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Analysis of Correlation using Fractional Fourier Transform in Images

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Abstract

Image Correlation is widely used in Image processing for finding similarity, locating patterns or some sub image matching. Robust Correlation technique is needed in case the Signal to Noise Ratio (SNR) is not appropriate. Correlation is largely found in spatial domain using Fast Fourier Transform (FFT) due to lower complexity and space requirements. It is either used directly on the samples or some signature features are extracted and then those features are correlated on sample and test sequence / image to reduce space requirements. Fractional Fourier Transform (FrFT) has been used recently to find the intermediatery details of a signal between the time and frequency domains and circumvent noise. In this work, Correlation of Images using FrFT has been found In particular, the Correlation obtained through FFT and FrFT at varied noise levels is compared. Experimental results demonstrate that FrFT is better as compared to FFT, for finding Correlation, by optimizing Fractional Factor (Rotation Angle). It is established that at some Rotation angles, results show greater immunity to noise. The Results have been further verified by using Coefficient of Variance for finding variance in the series obtained.

Keywords: Analysis of Variance, Computer Vision, Correlation, Fast Fourier Transform, Fractional Fourier Transform, Image Processing, Pattern Recognition, Template Matching.

I INTRODUCTION

Correlating different images or patterns is a task that is vital for many Engineering, Medical, Physics, Agricultural, Weather etc. applications[1]–[7]. Speed and complexity demand has increased and new algorithms and techniques are now being thought of [8] for want of Lower Space, memory and computation cost. Traditionally Correlation Filters are used to produce an indicator when matching gives a peak. Correlation is an accepted parameter for describing the resemblance between a reference pattern r(x,y) and a test pattern t(x,y),. Correlation can be found for one Dimensional or Multi-Dimensional Signals. In Image processing applications, the pattern can be plain image of target, or it can be some signature features of the target. Time and again, the two images being compared contain relative shifts and therefore the cross-correlation c(p, q) between the two images for various possible shifts p and q is computed. Then, its maximum as a metric of the comparison between the two images is selected and the position of the correlation peak is used as the estimated shift of one pattern with respect to the other. Mathematically

$$C(P,Q)=\sum_{i=0}^{n-1}\sum_{j=0}^{m-1}P(i,\mathrm{j})\,\mathrm{Q(i,\,j)} \tag{1}$$

Where C(P,Q) finds the Correlation between Images P and Q.[9]

Eq (1) can also be expressed as Template Image P rotated on Search Image Q in horizontal as well as vertical directions

$$C(\mathrm{x,\,y})=\sum_{i=0}^{n-1}\sum_{j=0}^{m-1}P(i,j)Q(i+x,\mathrm{j+\,y}) \tag{2}$$

The Correlation C, gets maximum value when "match" occurs at corresponding x and y. An efficient implementation of Cross Correlation can also be obtained by using the correlation theorem. This theorem states that computing the correlation in the spatial domain is equivalent to performing a point-wise multiplication in the Fourier domain[10]

$$C(x,y)=F^{-1}(F(\mathrm{P})*\mathrm{F}^{*}(Q)) \tag{3}$$

Here F denotes Fourier Transform, F^{-1} denotes Inverse Fourier Transform and F^{*} indicates complex conjugate. Equation (3) can be understood as the test pattern P being filtered by a filter with frequency response H(x,y) = F*(Q) to produce the output C(x y) and hence the terminology "correlation filtering" is referred to for this operation.

Some Inherent Advantages of Using FFT for finding Correlation Include Less Computational Complexity and Lesser Memory Requirements.[11]

FT has some inadequacies such as non-localization of time frequency property due to which alternative methods were considered more suitable. [12][13]. FrFT is a tool for time frequency analysis. As the signal changes, more particularly in the presence of noise, FrFT gives promising results as compared to FT.[12], [14], [15]. An effort is made in this work to find correlation using FrFT and compare the findings with FFT Rest of the paper is organized as:

Section II describes the methodology to obtain the Correlation FrFT in 2-D. Section III Describes experimental setup and method to obtain results using FrFT and FT and Section IV gives discussion and analysis about the results obtained.

II. FRACTIONAL FOURIER TRANSFORM

Fractional Fourier Transform can be defined as following [12], [14], [16]–[19] :

$$F_\alpha(\omega) = F^\alpha[f(t)](\omega) = \int_{-\infty}^{\infty} f(t) K_\alpha(\omega, t) dt \quad (4)$$

$$f(t) = \int_{-\infty}^{\infty} F_\alpha(\omega) K'_\alpha(\omega, t) d\omega \quad (5)$$

where $K_\alpha(\omega, t)$ is called transform kernel and is given by

$$K_\alpha(\omega, t) = \begin{cases} A_\alpha / \sqrt{2\pi} e^{j(t^2+u^2)/2)\cot\alpha - \mathrm{jtucsc}\alpha}, & \alpha \neq k\pi \\ \delta(t-u), & \alpha = 2k\pi \\ \delta(t+u), & \alpha = (2k+1)\pi \end{cases} \quad (6)$$

where the angle α is called as rotation angle and is similar to the factor given in Wigner Domain. and the ' denotes complex conjugation, $A\alpha = \sqrt{1 - icot\alpha}$ and Fα gives FrFT with angle α

FrFT is same as FT with $\alpha = \frac{\pi}{2}$

The FrFT definition as above can readily be applied to two dimensions by considering that the kernel is separable. The separable 2D-FrFT is reiteration of the transform in horizontal and vertical directions independently and is not the unique definition likely in two dimensions [20]. These definitions have separable kernels and their properties are similar to 1D–FrFT. The separable 2D-FrFT of orders ϕ for x-axis and γ for y-axis for 0 <ϕ <π/2 and 0 <γ <π/2, respectively, is defined as:

$$F_{\varphi,\psi}[f(x,y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) K_{\varphi\psi}(x,y,u_\varphi,v_\psi) dxdy \quad (7)$$

Here φ and ψ indicate rotation angles in x and y direction. The Integral Version of this equation as given in [16][21]

$$h(t) = \frac{A_\alpha}{\sqrt{\pi}} \int_{-\infty}^{\infty} f(\eta) g^*(\eta - \sqrt{2}t) e^{j(\cot\alpha/2)(t-\sqrt{2}\eta)^2} d\eta \quad (8)$$

Two versions are in practice to implement FrfT on digital computers. Sampling Type and Eigendecomposition Type. [12]

I : Sampling Type:

$$X_\alpha(\omega) = A_\varphi \int_{-\infty}^{\infty} e^{(j\pi(t^2\cot\varphi - 2tt'\csc\varphi + t'^2\cot\varphi))} x(t') dt' \quad (9)$$

Where

$$A_\varphi = \frac{e^{(-j\pi \mathrm{sgn}(\sin\varphi)/4 + \mathrm{j}\varphi/2)}}{\sin\varphi^{1/2}} \quad (10)$$

Where $\varphi = \alpha\pi/2$

Now the FRFT is implemented by breaking it down into chirp multiplication, after it chirp convolution and then followed by chirp multiplication.

II : Eignedecomposition Type as given in [22] [12][23]

$$F_{2\alpha/\pi} VD^{2\alpha/\pi} V^T = \begin{cases} \sum_{k=0}^{N-1} e^{-jk\alpha} v_k v_k^T, & \text{N odd} \\ (\sum_{k=0}^{N-2} e^{-jk\alpha} v_k v_k^T) + e^{-jN\alpha} v_{N-1} v_{N-1}^T & \text{N } Even \end{cases} \quad (11)$$

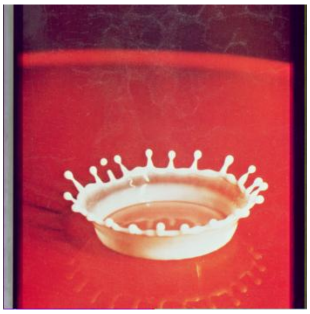
Where vk denotes the eigen vector of matrix S , vk requires k sign changes for the DFT Shifted case.

This work has used Eigen Decomposition type FrFT as it gives visually satisfying results on FrFT and Inverse FrFT.

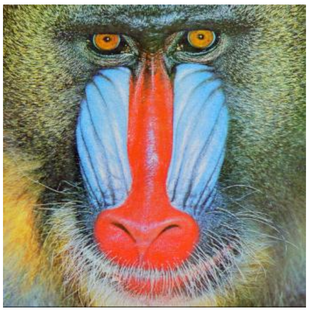
III. EXPERIMENTAL MODEL AND TESTING

Images used for Experiment Seven images from database of University of Southern California, Signal and Image processing Institute (USC-SIPI) have been used, which are provided for research work. These images are chosen at random from the database do not belong to any particular area. All images have size of 512 X 512 pixels and 8 bits / pixel for gray image and 24 bits / pixel for color images These images can be accessed at : http://sipi.usc.edu/database/ and the copyright information is provided at http://sipi.usc.edu/database/copyright.php

The Seven images used are as given below



File 1. Splash



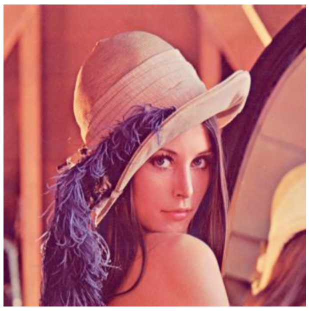
File 2. Baboon

File 3. Airplane

File 4. Sailboat

File 5. Peppers

File 6. Goldhill



File 7. Lenna

Methodology to obtain the results:

101 Load Image
102 Take section
103 Select angle of Fraction α=0.1
104 loop
105 Add Gaussian Noise to section with variance 0.01
106 Find Correlation using FFT
107 Find Correlation using FrFt at angle α
108 increase Variance of Gaussian Noise by 0.01
109 Go To Step 106
110 α= α+0.1
111 Go to Step 104
112 Repeat with different Image

Gaussian Noise has been added to the Subsection of the Image with Variance 0.01 to 0.2 in the steps of 0.1. and results calculated for each value of α (ranging from 0.1 to 0.9).

IV. RESULTS AND ANALYSIS

Results are obtained and summarized in the following figures and Tables. Different Images used in experiment have been labelled as file1 to file7.

Correlation values obtained for different α were on different scales for the same image, hence they were normalized for comparison.

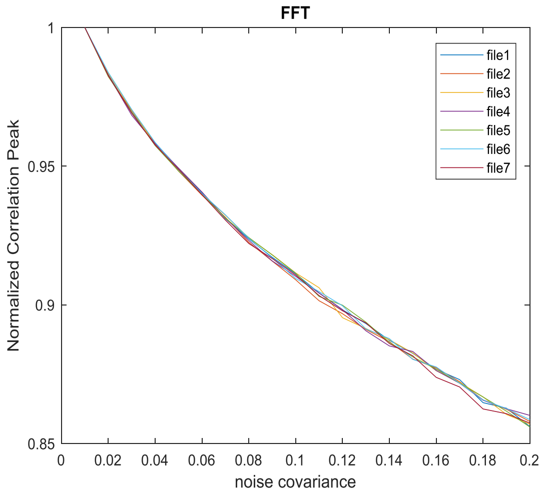


Figure 1 Normalized correlation peak using FFT

In Fig 1, correlation results using FFT for seven images have been compared. using equation (3). The Correlation peak has been plotted vs noise variance.

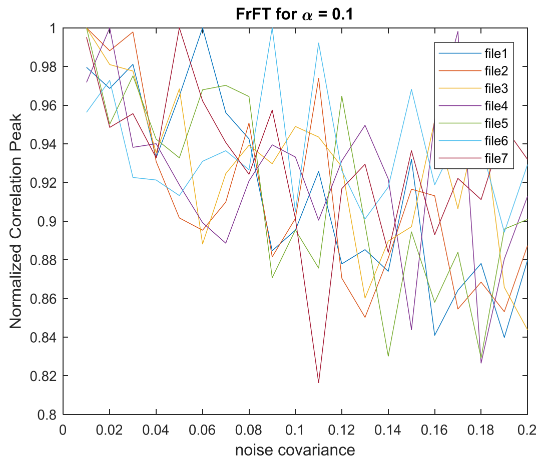


Figure 2 Normalized correlation peak using FrFT for α = 0.1

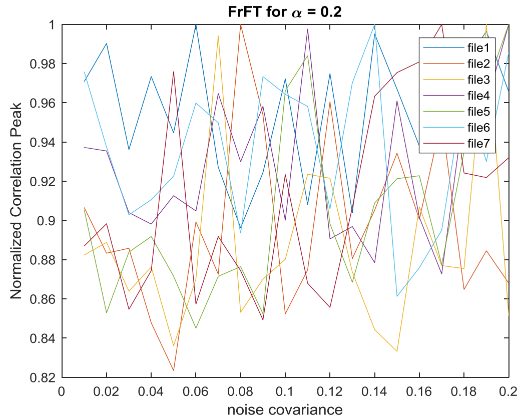


Figure 3 Normalized correlation peak using FrFT for α = 0.2

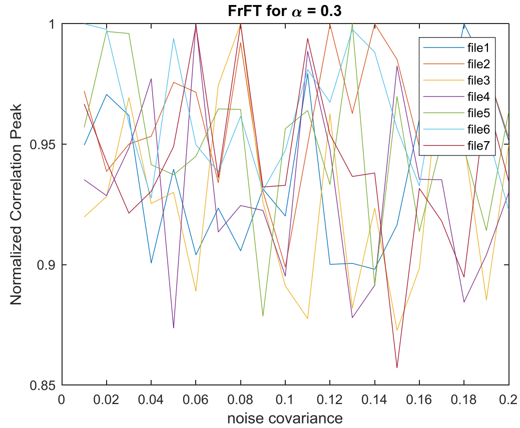


Figure 4 Normalized correlation peak using FrFT for α = 0.3

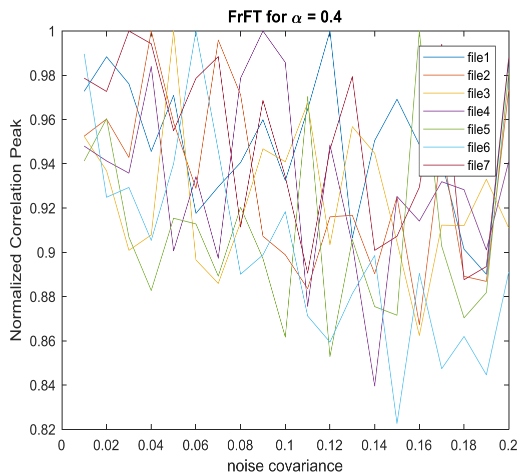


Figure 5 Normalized correlation peak using FrFT for α = 0.4

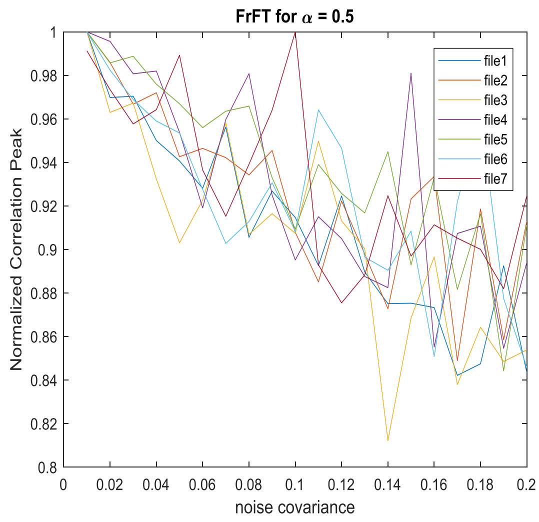


Figure 6 Normalized correlation peak using FrFT for α = 0.5

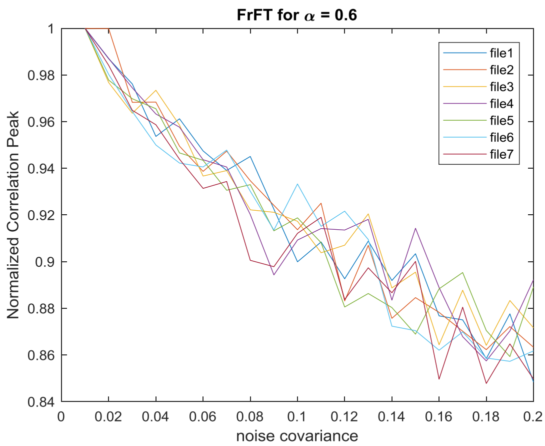


Figure 7 Normalized correlation peak using FrFT for α = 0.6

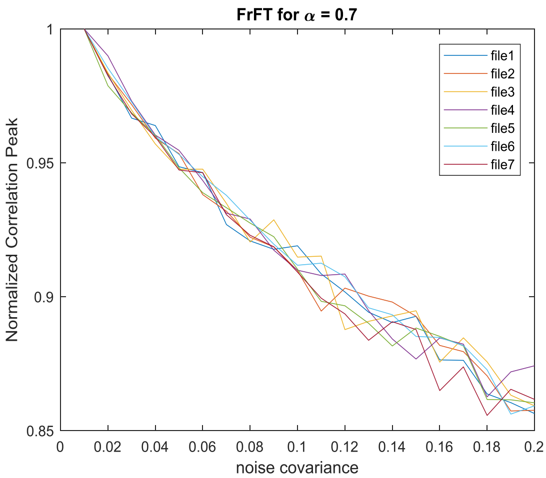


Figure 8 Normalized correlation peak using FrFT for α = 0.7

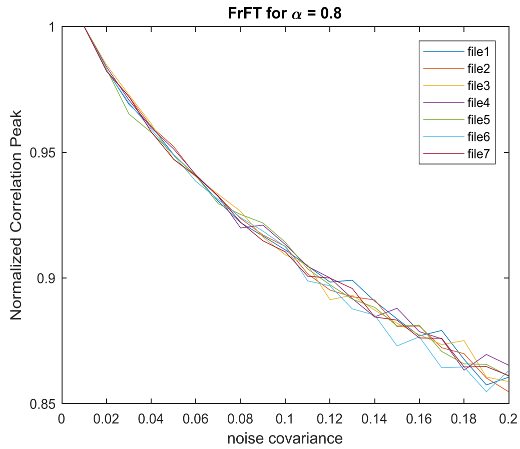


Figure 9 Normalized correlation peak using FrFT for α = 0.8

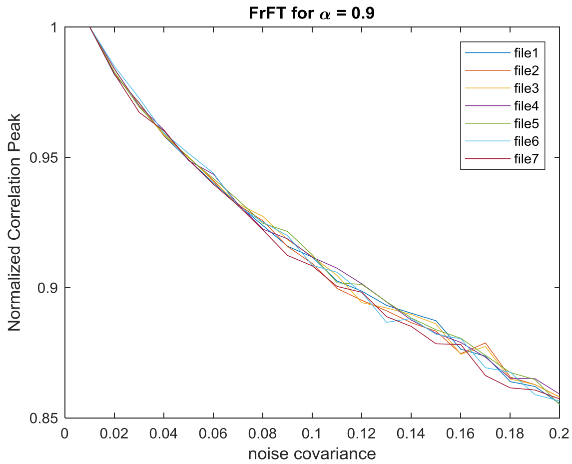


Figure 10 Normalized correlation peak using FrFT for α = 0.9

In Fig 1 it is noted that peak correlation is getting lower, as expected, when noise variance is increased for images in the experiment. Fig 2-Fig10 show the values of correlation peak obtained vs different values of noise variance using FrFT for α 0.1 to 0.9 respectively. On visual inspection it is observed that for α 0.2 and 0.3 , the graph does not show any trend in particular and is independent of the noise variance. Whereas for higher values of α, graph is showing trend similar to the one given by FFT. Table 1 summarized the above results in tabular form for one image (File 6 Goldhill) Values have been put in bold where peak of correlation is obtained for given value of noise variance. Values in the first row of table 1 are not kept bold as they already represent the value of correlation, with noise variance being minimum.

Interesting results are obtained when correlation peak is found using FrFT at α=0.2 and 0.3, it is observed that at these angles, we get higher number of normalized correlation peak values for different values of noise variance. Here, the noise variance is increasing but correlation is getting maximum values, this is in contrast to FFT where a dip in peak value of Correlation at higher noise variance is observed. For Higher Values of α, the results are very similar to Correlation using FFT.

The stability of series obtained for different values of noise variance is further investigated by Coefficient of Variance (CoV) which is widely used tool for relative measure of dispersion. It does not have any units and thus can be used to measure variability between different set of measurements better than standard deviation. It is defined as following:

Table 1. *Indicating Normalized Peak of Correlation with respected to alpha and n*

	FFT	FrFT α=0.1	FrFT α=0.2	FrFT α=0.3	FrFT α=0.4	FrFT α=0.5	FrFT α=0.6	FrFT α=0.7	FrFT α=0.8	FrFT α=0.9
n=0.01	1	0.9563	0.9759	1	0.9896	1	1	1	1	1
n=0.02	0.9838	0.9728	0.9374	**0.9976**	0.9249	0.9824	0.9803	0.9854	0.9827	0.9849
n=0.03	0.9705	0.9226	0.9029	0.9588	0.9293	0.9689	0.9644	**0.9727**	0.9697	**0.9727**
n=0.04	0.9586	0.9213	0.9106	0.9277	0.9054	0.959	0.95	**0.9598**	0.959	0.9594
n=0.05	0.9487	0.9132	0.9226	**0.9938**	0.9401	0.9535	0.9423	0.9532	0.9488	0.9515
n=0.06	0.9401	0.931	0.9599	0.9498	**1**	0.9275	0.9406	0.9448	0.9384	0.9439
n=0.07	0.9325	0.9364	**0.9499**	0.9385	0.9459	0.9028	0.9478	0.9379	0.9312	0.9313
n=0.08	0.924	0.9266	0.8936	**0.9616**	0.8901	0.9129	0.9302	0.9286	0.924	0.9251
n=0.09	0.9169	**1**	0.9734	0.9308	0.8988	0.9307	0.9135	0.9196	0.9187	0.9199
n=0.10	0.9095	0.9049	**0.9644**	0.9473	0.9183	0.9084	0.9333	0.9118	0.9124	0.9085
n=0.11	0.9048	**0.9922**	0.9582	0.981	0.8712	0.9641	0.9151	0.9126	0.8989	0.9059
n=0.12	0.8997	0.9262	0.9059	**0.9674**	0.8594	0.9465	0.9216	0.9075	0.8969	0.8981
n=0.13	0.8913	0.9011	0.9702	**0.9976**	0.8813	0.8965	0.9095	0.8959	0.8877	0.8867
n=0.14	0.8878	0.9176	**1**	0.9882	0.8985	0.8905	0.8724	0.8933	0.8852	0.8881
n=0.15	0.8804	**0.9682**	0.8613	0.9566	0.8226	0.9085	0.8704	0.8852	0.873	0.8821
n=0.16	0.8776	0.9188	0.8759	**0.9327**	0.8905	0.8509	0.862	0.8848	0.8766	0.8804
n=0.17	0.8718	0.9435	0.895	**0.9883**	0.8474	0.9218	0.8699	0.8818	0.8643	0.8692
n=0.18	0.8658	0.9391	**0.9666**	0.9533	0.862	0.9625	0.8587	0.8728	0.8646	0.8675
n=0.19	0.8626	0.8943	0.9301	**0.956**	0.8446	0.8775	0.8573	0.8562	0.8547	0.8589
n=0.20	0.8586	0.9293	**0.9855**	0.9226	0.891	0.846	0.8617	0.8593	0.8632	0.8565

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Coefficient of Variance : As per the definition given in [24]–[27], The Coefficient of Variance finds the variability of series of number. It does so independently of the unit of measurement used for the numbers. This is achieved by dividing Standard Deviation of the numbers by mean of the numbers. [25]

Coefficient of Variance is given as :

$$C_v = \frac{S}{M} \tag{12}$$

Where S is Standard Deviation and M is Mean

Table 2. CoV values for correlation using FFT and FrFT for image Goldhill

Method	α	CoV
FFT	-	4.6390
FrFT	0.1	4.2919
FrFT	0.2	4.2454
FrFT	**0.3**	**3.5988**
FrFT	0.4	4.1638
FrFT	0.5	4.1810
FrFT	0.6	4.4508
FrFT	0.7	4.6966
FrFT	0.8	4.5355
FrFT	0.9	4.6881

Table 3. *CoV values for Correlation using FFT and FrFT for image Lenna*

Method	α	CoV
FFT	-	4.5972
FrFT	0.1	3.0864
FrFT	0.2	4.1851
FrFT	**0.3**	**2.6526**
FrFT	0.4	5.0926
FrFT	0.5	4.534
FrFT	0.6	4.773
FrFT	0.7	4.508
FrFT	0.8	4.7402
FrFT	0.9	4.6993

Table 2 and Table 3. Summarize the Coefficient of Variance (CoV) for FFT and differnet values of α for FrFT for Image file 6 and 7, respectively

From Table 2 and Table 3 , at angles 0.3 the values of correlation show minimum variance, which indicates that at this angle the noise variance has little effect on correlation peak.

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