

Vectors of Identity Matrix are the only Solution for a Special Linear System of Equations and Linear Programming Problems

Dr. Enagandula Prasad

Associate Professor, Dept of Mathematics,
 Vignana Bharathi Institute of Technology, Hyderabad, India-501301

Abstract

This article is going to prove that the vectors of identity matrix are the only solutions for special kind of linear system of equations with n equations n unknown problems and for the Linear Programming Problems (LPP) problems of n variables if the objective function is Maximize and also we are also going to prove if it is given in LPP to Minimize objective function the optimal solution is trivial solution (zero solution) and therefore $\text{Min}Z=0$.

Keywords: C-Matrix, LPP, System of Linear Equations

I. INTRODUCTION

Definition 1.1: A square matrix is said to be C -Matrix if all the principal diagonal elements are same, in which rest of the elements are different from Principal diagonal element and at least two elements are different.

Examples: 1. $A = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 1 & 3 \\ 2 & 3 & 1 \end{bmatrix}$ is C -Matrix

2. $A = \begin{bmatrix} 2 & 3 & 5 & 4 \\ 3 & 2 & 6 & 8 \\ 5 & 6 & 2 & 8 \\ 6 & 6 & 7 & 2 \end{bmatrix}$ is C -Matrix

3. $A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 3 \\ 2 & 3 & 1 \end{bmatrix}$ is **not** C -Matrix because

1 is the principal diagonal element that should not present in other than principal diagonal element

4. $A = \begin{bmatrix} 2 & 2 & 5 & 4 \\ 3 & 2 & 6 & 8 \\ 5 & 6 & 2 & 8 \\ 6 & 6 & 2 & 2 \end{bmatrix}$ is **not** C -Matrix

because 2 is the principal diagonal element that should not present in other than principal diagonal element

Definition 1.2: A general system of n linear equations with n unknowns can be written as

can be written as

$$AX=B$$

$$\text{Where } A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}_{n \times n}, X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \text{ and } B =$$

$$\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

and A is called Coefficient Matrix with constants, X is a matrix with unknowns and B is a constant matrix

Definition 1.3: A Linear programming problem (LPP) that can be expressed in

$$\text{Minimize/Max } Z = c^T x$$

$$\text{Subject to } Ax \leq b$$

$$\text{and } x \geq 0$$

where x represents the vector of variables (to be determined), c and b are vectors of (known) coefficients, A is a (known) matrix of coefficients,

Definition 1.4: An n-tuple (x_1, x_2, \dots, x_n) of numbers which satisfies the constraints given by (b) of G.L.P.P. is called a solution to G.L.P.P.

Definition 1.5: Any solution to a G.L.P.P. which satisfies the non-negativity restrictions of the problem is called a feasible solution to a general L.P.P.

Definition 1.6: Any Feasible solution to a G.L.P.P. which optimizes (maximizes/ minimizes) the objective function of G.L.P.P. is called an optimum solution to the G.L.P.P.

Definition 1.7: A basic solution to the system is called degenerate if one or more of the basic variables vanish i.e. if any of the basic variable has zero value then it is called degenerate basic solution.

Definition 1.8: A basic solution the system is called non-degenerate if all the basic variables are non-zero (either positive or negative)

Definition 1.9: A feasible solution to L.P.P. which is also a basic solution to the problem is called a basic feasible solution (B.F.S.) to the L.P.P.

Problem 1.1: If A is C matrix and AX=B is the linear system of Equations (1) Then

i) If B is replaced with 1st column of A in AX=B then the solution is (1,0,0,..0)^T

ii) If B is replaced with 2nd column of A in AX=B then the solution is (0,1,0,..0)^T

iii) If B is replaced with 3rd column of A in AX=B then the solution is (0,0,1,..0)^T

iii) If B is replaced with nth column of A in AX=B then the solution is (0,0,0,..1)^T

Solution:

Case i) Let a general system of 3 linear equations with 3 unknowns can be written as

$$1x+2y+3z=1$$

$$2x+1y+4z=2$$

$$3x+4y+1z=3$$

If we observe the coefficient matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 4 & 1 \end{bmatrix}$ is a C

matrix and last and first column are same in AX=B

Claim: The solution is x=1,y=0,z=0

The solution by Normal form method

The Augmented matrix

$$[AB] = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 1 & 4 & 2 \\ 3 & 4 & 1 & 3 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - 2R_1; R_3 \rightarrow R_3 - 3R_1;$$

$$\approx \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & -3 & -2 & 0 \\ 0 & -2 & -8 & 0 \end{bmatrix}$$

$$\approx \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & -3 & -2 & 0 \\ 0 & -2 & -8 & 0 \end{bmatrix}$$

$$R_2 \rightarrow R_2 / -3$$

$$\approx \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 0 & 2/3 & 0 \\ 0 & -2 & -8 & 0 \end{bmatrix}$$

$$R_1 \rightarrow R_1 - 2R_2$$

$$\approx \begin{bmatrix} 1 & 0 & 5/3 & 1 \\ 0 & 1 & 2/3 & 0 \\ 0 & -2 & -8 & 0 \end{bmatrix}$$

$$R_3 \rightarrow R_3 + 2R_2$$

$$\approx \begin{bmatrix} 1 & 0 & 5/3 & 1 \\ 0 & 1 & 2/3 & 0 \\ 0 & 0 & -20/3 & 0 \end{bmatrix}$$

$$R_3 \rightarrow (-3/20)R_3$$

$$\begin{bmatrix} 1 & 0 & 5/3 & 1 \\ 0 & 1 & 2/3 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$R_1 \rightarrow R_1 - (5/3)R_3$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 2/3 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - (2/3)R_3$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

It is in normal form Therefore the solution is x=1,y=0,z=0

Case ii) Let a general system of 3 linear equations with 3 unknowns can be written as

$$1x+2y+3z=2$$

$$2x+1y+4z=1$$

$$3x+4y+1z=4$$

If we observe the coefficient matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 4 & 1 \end{bmatrix}$ is a C

matrix and second and last column are same in $AX=B$

Claim: The solution is $x=0, y=1, z=0$

The solution by Normal form method

The Augmented matrix

$$[AB] = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 1 & 4 & 1 \\ 3 & 4 & 1 & 4 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 2R_1; R_3 \rightarrow R_3 - 3R_1;$

$$\approx \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & -3 & -2 & -3 \\ 0 & -2 & -8 & -2 \end{bmatrix}$$

$R_2 \rightarrow R_2 / -3$

$$\approx \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & 2/3 & 1 \\ 0 & -2 & -8 & -2 \end{bmatrix}$$

$R_1 \rightarrow R_1 - 2R_2$

$$\approx \begin{bmatrix} 1 & 0 & 5/3 & 0 \\ 0 & 1 & 2/3 & 1 \\ 0 & -2 & -8 & -2 \end{bmatrix}$$

$R_3 \rightarrow R_3 + 2R_2$

$$\approx \begin{bmatrix} 1 & 0 & 5/3 & 0 \\ 0 & 1 & 2/3 & 1 \\ 0 & 0 & -20/3 & 0 \end{bmatrix}$$

$R_3 \rightarrow (-3/20)R_3$

$$\begin{bmatrix} 1 & 0 & 5/3 & 1 \\ 0 & 1 & 2/3 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$R_1 \rightarrow R_1 - (5/3)R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 2/3 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$R_2 \rightarrow R_2 - (2/3)R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

It is in normal form Therefore the solution is $x=0, y=1, z=0$

Case iii) Let a general system of 3 linear equations with 3 unknowns can be written as

$$1x + 2y + 3z = 3$$

$$2x + 1y + 4z = 4$$

$$3x + 4y + 1z = 1$$

Claim: The solution is $x=0, y=0, z=1$

If we observe the coefficient matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 4 & 1 \end{bmatrix}$ is a C

matrix and 3rd and last column are same in $AX=B$

The solution by Normal form method

The Augmented matrix

$$[AB] = \begin{bmatrix} 1 & 2 & 3 & 3 \\ 2 & 1 & 4 & 4 \\ 3 & 4 & 1 & 1 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 2R_1; R_3 \rightarrow R_3 - 3R_1;$

$$\approx \begin{bmatrix} 1 & 2 & 3 & 3 \\ 0 & -3 & -2 & -2 \\ 0 & -2 & -8 & -8 \end{bmatrix}$$

$R_2 \rightarrow R_2 / -3$

$$\approx \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & 2/3 & 2/3 \\ 0 & -2 & -8 & -8 \end{bmatrix}$$

$R_1 \rightarrow R_1 - 2R_2$

$$\approx \begin{bmatrix} 1 & 0 & 5/3 & 5/3 \\ 0 & 1 & 2/3 & 2/3 \\ 0 & -2 & -8 & -8 \end{bmatrix}$$

$R_3 \rightarrow (-3/20)R_3$

$$\begin{bmatrix} 1 & 0 & 5/3 & 5/3 \\ 0 & 1 & 2/3 & 2/3 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$R_1 \rightarrow R_1 - (5/3)R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2/3 & 2/3 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$R_2 \rightarrow R_2 - (2/3)R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

It is in normal form Therefore the solution is $x=0, y=0, z=1$
 \therefore If the coefficient matrix is C matrix and B is changing column 1, 2, 3 in $AX=B$ then the solution is

$$X_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, X_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, X_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Problem1.2: Solve the linear Programming Problem using Simplex (Big M) method when the coefficient matrix is C matrix when objective function is Maximum and subjective to $AX \leq B, X \geq 0$

- i) If B is replaced with 1st column of A in $AX=B$ then the solution is $(1,0,0,..0)^T$
- ii) If B is replaced with 2nd column of A in $AX=B$ then the solution is $(0,1,0,..0)^T$
- iii) If B is replaced with 3rd column of A in $AX=B$ then the solution is $(0,0,1,..0)^T$
- iii) If B is replaced with nth column of A in $AX=B$ then the solution is $(0,0,0,..1)^T$

Case i) The last and first columns are same in $AX \leq B$ and A is C matrix in LPP

$$\text{Max } Z = 5x_1 + 10x_2 + 8x_3$$

Subject to,

$$x_1 + 5x_2 + 2x_3 \leq 1;$$

$$4x_1 + 1x_2 + 4x_3 \leq 4;$$

$$2x_1 + 4x_2 + x_3 \leq 2$$

Where $x_1, x_2, x_3 \geq 0$

Sol. Given Linear Programming Problem

$$\text{Max } Z = 5x_1 + 10x_2 + 8x_3$$

Subject to, $x_1 + 5x_2 + 2x_3 \leq 1;$

$$4x_1 + 1x_2 + 4x_3 \leq 4;$$

$$2x_1 + 4x_2 + x_3 \leq 2$$

Where $x_1, x_2, x_3 \geq 0$

If we observe the coefficient matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 4 & 1 \end{bmatrix}$ is a C

matrix and last and first column are same in $AX=B$

Claim: The solution is $x=1, y=0, z=0$

The Problem is converted to canonical form by adding slack, surplus and artificial variables

1. As the constraint -1 of the type \leq we should add slack variable S1

1. As the constraint -2 of the type \leq we should add slack variable S2

1. As the constraint -1 of the type \leq we should add slack variable S3

After introducing slack variables

$$\text{Max } Z = 5x_1 + 10x_2 + 8x_3 + 0S_1 + 0S_2 + 0S_3$$

Subject to

$$x_1 + 5x_2 + 2x_3 + S_1 = 1$$

$$4x_1 + 1x_2 + 4x_3 + S_2 = 4$$

$$2x_1 + 4x_2 + x_3 + S_3 = 2$$

Where $x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$

Iteration		C_j	5	10	8	0	0	0	
1									
B	CB	XB	X1	X2	X3	S1	S2	S3	Min Ratio $\frac{XB}{x_2}$
S1	0	1	1	(5)	2	1	0	0	1/5 ←
S2	0	4	4	1	4	0	1	0	4/1
S3	0	2	2	4	1	0	0	1	2/4
Z=0		Z_j	0	0	0	0	0	0	
		$Z_j - C_j$	-5	-10	-8	0	0	0	

$$R_1(\text{new}) \Rightarrow R_1(\text{old})/5$$

$$R_2(\text{new}) \Rightarrow R_2(\text{old}) - R_1(\text{new})$$

$$R_3(\text{new}) \Rightarrow R_3(\text{old}) - 4R_1(\text{new})$$

Negative minimum $Z_j - C_j$ is -10 and its column index is 2. So, the entering variable is x_2 .

Minimum ratio is 0.2 and its row index is 1. So, the leaving basis variable is S1.

\therefore The pivot element is 5.

Entering = x_2 , Departing = S1, Key Element = 5

Iteration 2		C _j	5	10	8	0	0	0	
B	C B	XB	X1	X2	X3	S1	S2	S3	Min Ratio $\frac{O_{XB}}{x2}$
X2	10	1/5	1/5	1	2/5	1/5	0	0	1/2 ←
S2	0	19/5	19/5	0	18/5	-1/5	1	0	19/18
S3	0	6/5	6/5	0	-3/5	-4/5	0	1	2/4
Z=2		Z _j	2	10	4	2	0	0	
		Z _j -C _j	-3	0	4 ↑	2	0	0	

$R1(new) \Rightarrow R1(old) * 5/2$

$R2(new) \Rightarrow R2(old) - (18/5)R1(new)$

$R3(new) \Rightarrow R3(old) + (3/5)R1(new)$

Negative minimum $Z_j - C_j$ is -4 and its column index is 3. So, the entering variable is x_3 .

Minimum ratio is 0.5 and its row index is 1. So, the leaving basis variable is x_2 .

∴ The pivot element is 2/5

Entering = x_3 , Departing = x_2 , Key Element = 2/5

Iteration 3		C _j	5	10	8	0	0	0	
B	C B	XB	X1	X2	X3	S1	S2	S3	Min Ratio $\frac{O_{XB}}{x2}$
X3	8	1/2	1/2	5/2	1	1/2	0	0	1
S2	0	2	2	-9	0	-2	1	0	1
S3	0	3/2	3/2	3/2	0	-1/2	0	1	1 ←
Z=4		Z _j	4	20	8	4	0	0	
		Z _j -C _j	-1	10 ↑	0	4	0	0	

$R3(new) \Rightarrow R3(old) * 2/3$

$R1(new) \Rightarrow R1(old) - (1/2)R3(new)$

$R2(new) \Rightarrow R2(old) - 2R3(new)$

Negative minimum $Z_j - C_j$ is -1 and its column index is 1.

So, the entering variable is x_1 .

Minimum ratio is 1 and its row index is 3. So, the leaving basis variable is S_3 .

∴ The pivot element is 3/2 .

Entering = x_1 , Departing = S_3 , Key Element = 3/2

Iteration 4		C _j	5	10	8	0	0	0	
B	C B	XB	X1	X2	X3	S1	S2	S3	Min Ratio $\frac{O_{XB}}{x2}$
X3	8	0	0	2	1	2/3	0	-1/3	
S2	0	0	0	-11	0	-4/3	1	-4/3	
X1	5	1	1	1	0	-1/3	0	2/3	
Z=5		Z _j	5	21	8	11/3	0	2/3	
		Z _j -C _j	0	11	0	11/3	0	2/3	

Since all $Z_j - C_j \geq 0$

Hence, optimal solution is arrived with value of variables as : $x_1=1, x_2=0, x_3=0$

Max $Z=5$

Caseii) The last and second columns are same in $AX \leq B$, and A is C matrix in LPP

Solve the linear Programming Problem using Simplex (Big M) Method

Max $Z=5x_1+10x_2+8x_3$

Subject to,

$x_1+5x_2+2x_3 \leq 5;$

$4x_1+1x_2+4x_3 \leq 1;$

$2x_1+4x_2+x_3 \leq 4$

Where $x_1, x_2, x_3 \geq 0$

Sol. Given Linear Programming Problem

Max $Z=5x_1+10x_2+8x_3$

Subject to, $x_1+5x_2+2x_3 \leq 5;$

$4x_1+1x_2+4x_3 \leq 1;$

$2x_1+4x_2+x_3 \leq 4$

Where $x_1, x_2, x_3 \geq 0$

If we observe the coefficient matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 4 & 1 \end{bmatrix}$ is a C

matrix and last and second column are same in $AX=B$

Claim: The solution is $x=1, y=0, z=0$

The Problem is converted to canonical form by adding slack, surplus and artificial variables

1. As the constraint -1 of the type \leq we should add slack variable S1

1. As the constraint -2 of the type \leq we should add slack variable S2

1. As the constraint -1 of the type \leq we should add slack variable S3

After introducing slack variables

$$\text{Max } Z = 5x_1 + 10x_2 + 8x_3 + 0S_1 + 0S_2 + 0S_3$$

Subject to

$$x_1 + 5x_2 + 2x_3 + S_1 = 5$$

$$4x_1 + 1x_2 + 4x_3 + S_2 = 1$$

$$2x_1 + 4x_2 + x_3 + S_3 = 4$$

Where $x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$

Iteration 1		C_j	5	10	8	0	0	0	
B	C	X	X	X	X	S	S	S	Min Ratio $\frac{O_{XB}}{x_2}$
	B	B	1	2	3	1	2	3	
S1	0	5	1	5	2	1	0	0	5/5
S2	0	1	4	1	4	0	1	0	1/1
S3	0	4	2	(4)	1	0	0	1	4/4 ←
Z=0		Z_j	0	0	0	0	0	0	
		$Z_j - C_j$	-5	-10 ↑	-8	0	0	0	

Negative minimum $Z_j - C_j$ is -10 and its column index is 2. So, the entering variable is x_2 .

Minimum ratio is 1 and its row index is 3. So, the leaving basis variable is S3.

∴ The pivot element is 4.

Entering = x_2 , Departing = S3, Key Element = 4

$$R_3(\text{new}) \Rightarrow R_3(\text{old})/4$$

$$R_1(\text{new}) \Rightarrow R_1(\text{old}) - 5R_3(\text{new})$$

$$R_2(\text{new}) \Rightarrow R_2(\text{old}) - R_3(\text{new})$$

Iteration 2		C_j	5	10	8	0	0	0	
B	C	X	X	X	X3	S	S	S3	Min Ratio $\frac{O_{XB}}{x_2}$
	B	B	1	2		1	2		
S1	0	0	-3/2	0	3/4	1	0	-5/4	0
S2	0	0	7/2	0	(15/4)	0	1	-1/4	0 →
X2	10	1	1/2	1	1/4	0	0	1/4	4
Z=10		Z_j	5	10	5/2	0	0	5/2	
		$Z_j - C_j$	0	0	-11/2 ↑	0	0	5/2	

Negative minimum $Z_j - C_j$ is -11/2 and its column index is 3. So, the entering variable is x_3 .

Minimum ratio is 0 and its row index is 2. So, the leaving basis variable is S2.

∴ The pivot element is 15/4

Entering = x_3 , Departing = S2, Key Element = 15/4

$$R_2(\text{new}) \Rightarrow R_2(\text{old}) * 4/15$$

$$R_1(\text{new}) \Rightarrow R_1(\text{old}) - (3/4)R_2(\text{new})$$

$$R_3(\text{new}) \Rightarrow R_3(\text{old}) - (1/4)R_2(\text{new})$$

Iteration 3		C_j	5	10	8	0	0	0	
B	C	X	X	X	X	S1	S2	S	Min Ratio $\frac{O_{XB}}{x_2}$
	B	B	1	2	3			3	
S1	0	0	-1/5	0	0	1	-1/5	-6/5	
X3	8	0	1/5	0	1	0	4/5	-1/5	
X2	10	1	4 /5	1	0	0	-1/5	4/5	

			5				15	5	
Z=10		Zj	1 5 2 / 1 5	10	8	0	22 /1 5	3 2 / 1 5	
		Zj - Cj	7 7 / 1 5	8	0	0	22 /1 5	3 2 / 1 5	

Since all $Z_j - C_j \geq 0$
 Hence, optimal solution is arrived with value of variables as :

$x_1=0, x_2=1, x_3=0$
 Max Z=10

Caseiii) The last and third columns are same in $AX \leq B$, and A is C matrix in LPP

the linear Programming Problem using Simplex (Big M) Method

Max $Z=5x_1+10x_2+8x_3$

Subject to,

$x_1+5x_2+2x_3 \leq 2;$

$4x_1+1x_2+4x_3 \leq 4;$

$2x_1+4x_2+x_3 \leq 1;$

Where $x_1, x_2, x_3 \geq 0$

Sol. Given Linear Programming Problem

Max $Z=5x_1+10x_2+8x_3$

Subject to, $x_1+5x_2+2x_3 \leq 2;$

$4x_1+1x_2+4x_3 \leq 4;$

$2x_1+4x_2+x_3 \leq 1;$

Where $x_1, x_2, x_3 \geq 0$

If we observe the coefficient matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 4 & 1 \end{bmatrix}$ is a C

matrix and last and third column are same in $AX=B$

Claim: The solution is $x=1, y=0, z=0$

The Problem is converted to canonical form by adding slack, surplus and artificial variables

1.As the constraint -1 of the type \leq we should add slack variable S1

1.As the constraint -2 of the type \leq we should add slack variable S2

1.As the constraint -1 of the type \leq we should add slack

variable S3

After introducing slack variables

Max $Z=5x_1+10x_2+8x_3+0S_1+0S_2+0S_3$

Subject to

$x_1+5x_2+2x_3+S_1 = 2$

$4x_1+1x_2+4x_3 + S_2 = 4$

$2x_1+4x_2+x_3 + S_3 = 1$

Where $x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$

Iterati on 1		Cj	5	10	8	0	0	0		
B	C	X B	X B	X 1	X 2	X 3	S 1	S 2	S 3	Min Rati $\frac{0 \cdot XB}{x_2}$
S1	0	2	1	5	2	1	0	0	0.4	
S2	0	4	4	1	4	0	1	0	4	
S3	0	1	2	(4)	1	0	0	1	0.25 →	
Z=0		Zj	0	0	0	0	0	0		
		Zj - Cj	-5	-10 ↑	-8	0	0	0		

Negative minimum $Z_j - C_j$ is - 10 and its column index is 2. So, the entering variable is x_2 .

Minimum ratio is 0.25 and its row index is 3. So, the leaving basis variable is S_3 .

∴ The pivot element is 4.

Entering = x_2 , Departing = S_3 , Key Element = 4

$R_3(\text{new}) \Rightarrow R_3(\text{old})/4$

$R_1(\text{new}) \Rightarrow R_1(\text{old}) - 5R_3(\text{new})$

$R_2(\text{new}) \Rightarrow R_2(\text{old}) - R_3(\text{new})$

Iterati on 2		Cj	5	10	8	0	0	0	
B	C	X B	X 1	X 2	X 3	S 1	S 2	S 3	Min Rati $\frac{0 \cdot XB}{x_2}$
S1	0	3/4	-3/2	0	3/4	1	0	-5/4	1
S2	0	(15/4)	7/2	0	(15/4)	0	1	-1/4	1 →
X2	10	1/4	1/1	1	1/4	0	0	1/1	1

			2				4	
Z=5/2		Zj	5	1 0	5/2	0	0	5/ 2
		Zj- Cj	0	0	- 1/2	0	0	5/ 2

Negative minimum $Z_j - C_j$ is $-11/2$ and its column index is 3. So, the entering variable is x_3 .

Minimum ratio is 0 and its row index is 2. So, the leaving basis variable is s_2 .

∴ The pivot element is $15/4$

Entering = x_3 , Departing = s_2 , Key Element = $15/4$

$R_2(\text{new}) \Rightarrow R_2(\text{old}) * 4/15$

$R_1(\text{new}) \Rightarrow R_1(\text{old}) - (3/4)R_2(\text{new})$

$R_3(\text{new}) \Rightarrow R_3(\text{old}) - (1/4)R_2(\text{new})$

Since all $Z_j - C_j \geq 0$

Hence, optimal solution is arrived with value of variables as :

$x_1=0, x_2=0, x_3=1$

Max $Z=8$

∴ The vectors of Identity matrix are the solutions of LPP spacial problems

Problem1.3: Solve the linear Programming Problem using Simplex (Big M) method when the coefficient matrix is C matrix when objective function is Minimum and subjective to $AX \leq B, X \geq 0$, the solution of is trivial solution and $\text{Min} Z=0$ for each case given below

i) If B is replaced with 1st column of A in $AX=B$ then the solution is $(0,0,0,..0)^T$

ii) If B is replaced with 2nd column of A in $AX=B$ then the solution is $(0,0,0,..0)^T$

iii) If B is replaced with 3rd column of A in $AX=B$ then the solution is $(0,0,0,..0)^T$

iii) If B is replaced with nth column of A in $AX=B$ then the solution is $(0,0,0,..0)^T$

Case i) The last and first columns are same in $AX \leq B$

and A is C matrix in LPP

Min $Z=5x_1+10x_2+8x_3$

Subject to,

$x_1+5x_2+2x_3 \leq 1$;

$4x_1+1x_2+4x_3 \leq 4$;

$2x_1+4x_2+x_3 \leq 2$

Where $x_1, x_2, x_3 \geq 0$

Sol. Given Linear Programming Problem

Min $Z=5x_1+10x_2+8x_3$

Subject to, $x_1+5x_2+2x_3 \leq 1$;

$4x_1+1x_2+4x_3 \leq 4$;

$2x_1+4x_2+x_3 \leq 2$;

Where $x_1, x_2, x_3 \geq 0$

If we observe the coefficient matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 4 & 1 \end{bmatrix}$ is a C

matrix and last and first column are same in $AX=B$

Claim: The solution is $x=0, y=0, z=0$

The Problem is converted to canonical form by adding slack, surplus and artificial variables

1. As the constraint -1 of the type \leq we should add slack variable S_1

1. As the constraint -2 of the type \leq we should add slack

Iteration 3		C _j	5	10	8	0	0	0	
B	C B	X B	X 1	X 2	X 3	S ₁	S 2	S ₃	Min Rati o $\frac{XB}{x_2}$
S ₁	0	0	- 1 1 / 5	0	0	1	- 1 / 5	- 6/ 5	
X ₃	8	0	1 4 / 5	0	1	0	4 / 5	- 1/ 15	
X ₂	10	1	4 / 5	1	0	0	- 1 / 1 5	4/ 15	
Z=8		Z _j	1 5 2 / 1 5	10	8	0	2 2 / 1 5	32 /1 5	
		Z _j - C _j	7 7 / 1 5	0	0	0	2 2 / 1 5	32 /1 5	

variable S2

1.As the constraint -1 of the type \leq we should add slack variable S3

After introducing slack variables

$$\text{Max } Z=5x_1+10x_2+8x_3+0S_1+0S_2+0S_3$$

Subject to

$$x_1+5x_2+2x_3+S_1 = 1$$

$$4x_1+1x_2+4x_3 + S_2 = 4$$

$$2x_1+4x_2+x_3 + S_3 = 2$$

Where $x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$

Since all $Z_j - C_j \leq 0$

Hence, optimal solution is arrived with value of variables as :

$$x_1=0, x_2=0, x_3=0;$$

$$\text{Min } Z=0$$

Case ii) The last and second columns are same in $AX \leq B$ and A is C matrix in LPP

$$\text{Min } Z=5x_1+10x_2+8x_3$$

Subject to,

$$x_1+5x_2+2x_3 \leq 5;$$

$$4x_1+1x_2+4x_3 \leq 1;$$

$$2x_1+4x_2+x_3 \leq 4;$$

Where $x_1, x_2, x_3 \geq 0$

Sol. Given Linear Programming Problem

$$\text{Min } Z=5x_1+10x_2+8x_3$$

Subject to, $x_1+5x_2+2x_3 \leq 5;$

$$4x_1+1x_2+4x_3 \leq 1;$$

$$2x_1+4x_2+x_3 \leq 4;$$

Where $x_1, x_2, x_3 \geq 0$

If we observe the coefficient matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 4 & 1 \end{bmatrix}$ is a C

matrix and last and first column are same in $AX=B$

Claim: The solution is $x=0, y=0, z=0$

The Problem is converted to canonical form by adding slack, surplus and artificial variables

1.As the constraint -1 of the type \leq we should add slack variable S1

1.As the constraint -2 of the type \leq we should add slack variable S2

1.As the constraint -1 of the type \leq we should add slack variable S3

After introducing slack variables

$$\text{Max } Z=5x_1+10x_2+8x_3+0S_1+0S_2+0S_3$$

Subject to

$$x_1+5x_2+2x_3+S_1 = 1$$

$$4x_1+1x_2+4x_3 + S_2 = 4$$

$$2x_1+4x_2+x_3 + S_3 = 2$$

Where $x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$

Since all $Z_j - C_j \leq 0$

Hence, optimal solution is arrived with value of variables as

Iterati on 1		C _j	5	10	8	0	0	0	
B	C B	X B	X 1	X 2	X 3	S 1	S 2	S 3	Min Rati $\frac{0 \text{ } XB}{x_2}$
S1	0	1	1	(5)	2	1	0	0	
S2	0	4	4	1	4	0	1	0	
S3	0	2	2	4	1	0	0	1	
Z=0		Z _j	0	0	0	0	0	0	

Iterati on 1		C _j	5	10	8	0	0	0	
B	C B	X B	X 1	X 2	X 3	S 1	S 2	S 3	Min Rati $\frac{0 \text{ } XB}{x_2}$
S1	0	5	1	(5)	2	1	0	0	
S2	0	1	4	1	4	0	1	0	
S3	0	4	2	4	1	0	0	1	
Z=0		Z _j	0	0	0	0	0	0	
		Z _j - C _j	-	-	-8	0	0	0	
		Z _j - C _j	-	-	-8	0	0	0	

:

$$x_1=0, x_2=0, x_3=0;$$

$$\text{Min } Z=0$$

Case iii) The last and third columns are same in $AX \leq B$

and A is C matrix in LPP

$$\text{Min } Z=5x_1+10x_2+8x_3$$

Subject to,

$$x_1+5x_2+2x_3 \leq 2;$$

$$4x_1+1x_2+4x_3 \leq 4;$$

$$2x_1+4x_2+x_3 \leq 1;$$

Where $x_1, x_2, x_3 \geq 0$

Sol. Given Linear Programming Problem

$$\text{Min } Z=5x_1+10x_2+8x_3$$

Subject to, $x_1+5x_2+2x_3 \leq 2;$

$$4x_1+1x_2+4x_3 \leq 4;$$

$$2x_1+4x_2+x_3 \leq 1;$$

Where $x_1, x_2, x_3 \geq 0$

If we observe the coefficient matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 4 & 1 \end{bmatrix}$ is a C

matrix and last and first column are same in $AX=B$

Claim: The solution is $x=0, y=0, z=0$

The Problem is converted to canonical form by adding slack, surplus and artificial variables

1. As the constraint -1 of the type \leq we should add slack variable S1

1. As the constraint -2 of the type \leq we should add slack variable S2

1. As the constraint -1 of the type \leq we should add slack variable S3

After introducing slack variables

$$\text{Max } Z=5x_1+10x_2+8x_3+0S_1+0S_2+0S_3$$

Subject to

$$x_1+5x_2+2x_3+S_1 = 2$$

$$4x_1+1x_2+4x_3 + S_2 = 4$$

$$2x_1+4x_2+x_3 + S_3 = 1$$

Where $x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$

Since all $Z_j - C_j \leq 0$

Hence, optimal solution is arrived with value of variables as

$$: x_1=0, x_2=0, x_3=0; \text{Min } Z=0$$

Iteration 1		C_j	5	10	8	0	0	0	
B	C	X	X	X	X	S	S	S	Min Rati
	B	B	1	2	3	1	2	3	$\frac{0 \cdot XB}{x_2}$
S1	0	5	1	(5)	2	1	0	0	
S2	0	1	4	1	4	0	1	0	
S3	0	4	2	4	1	0	0	1	
Z=0		Z_j	0	0	0	0	0	0	
		$Z_j - C_j$	-5	-10	-8	0	0	0	

II.CONCLUSION

Therefore we have proved the vectors of identity matrix are the only solutions for special kind of linear system of equations with n equations n unknown problems and for the Linear Programming Problems (LPP) problems of n variables if the objective function is Maximize and also we have proved if it is given in LPP to Minimize objective function the optimal solution is trivial solution (zero solution) and therefore $\text{Min } Z=0$.

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Prasad Enagandula received the PhD degree in Mathematics from Jawaharlal Nehru Technological University, Hyderabad, India. His research interests are in the areas of Linear Algebra and the role technology and social media in Mathematics education. He has published research articles in reputed national and international journals of Mathematical and engineering sciences. At present he is working as Associate Professor in the Department of Mathematics, Vignana Bharathi Institute of Technology, Hyderabad –India.