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A Study on Multi Anti L-Fuzzy Normal Subgroup

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1. INTRODUCTION

Applying the concept of fuzzy sets introduced by Zadeh [1], Azriel Rosenfeld [2] defined fuzzy subgroup of a given group and derived some of their properties. The concept of anti – fuzzy subgroup was introduced by Biswas [3]. Sabu and Ramakrishnan [4] introduce the concept of multi-fuzzy sets. In all these studies, the closed unit interval [0, 1] is taken as the Membership lattice.

In this paper, we introduce the notion of a multi anti L-fuzzy normal subgroup and discuss some of its properties.

2. PRELIMINARIES

Throughout this paper G denotes an arbitrary multiplicative group with "e" is an identity element and L denotes an arbitrary Lattice with least element 0 and greatest element 1. The join and meet operations in L are denoted by $\wedge$ and $\vee$ respectively. A function B: G $\rightarrow$ L is called multi L – fuzzy subset of G

2.1 Definition:

Let Y be any nonempty set. A fuzzy set A of Y is $B : Y \rightarrow [0, 1]$.

2.2 Definition:

Let (G, **.**) be a group. A fuzzy subset B of G is said to be a fuzzy subgroup (FSG) of G if the following conditions are satisfied:

i. $B(xy) \geq \min\{ B(x), B(y) \}$,

ii. $B(x^{-1}) = B(x)$, for all x and $y \in G$.

2.3 Definition:

Let (G, **.**) be a group. A fuzzy subgroup B of G is said to be a normal fuzzy subgroup of G if $B(xy) = B (yx)$, for all x and $y \in G$.

2.4 Definition:

A fuzzy subset B of G is said to be a anti fuzzy group of G, if for all $x,y \in G$

i. $B(xy) \leq \max \{B(x), B(y)\}$

ii. $B(x^{-1}) = B (x)$.

2.5 Definition:

A anti fuzzy subgroup B of G is called a anti fuzzy normal subgroup (AFNS) of G if for every $x, y \in G$, $B(xyx^{-1}) \leq B(y)$.

2.6 Definition:

Let Y be a non – empty set. A multi L-fuzzy set B in Y is defined as a set of ordered sequences, $B = \{(x, B_1(x), B_2(x), ..., B_i(x), ...) : x \in X\}$, where $B_i : Y \rightarrow L$ for all i.

2.7 Definition:

A multi L-fuzzy subset B of G is called a multi L-fuzzy subgroup (MLFS) of G if for every $x,y \in G$,

i. $B(xy) \geq \min\{B(x), B(y)\}$

ii. $B(x^{-1}) = B(x)$.

2.8 Definition:

A multi L-fuzzy subset B of G is called a multi anti L-fuzzy subgroup (MALFS) of G if for every $x,y \in G$,

i. $B(xy) \leq \max\{B(x), B(y)\}$

ii. $B(x^{-1}) = B (x)$.

3. PROPERTIES OF MULTI ANTI L-FUZZY NORMAL SUBGROUP:

In this section, we introduce the notion of multi L-fuzzy normal subgroup and discuss some of its properties.

3.1 Definition:

A multi L-fuzzy subgroup B of G is called a multi L-fuzzy normal subgroup (MLFNS) of G if for every $x, y \in G$, $B(xyx^{-1}) \geq B(y)$.

3.2 Theorem:

Let G be a group and B be a multi L-fuzzy subgroup of G, then the following conditions are equivalent.

i. B is a multi L-fuzzy normal subgroup of G.

ii. $B(xyx^{-1}) = B(y)$, for all $x, y \in G$.

iii. $B(xy) = B(yx)$, for all $x, y \in G$.

Proof:

$i \Rightarrow ii.$

Let B is a multi L-fuzzy normal subgroup of G.

Then $B(xyx^{-1}) \geq B(y)$ for all $x, y \in G$. By taking advantage of the arbitrary property of x, we have,

$$B(x^{-1}y(x^{-1})^{-1}) \geq B(y).$$

Now, $B(y) = B(x^{-1}(xyx^{-1})(x^{-1})^{-1})$

$= B(xyx^{-1})$

$\geq B(y).$

Hence, $B(xyx^{-1}) = B(y)$ for all $x, y \in G$.

$ii \Rightarrow iii.$

Let $B(xyx^{-1}) = B(y)$, for all $x, y \in G$.

Take $y = yx$, we get,

$$B(xy) = B(yx), \text{ for all } x, y \in G.$$

$iii \Rightarrow i.$

Let $B(xy) = B(yx)$, for all $x, y \in G$.

$B(xyx^{-1}) = B(yxx^{-1}) = B(y) \geq B(y).$

Hence, B is a multi L-fuzzy normal subgroup of G.

3.3 Theorem:

Let B be a multi L-fuzzy subset of a group $(G,\cdot)$. If $B(e) = 1$ and $B(xy^{-1}) \geq B(x) \wedge A(y)$ and $B(xy) = B(yx)$, for all x and y in G, then B is a multi L-fuzzy normal subgroup of a group G, where e is the identity element of G.

Proof:

Let e be identity element of G and x and y in G.

Let $B(e) = 1$ and $B(xy^{-1}) \geq B(x) \wedge B(y)$, for all x and y in G.

Now, $B(x^{-1}) = B(ex^{-1})$

$\geq B(e) \wedge B(x)$

$\geq 1 \wedge B(x)$

$= B(x).$

Therefore, $B(x^{-1}) \geq B(x)$, for all x in G.

Hence, $B((x^{-1})^{-1}) \geq B(x^{-1})$ and $B(x) \geq B(x^{-1})$.

Therefore, $B(x^{-1}) = B(x)$, for all x in G.

Now, replace y by y^{-1}, then

$B(xy) = B(x(y^{-1})^{-1})$

$\geq B(x) \wedge B(y^{-1})$

$= B(x) \wedge B(y)$, for all x and y in G.

$B(xy) \geq B(x) \wedge B(y)$, for all x and y in G.

Hence, B is a multi L-fuzzy subgroup of a group $(G,\cdot)$.

Since, $B(xy) = B(yx)$ for all x and y in G, B is a multi L-fuzzy normal subgroup of a group $(G,\cdot)$.

3.4 Theorem:

If M and N are two multi L-fuzzy normal subgroups of a group $(G,\cdot)$, then their intersection $M \cap N$ is a multi L-fuzzy normal subgroup of G.

Proof:

Let x and y belong to G.

i. $(M \cap N)(xy) = M(xy) \wedge N(xy)$

$\geq \{ M(x) \wedge M(y) \} \wedge \{ N(x) \wedge N(y) \}$

$\geq \{ M(x) \wedge N(x) \} \wedge \{ M(y) \wedge N(y) \}$

$= (M \cap N)(x) \wedge (M \cap N)(y).$

Therefore, $(M \cap N)(xy) \geq (M \cap N)(x) \wedge (M \cap N)(y)$, for all x and y in G.

ii. $(M \cap N)(x^{-1}) = M(x^{-1}) \wedge N(x^{-1})$

$= M(x) \wedge N(x)$

$= (M \cap N)(x).$

Therefore, $(M \cap N)(x^{-1}) = (M \cap N)(x)$, for all x in G.

Hence $M \cap N$ is a multi L-fuzzy subgroup of a group G.

Now, $(M \cap N)(xy) = M(xy) \wedge N(xy)$

$= M(yx) \wedge N(yx)$, since A and B are MLFNS of G.

$= (M \cap N)(yx).$

$(M \cap N)(xy) = (M \cap N)(yx).$

Hence $M \cap N$ is a multi L-fuzzy normal subgroup of a group G.

Remark:

The intersection of a family of multi L-fuzzy normal subgroups of a group $(G,\cdot)$ is a multi L-fuzzy normal subgroup of a group G.

3.5 Theorem:

If B is a multi L-fuzzy normal subgroup of a group (G,·) if and only if $B(x) = B(y^{-1}xy)$, for x and y in G.

Proof:

Let x and y be in G. Let B be a multi L-fuzzy normal subgroup of a group G.

Now, $B(y^{-1}xy) = B(y^{-1}yx)$

$= B(ex)$

$= B(x)$.

Therefore, $B(x) = B(y^{-1}xy)$, for all x and y in G.

conversely, assume that $B(x) = B(y^{-1}xy)$.

Now, $B(xy) = B(xyxx^{-1})$

$= B(yx)$

Therefore, B(xy) = B(yx), for all x and y in G.

Hence, B is a multi L-fuzzy normal subgroup of a group G.

3.6 Theorem:

Let B be a multi L-fuzzy subgroup of a group (G,·) with B(y) < B(x), for some x and y in G, then B is a multi L-fuzzy normal subgroup of a group G.

Proof:

Let B be a multi L-fuzzy subgroup of a group (G,·).

Given B(y) < B(x), for some x and y in G,

$B(xy) \geq B(x) \wedge B(y)$, as B is a MLFG of G

$= B(y)$; and

$B(y) = B(x^{-1}xy)$

$\geq B(x^{-1}) \wedge B(xy)$

$\geq B(x) \wedge B(xy)$, as B is a MLFS of G

$= B(xy)$.

$B(y) \geq B(xy) \geq B(y)$.

Therefore, B(xy) = B(y), for all x and y in G.

and, $B(yx) \geq B(y) \wedge B(x)$, as B is a MLFS of G

$= B(y)$; and

$B(y) = B(yxx^{-1})$

$\geq B(yx) \wedge B(x^{-1})$

$\geq B(yx) \wedge B(x)$, as B is a MLFS of G

$= B(yx)$.

$B(y) \geq B(yx) \geq A(y)$.

Therefore, B(yx) = B(y), for all x and y in G.

Hence, A(xy) = B(y) = B(yx), for all x and y in G.

Hence, B(xy) = B(yx), for all x and y in G.

Hence, B is a multi L-fuzzy normal subgroup of a group of G.

3.7 Theorem:

Let B be a multi L-fuzzy subgroup of a group (G,·) with B(y) > B(x) for some x and y in G, then B is a multi L-fuzzy normal subgroup of a group G.

Proof:

Let B be a multi L-fuzzy subgroup of a group G.

Given B(y) > B(x), for some x and y in G,

$B(xy) \geq B(x) \wedge B(y)$, as A is a MLFS of G

$= B(x)$; and

$B(x) = B(xy\,y^{-1})$

$\geq B(xy) \wedge B(y^{-1})$

$\geq B(xy) \wedge B(y)$, as A is a MLFS of G

$= B(xy)$.

$B(x) \geq B(xy) \geq B(x)$.

Therefore, B(xy) = B(x), for all x and y in G.

and, $B(yx) \geq B(y) \wedge B(x)$, as B is a MLFS of G

$= B(x)$; and

$B(x) = B(y^{-1}yx)$

$\geq B(y^{-1}) \wedge B(yx)$

$\geq B(y) \wedge B(yx)$, as B is a MLFS of G

$= B(yx)$.

$B(x) \geq B(yx) \geq B(x)$.

Therefore, B(yx)= B(x), for all x and y in G.

Hence, B(xy) = B(x) = B(yx), for all x and y in G.

Hence, B(xy) = B(yx), for all x and y in G.

Hence, B is a multi L-fuzzy normal subgroup of a group of G.

3.8 Definition:

A multi L-fuzzy subset B of a set X is said to be normalized if there exist $x \in X$ such that B(x) =1.

3.9 Theorem:

Let b be a multi L-fuzzy normal subgroup of a group G. Then for any $y \in G$ we have $B(yxy^{-1}) = B(y^{-1}xy)$ for every $x \in G$.

Proof:

Let B be a multi L-fuzzy normal subgroup of a group G.

for any $y \in G$, we have,

$$B(yxy^{-1}) = b(x), \text{ since B is a MLFNS of G}$$
$$= B(xyy^{-1}), \text{ since B is an MLFNS of G}$$
$$= B(y^{-1}xy)$$

Hence, $B(yxy^{-1}) = B(y^{-1}xy)$.

3.10 Theorem:

A multi L-fuzzy subgroup B of a group G is normalized if and only if B(e) = 1, where e is the identity element of the group G.

Proof:

If B is normalized, then there exists $x \in G$ such that $B(x) = 1$ but by properties of a multi L-fuzzy subgroup B of the group G, $B(x) \le B(e)$, for every $x \in G$.

Since $B(x) = 1$ and $B(x) \le B(e)$ and $1 \le B(e)$. But $1 \ge B(e)$.

Hence, $B(e) = 1$. Conversely, if $B(e) = 1$, then by the definition of normalized multi L-fuzzy subset, B is normalized.

3.11 Theorem:

If L is a chain and B is an MLFNS of G, then the following are equivalent.

i. $B(xy) = B(x) \wedge B(y) = B(yx)$ whenever $B(x) = B(y)$,

ii. B is a constant.

Proof:

Let x and y belongs to G.

Assume that $B(y) < B(x)$.

Now, $B(y) = B(x^{-1}xy)$

$$\ge B(x^{-1}) \wedge B(xy)$$
$$\ge B(x) \wedge B(xy)$$
$$= B(xy)$$
$$\ge B(x) \wedge B(y)$$
$$\ge B(y).$$

$$B(y) \ge B(xy) \ge B(x) \wedge B(y) \ge B(y).$$

Therefore, $B(xy) = B(y) = B(x) \wedge B(y)$, for all x and y in G.

Implies that $B(xy) = B(x) \wedge B(y) = B(yx)$ for every $x, y \in G$.

Putting $y = x^{-1}$ we get $B(x) = B(e)$ for every $x \in G$.

Hence $i \Rightarrow ii$.

The converse is trivial.

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