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Solving FFLPP Problem with Hexagonal Fuzzy Numbers by New Ranking Method

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Abstract:

This paper presents fully fuzzy linear programming problem (FFLPP) with hexagonal fuzzy number (HFN) is solved by new ranking function. We convert the fully fuzzy linear programming problem to a crisp valued problem then can be solved using Simplex / Big-M method. Optimal solution obtained by solving numerical examples.

Keywords: Fully fuzzy linear programming problem, Hexagonal fuzzy numbers, Ranking function, Simplex method / Big-M method.

1. Introduction:

Linear programming problem is a tool which is used in operation research. It is a special class of optimization technique. In real life situation it is important and gives a efficiency and simplicity. George B. Dantizig [4] developed linear programming problem in primary level for solving military logistic problem during world war II and he is the founder of the Simplex method of linear programming. He was published his work in 1947 and developed many techniques to solve LPP. LPP is versatile technique useful for variety of problems occurs in education, business, industry, advertising, transportation investment etc. LPP have a power to faced complexity and gives a best solution.

The classical set is strongly limited according to the two valued logic which is only allows yes/no statement, each object has to be either an element of the set or not. This strong delimitation of a set which is characterized as crisp set quite often cases difficulties in the case of application to real problem. Zadeh [10] introduced fuzzy set theory in which fuzzy set is directly an extension of the set definition. Fuzzy set theory deals with reasoning that gives approximate and exact solution.

George j. Klir [5] apply his logic in fuzzy sets and fuzzy logic. He make many methods as well as models and gives a authenticate solutions.

Fuzzy linear programming problem was introduced by Zimmermann [6]. Further Tanaka et al. [8] solve fuzzy linear programming problem with fuzzy numbers. Cadenas et al. [15] used fuzzy numbers in linear programming problem. Qiu-Peng Gu et al. [11] Approach to linear programming with fuzzy coefficients based on fuzzy numbers distance are discussed. Abbasbandy et al. [12] rank a fuzzy numbers and they solved by sign distance formula. Nasseri [13] has proposed new method for solving fuzzy linear programming problem by using ranking method. Maleki [7] discussed about ranking functions and their applications to fuzzy linear Programming. Amitkumar [1] solve fuzzy linear programming problem with trapezoidal fuzzy numbers.

Fully fuzzy linear programming problem is a linear programming problem in which parameters and variables are all fuzzy numbers. Allahviranloo et al. [14] solved a new method for fully fuzzy linear programming problem by ranking function with fuzzy integer programming problem and finally they got a crisp solution. Amitkumar et al. [2] used fully fuzzy linear programming problem with inequality constraints. Karpagam et al. [3] discussed symmetric trapezoidal fuzzy numbers in fully fuzzy linear programming problems. Dhurai et al. [9] compared three different ranking functions by solving some fully fuzzy linear programming problem which are tabulated.

2. Priliminaries:

Definition 2.1

The characteristic function μ_A of a crisp set $A \subset X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range i.e. $\mu_{\tilde{A}} : X \rightarrow [0,1]$. The assigned value indicate the membership function and the set $A = \{(x, \mu_{\tilde{A}}(x); x \in X\}$ defined by $\mu_{\tilde{A}}(x)$ for $x \in X$ is called fuzzy set.

Definition 2.2

A fuzzy number is a generalization of a regular real number and which does not refer to a single value but rather to a connected a set of possible values, where each possible value has its weight between 0 and 1. This weight is called the membership function.

A fuzzy number is a convex normalized fuzzy set on the real line R such that:

1) There exist at least one $x \in R$ $with$ $\mu_{\tilde{A}}(x) = 1$.
2) $\mu_{\tilde{A}}(x)$ is piecewise continuous.

Definition 2.3

A fuzzy number $\widetilde{A_H}$ is a hexagonal fuzzy number denoted by $\widetilde{A_H} = (a_1, a_2, a_3, a_4, a_5, a_6)$ where $a_1, a_2, a_3, a_4, a_5, a_6$ are real numbers and its membership function $\mu_{\tilde{A}}(x)$ is given below,

$$\mu_{\widetilde{A_H}}(x) = \begin{cases} 0, & for\ x < a_1 \\ \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right), & for\ a_1 \le x \le a_2 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-a_2}{a_3-a_2}\right), & for\ a_2 \le x \le a_3 \\ 1, & for\ a_3 \le x \le a_4 \\ 1-\frac{1}{2}\left(\frac{x-a_4}{a_5-a_4}\right), & for\ a_4 \le x \le a_5 \\ \frac{1}{2}\left(\frac{x-a_4}{a_5-a_4}\right), & for\ a_5 \le x \le a_6 \\ 0, & for\ x > a_6 \end{cases}$$

Definition 2.4

An effective approach for ordering the elements of F(R) is also to define a ranking function R: F(R) → R which maps each fuzzy number into the real line, where a natural order exists. We define orders on F(R) by,

$\tilde{a} \ge \tilde{b}$ if and only if R $(\tilde{a}) \ge$ R $(\tilde{b})$,

$\tilde{a} \le \tilde{b}$ if and only if R $(\tilde{a}) \le$ R $(\tilde{b})$,

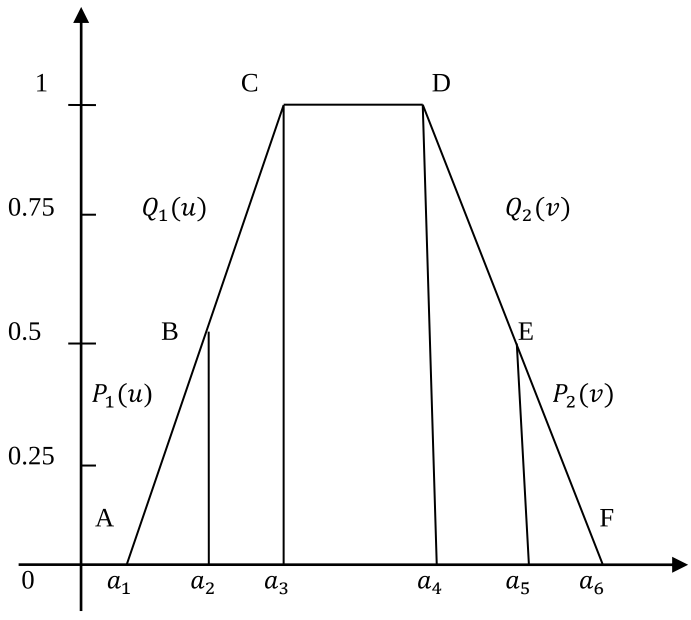
$\tilde{a} = \tilde{b}$ if and only if R $(\tilde{a}) =$ R $(\tilde{b})$.

Proposed formula:

In this paper, for hexagonal fuzzy number $\widetilde{A_H} = (a_1, a_2, a_3, a_4, a_5, a_6)$ a ranking method is given by following formula,

$$\mathbf{R}\ (\widetilde{A_H}) = \frac{a_1 + a_2 + a_3 + a_4 + a_5 + a_6}{3} \qquad (2.1.1)$$

3. Graphical representation of a hexagonal fuzzy number:



4. Numerical Example:

4.1 Max $\tilde{Z}$ = (17, 19, 21, 23, 25, 27) $\widetilde{x_1}$ + (37, 39, 41, 43, 45, 47) $\widetilde{x_2}$

Subject to
(47, 49, 51, 53, 55, 57) $\widetilde{x_1}$ +(67, 69, 71, 73, 75, 77) $\widetilde{x_2}$ ≤ (157, 159, 161, 163, 165, 167)
(87, 89, 91, 93, 95, 97) $\widetilde{x_1}$ + (107,109,111,113,115, 117) $\widetilde{x_2}$ ≤ (277,279,281283,285,287)

Solution:

Maximize (17, 19, 21, 23, 25, 27) $\widetilde{x_1}$ +
(37, 39, 41, 43, 45, 47) $\widetilde{x_2}$
Subject to

(47, 49, 51, 53, 55, 57) $\widetilde{x_1}$ + (67, 69, 71, 73, 75, 77) $\widetilde{x_2}$ +(1, 1, 1, 1, 1, 1) $\widetilde{x_3}$ = (157, 159, 161, 163, 165, 167)

(87, 89, 91, 93, 95, 97) $\widetilde{x_1}$ + (107, 109, 111, 113, 115, 117) $\widetilde{x_2}$ + (1, 1, 1, 1, 1, 1) $\widetilde{x_4}$

= (277, 279, 281 283, 285, 287)

$\widetilde{x_1}, \widetilde{x_2}, \widetilde{x_3}, \widetilde{x_4} \geq 0$

Using ranking function (2.1.1) we get,

Maximize 44 $\widetilde{x_1}$ + 84 $\widetilde{x_2}$ + 0 $\widetilde{x_3}$+ 0 $\widetilde{x_4}$

Subject to

104 $\widetilde{x_1}$ +144 $\widetilde{x_2}$ + $\widetilde{x_3}$ = (157, 159, 161, 163, 165, 167)

184 $\widetilde{x_1}$ + 224 $\widetilde{x_2}$ + $\widetilde{x_4}$ = (277, 279, 281 283, 285, 287)

$\widetilde{x_1}, \widetilde{x_2}, \widetilde{x_3}, \widetilde{x_4} \geq 0$

Using Simplex method we get,

Initial table

Basis	$\widetilde{x_1}$	$\widetilde{x_2}$	$\widetilde{x_3}$	$\widetilde{x_4}$	R.H.S	R($\widetilde{B_1}$)
$\widetilde{x_3}$	104	144	1	0	(157, 159, 161, 163, 165,167)	324
$\widetilde{x_4}$	184	224	0	1	(277, 279, 281 283, 285, 287)	564
$\tilde{z}$	-44	-84	0	0	(0, 0, 0, 0, 0, 0)	

Here -84 is the most positive in $\tilde{z}$. So $\widetilde{x_2}$ is the entering variable and $\min\{R(157, 159, 161, 163, 165,167), R(277, 279, 281\ 283, 285, 287)\}$ is 324. So $\widetilde{x_3}$ is a leaving variable, pivote element is 144.

First iterations

Basis	$\widetilde{x_1}$	$\widetilde{x_2}$	$\widetilde{x_3}$	$\widetilde{x_4}$	R.H.S
$\widetilde{x_2}$	$\frac{104}{144}$	1	$\frac{1}{144}$	0	$\frac{157}{144}, \frac{159}{144}, \frac{161}{144}, \frac{163}{144}, \frac{165}{144}, \frac{167}{144}$
$\widetilde{x_4}$	$\frac{3200}{144}$	0	$\frac{-224}{144}$	1	$\frac{4720}{144}, \frac{4560}{144}, \frac{4400}{144}, \frac{4240}{144}, \frac{4080}{144}, \frac{3920}{144}$
$\tilde{z}$	$\frac{2400}{144}$	0	$\frac{84}{144}$	0	$\frac{13188}{144}, \frac{13356}{144}, \frac{13524}{144}, \frac{13692}{144}, \frac{13860}{144}, \frac{14028}{144}$

Since $\tilde{z} \geq 0$, the fuzzy optimal solution of FFLPP is,

$\widetilde{x_1}$ = (0, 0, 0, 0, 0, 0),

$\widetilde{x_2} = \left(\frac{157}{144}, \frac{159}{144}, \frac{161}{144}, \frac{163}{144}, \frac{165}{144}, \frac{167}{144}\right)$ and

$\tilde{z} = \left(\frac{13188}{144}, \frac{13356}{144}, \frac{13524}{144}, \frac{13692}{144}, \frac{13860}{144}, \frac{14028}{144}\right) \cong 567$

4.2 Minimize Z = (0, 1, 2, 3, 7, 8) $\widetilde{x_1}$ +(2, 4, 6, 8, 10, 15) $\widetilde{x_2}$ + (4, 6, 8, 10, 15, 17) $\widetilde{x_3}$

Subject to
(0, 1, 1, 1, 1, 2) $\widetilde{x_1}$ +(0, 1, 2, 2, 3, 4) $\widetilde{x_2}$ +

(0, 1, 2, 4, 5, 6) $\widetilde{x_3}$ $\geq$ (7, 8, 10, 12, 15, 20)

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(0, 1, 1, 2, 2, 3) $\widetilde{x_1}$ + (2, 3, 4, 5, 6, 7) $\widetilde{x_2}$ +

(0, 1, 2, 4, 5, 6) $\widetilde{x_3}$ ≥ (8, 9, 12, 16, 20, 25)

Solution: Using ranking function (2.1.1) we get,

Min Z = 7 $\widetilde{x_1}$ +15 $\widetilde{x_2}$ +20 $\widetilde{x_3}$ +0 S_1 + 0 S_2 + M A_1 + M A_2
Subject to,

$2\widetilde{x_1} + 4\widetilde{x_2} + 6\widetilde{x_3} - S_1 + A_1 = (7, 8, 10, 12, 15, 20)$

$3\widetilde{x_1} + 9\widetilde{x_2} + 6\widetilde{x_3} - S_2 + A_2 = (8, 9, 12, 16, 20, 25)$

$\widetilde{x_1}, \widetilde{x_2}, \widetilde{x_3}, \widetilde{x_4}, S_1, S_2, A_1, A_2 \geq 0$

Initial Table,

	C_j	7	15	20	0	0	M	M	R.H.S	Ratio
CB	BV	$\widetilde{x_1}$	$\widetilde{x_2}$	$\widetilde{x_3}$	S_1	S_2	A_1	A_2		
M	A_1	2	4	6	-1	0	1	0	(7, 8, 10, 12, 15, 20)	$\frac{24}{4}$
M	A_2	3	9	6	0	-1	0	1	(8, 9, 12, 16, 20, 25)	$\frac{30}{9}$
	Z_j	5M	13M	12M	-M	-M	M	M		
	$C_j - \tilde{Z}_j$	$7 - 5$ M	$15 - 13$M	$20 - 12$M	M	M	0	0		

Using Big-M method we get,

Final table

	C_j	7	15	20	0	0	M	M	Solution
CB	BV	$\widetilde{x_1}$	$\widetilde{x_2}$	$\widetilde{x_3}$	S_1	S_2	A_1	A_2	
20	$\widetilde{x_3}$	$\frac{1}{5}$	0	1	$\frac{-3}{10}$	$\frac{2}{15}$	-	-	$\left(\frac{31}{30}, \frac{36}{30}, \frac{42}{30}, \frac{44}{30}, \frac{55}{30}, \frac{80}{30}\right)$
15	$\widetilde{x_2}$	$\frac{1}{5}$	1	0	$\frac{1}{5}$	$\frac{-1}{5}$	-	-	$\left(\frac{81}{405}, \frac{9}{405}, \frac{162}{405}, \frac{324}{405}, \frac{405}{405}, \frac{405}{405}\right)$
	$\tilde{Z}_j$	7	15	20	-3	$\frac{-1}{3}$	-	-	(17.66 , 24.33, 34, 41.33, 51.66, 68.33)
	$C_j - \tilde{Z}_j$	0	0	0	3	$\frac{1}{3}$	-	-	

Since all $C_j - \tilde{Z}_j \geq 0$. The fuzzy optimal solution of FFLPP is,

$\widetilde{x_1}$ = (0, 0, 0, 0, 0, 0),

$\widetilde{x_2} = \left(\frac{81}{405}, \frac{9}{405}, \frac{162}{405}, \frac{324}{405}, \frac{405}{405}, \frac{405}{405}\right)$, $\widetilde{x_3} = \left(\frac{31}{30}, \frac{36}{30}, \frac{42}{30}, \frac{44}{30}, \frac{55}{30}, \frac{80}{30}\right)$

and $\tilde{z} = (17.66\,, 24.33,\ 34,\ 41.33,\ 51.66,\ 68.33\ ) \cong 79.10$

5. Conclusion:

In this paper, A new ranking technique obtained an optimal solution for fully fuzzy linear programming problem using hexagonal fuzzy number. This ranking function has less calculation comparatively to other ranking functions. The numerical example is solved for minimization and maximization problem. This method is more efficient and easy and gives the best optimal solution.

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