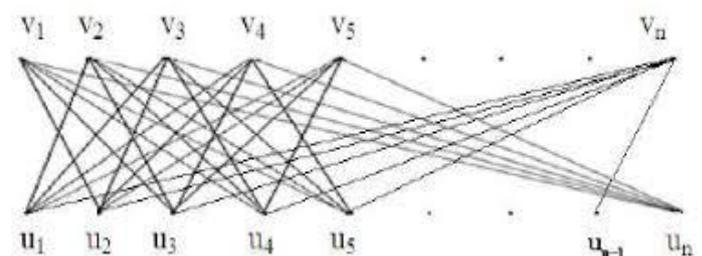
k-powerdomination in crown graph network:

Input: the crown graph network Hn,n ,n≥ $k + 2$,

Algorithm: Name the vertex in the ith row j^{th} column position as $v_{i,j}$ 1 ≤i≤ m, 1 ≤j≤ n, and select the vertices $v_{i1}, 1$ ≤i≤ $m - k$ in S.

Output: $\gamma_{p,k}(G)=\gamma^t_{p,k}(G)=\gamma^c_{p,k}(G)$=n-k.

Proof of correctness: $M^0(S)$= N[s] contains all vertices in the first column and all vertices in the first m-k rows of G . Each vertex in $M^0(S)$ other than the first column is adjacent to k vertices V(G) /N[S].Hence $M^1(S)$=V(G). Since the subgraph induced by S is connected,We have $\gamma^c_{p,k}(G)$=n-k.

A biclique cover of the n- vertices crown graph.

CONCLUSION

In this paper we found the representation power domination of any crown graph . We suspect that crown graphs are (among) the hardest graphs to be represented in the class of bipartite graphs .

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