

# Magnetohydrodynamic Boundary Layer Flow with Soret/Dufour effects in presence of Heat source and Chemical Reaction

Utpal Jyoti Das<sup>1</sup> and Sonam Dorjee<sup>2</sup>

<sup>1</sup>Department of Mathematics, Gauhati University, Assam, India.

<sup>2</sup>Department of Mathematics, Rajiv Gandhi University, Arunachal Pradesh, India.

## Abstract:

This paper aims to investigate the effects of Soret and Dufour numbers in presence of heat source and chemical reaction parameter on magnetohydrodynamic boundary layer flow of an incompressible fluid past a moving porous vertical plate. A similarity transformation has been used to convert the governing partial differential equations into ordinary differential equations. The effects of physical parameters on the velocity, temperature and concentration profiles are analyzed graphically. Also, skin friction, heat transfer and mass transfer are presented in tabular form.

**Keywords:** Magnetohydrodynamic, boundary layer, Soret number, Dufour number.

## 1. INTRODUCTION

The analysis of heat and mass transfer of boundary layer flow is of great theoretical interest because it has been applied in engineering, chemical technology and industry. Sakiadis [1] initiated the study of the boundary layer flow over a continuous solid surface moving with constant speed. Raptis [2] investigated the thermal radiation effect in a steady free convective flow through a porous medium bounded by a vertical infinite porous plate. Chamkha [3] investigated the effect of heat source on hydromagnetic three dimensional free convective flow over a vertical stretching surface. Jaiswal and Soundalgekar [4] analyzed an unsteady flow past an infinite vertical plate with constant suction and oscillating plate temperature in a porous medium. Bakier[5] investigated the thermal radiation effects on stationary mixed convection from vertical surfaces in saturated porous media. Raptis and Perdikis [6] studied the effects of thermal radiation on moving vertical plate in the presence of mass diffusion. Makinde [7] studied the problem of free convection boundary layer flow with mass transfer past a moving vertical permeable plate. Cortell [8] studied the steady laminar boundary layer flow of an electrically conducting fluid in a porous medium subject to a transverse uniform magnetic field incorporating chemical reaction past a semi-infinite impermeable stretching sheet. Choudhury and Das [9] presented a theoretical study for hydromagnetic convection of a visco-elastic fluid over a continuously moving vertical surface with uniform suction. Das [10, 11] analyzed the effects of visco-elasticity of a visco-elastic, incompressible, electrically conducting fluid past a vertical plate in different physical situation.

In the present paper, we have investigated the magnetohydrodynamic boundary layer flow of incompressible fluid past a moving vertical plate in presence of heat source and chemical reaction incorporating Soret and Dufour effects.

## MATHEMATICAL FORMULATION

We consider a steady two dimensional boundary layer flow with heat and mass transfer over a continuously moving porous vertical plate in presence of heat source and first-order chemical reaction incorporating Soret and Dufour effects. All the fluid properties are assumed to be constant except for the density variations in the buoyancy force term of the momentum equation. Here  $x$ -axis is taken along the direction of the plate and  $y$ -axis is perpendicular to it. A uniform magnetic field of strength  $B_0$  is applied normal to the plate and it is assumed that the induced magnetic field is negligible. Under these assumptions, the governing equations describing the flow are:

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + g\beta_1(C - C_\infty) - \frac{\mu}{\rho K_1} u - \frac{\sigma B_0^2}{\rho} u, \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\kappa}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho C_p} (T - T_\infty) + \frac{D_m K_T}{C_s C_p} \frac{\partial^2 C}{\partial y^2}, \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} - K'_r (C - C_\infty) + \frac{D_m K_T}{T_m} \frac{\partial^2 T}{\partial y^2} \quad (4)$$

with boundary conditions

$$u = U_0 x, v = V_0, T = T_w = T_\infty + Ax, C = C_w = Bx + C_\infty \text{ at } y = 0 \quad (5)$$

$$u \rightarrow 0, v \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } y \rightarrow \infty \quad (6)$$

Where  $(u, v)$  denote the velocity components of fluid in the  $(x, y)$  directions,  $\rho$  is the fluid density,  $\nu$  is the kinematic viscosity,  $g$  is the gravitational acceleration,  $\sigma$  is the electrical conductivity,  $\beta$  is the coefficient of heat transfer,  $\beta_1$  is the coefficient of mass transfer,  $T$  is the temperature,

$C$  is the concentration,  $\mu$  is the coefficient of viscosity,  $K$  is the permeability,  $Q_0$  is the heat source,  $\kappa$  is the thermal conductivity,  $C_p$  is the specific heat at constant pressure,  $C_s$  is the concentration susceptibility,  $K_T$  is the thermal diffusion ratio,  $D_m$  is the mass diffusivity,  $K_r$  is the rate of chemical reaction,  $U_0$  is a constant,  $V_0$  is a constant,  $C_w$  is the concentration of the fluid at the plate,  $C_\infty$  is the free stream concentration,  $A$  and  $B$  denotes the stratification rate of the gradient of ambient temperature and concentration profiles.

To obtain a similarity solution of the equations (1)-(4), we introduce the following dimensionless functions

$$\eta = y\sqrt{\frac{U_0}{\nu}}, \psi = \sqrt{\nu U_0} f(\eta),$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \quad (7)$$

with the stream function  $\psi(x, y)$  defined as  $u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}$  so that the equation (1) is satisfied.

In view of (6), the equations (1)-(4) reduces to the following non-dimensional equations

$$f''' + ff'' - f'^2 + Gr\theta + Gc\phi - (H + K)f' = 0 \quad (8)$$

$$\theta'' + Pr f\theta' + Q\theta + DuPr\phi'' = 0 \quad (9)$$

$$\phi'' + Scf\phi' - K_r Sc\phi + SrSc\theta'' = 0 \quad (10)$$

with the boundary conditions

$$f' = 1, f = -f_w, \theta = 1, \phi = 1 \quad \text{at} \quad \eta = 0 \quad (11)$$

$$f' = 0, \theta = 0, \phi = 0 \quad \text{as} \quad \eta \rightarrow \infty \quad (12)$$

Where the symbol prime (') denotes derivative with respect to  $\eta$ ,

$Gr = \frac{g\beta(T_w - T_\infty)}{xU_0^2}$  is the Grashof number for heat transfer,

$Gc = \frac{g\beta_1(C_w - C_\infty)}{xU_0^2}$  is the Grashof number for mass transfer,

$H = \frac{\sigma B_0^2}{\rho U_0}$  is the Hartmann number,  $K = \frac{\nu}{K_1 U_0}$  is the permeability parameter,

$Q = \frac{Q_0 \nu}{K U_0}$  is the local heat source parameter,

$K_r = \frac{K_r'}{U_0}$  is the local chemical reaction parameter,

$Pr = \frac{\mu C_p}{\kappa}$  is the Prandtl number,  $Sc = \frac{\nu}{D_m}$  is the Schmidt number,

$Sr = \frac{D_m K_T (T_w - T_\infty)}{\nu T_m (C_w - C_\infty)}$  is the Soret number,

$Du = \frac{D_m K_T (C_w - C_\infty)}{\nu C_s C_p (T_w - T_\infty)}$  is the Dufour number,  $f_w = -\frac{V_0}{\sqrt{\nu B}}$  is the suction velocity.

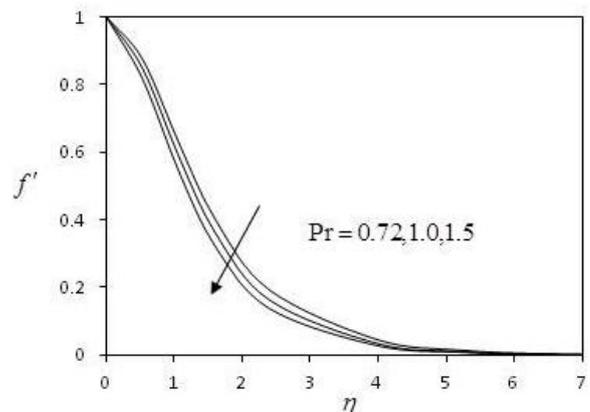
## RESULTS AND DISCUSSION

The governing equations (8)-(10) are solved under boundary conditions (11) and (12) by using MATLAB.

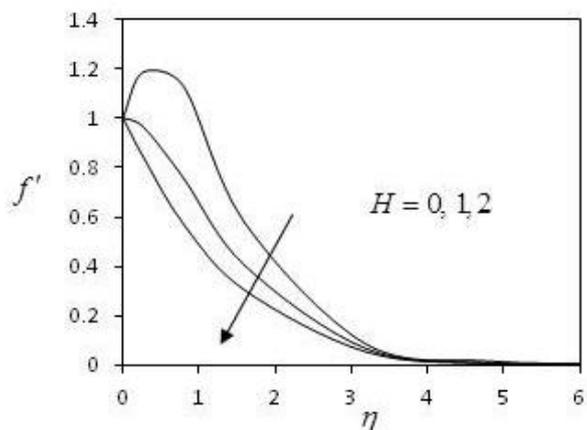
In order to get inside the problem, the following values of the parameters are considered

$H = 0.5, Gr = 1, Gc = 1, Du = 0.03, K = 0.5, K_r = 0.5, Sr = 2, Pr = 0.72, Sc = 0.62, Ec = 0.2, Q = 0.2, f_w = 0.1$  unless otherwise stated.

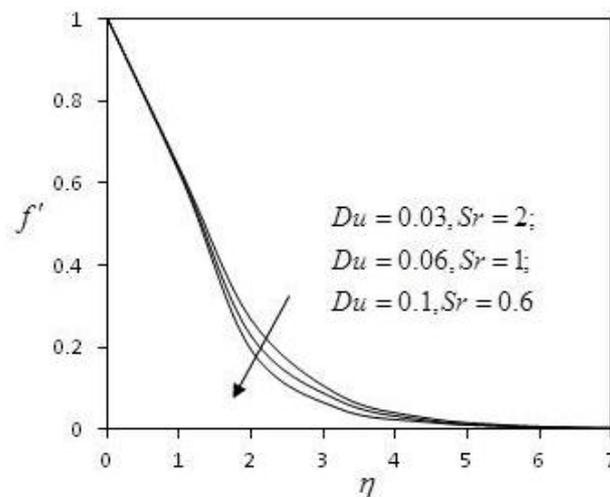
The effects of the Prandtl number ( $Pr$ ), magnetic field parameter or Hartmann number ( $H$ ), chemical reaction parameter ( $K_r$ ), heat source parameter ( $Q$ ), and Dufour/ Soret ( $Du/Sr$ ) number on velocity profiles are presented in figures 1-5, respectively. From Figure 1, it is seen that the velocity of the fluid decreases with the increasing values of Prandtl number. Figure 2 show that an increase in magnetic field parameter decreases the fluid velocity as the Lorentz force acts against the flow because the magnetic field is applied in the normal direction. Figure 3 exhibits that the fluid velocity decreases as the chemical reaction parameter increases. From Figure 4, it is seen that velocity of the fluid increases with the increasing values of heat source parameter. As the product of Dufour number and Soret number maintain a constant number, we assume that it is equal to 0.06. From the Figure 5, it is seen that an increasing values of Dufour number (or decreasing Soret number) decreases the fluid velocity.



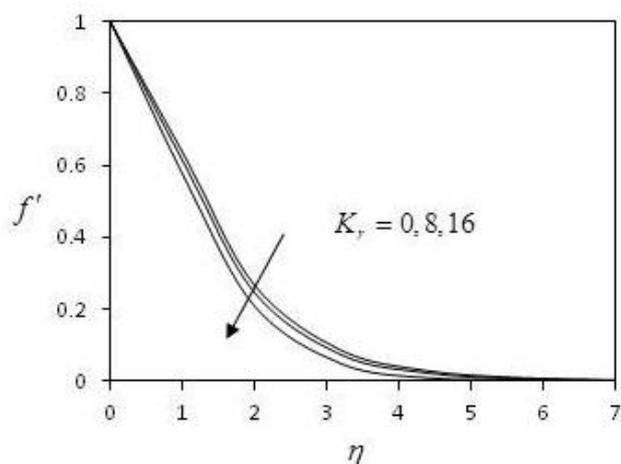
**Figure 1.** Velocity distribution for various values of Prandtl number.



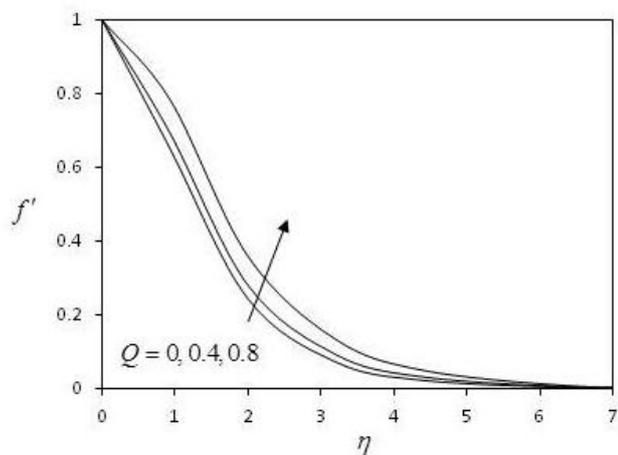
**Figure 2.** Velocity distribution for various values of Hartmann number



**Figure 5.** Velocity distribution for various values of Dufour/Soret numbers.

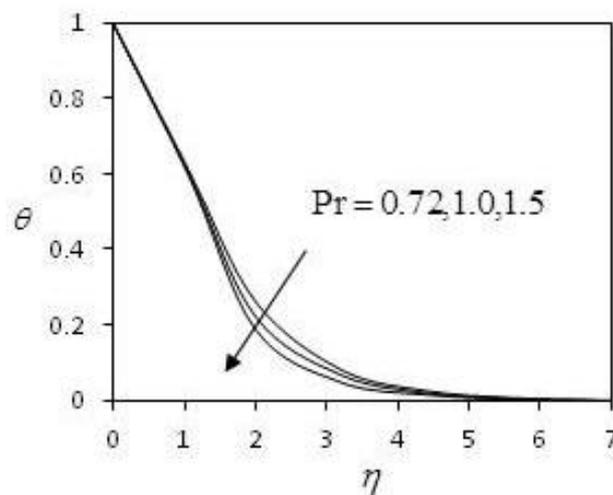


**Figure 3.** Velocity distribution for various values of chemical reaction parameter

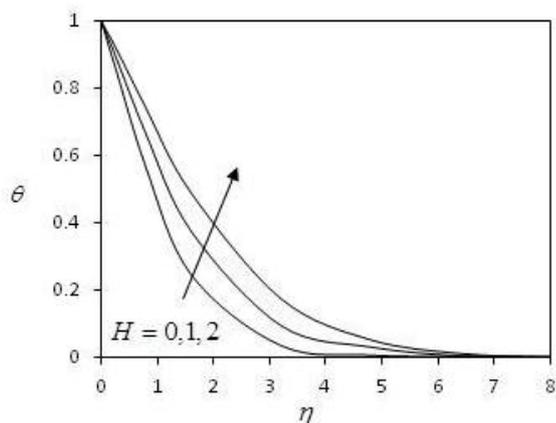


**Figure 4.** Velocity distribution for various values of heat source parameter.

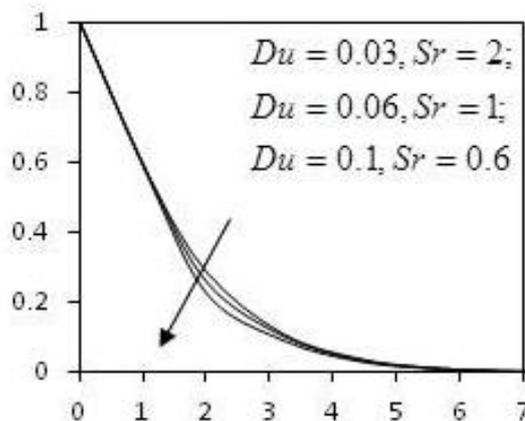
The effects of the Prandtl number ( $Pr$ ), magnetic field parameter ( $H$ ), chemical reaction parameter ( $K_r$ ), heat source parameter ( $Q$ ), and Dufour (or Soret) ( $Du$  or  $Sr$ ) number on temperature profiles are presented in Figures 6-10, respectively. From these figures, it is seen that temperature of the fluid increases with the increasing values of magnetic field parameter and heat source parameter whereas it decreases with an increasing values of Prandtl number, chemical reaction parameter, and Dufour number (or decreasing Soret number).



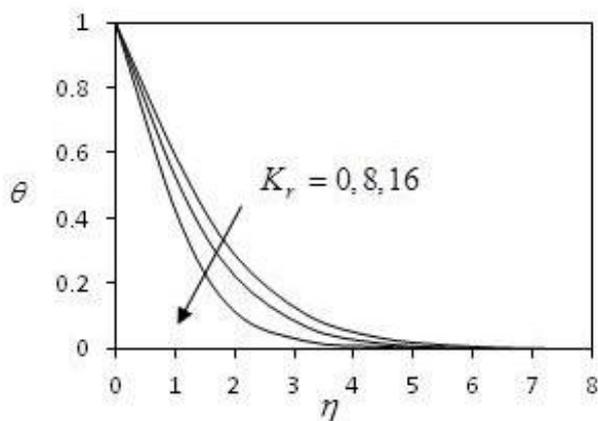
**Figure 6.** Temperature distribution for various values of Prandtl number.



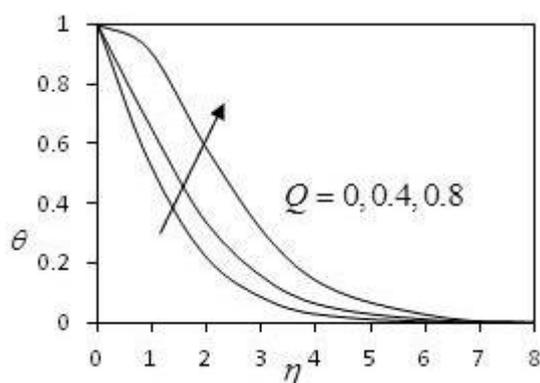
**Figure 7.** Temperature distribution for various values of Hartmann number



**Figure 10.** Temperature distribution for various values of Dufour/Soret numbers.

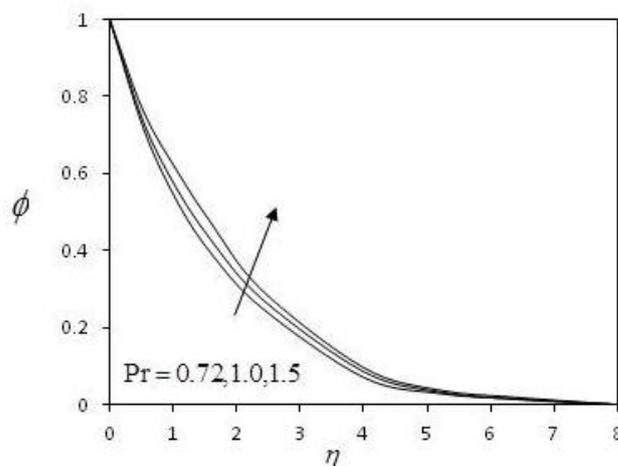


**Figure 8.** Temperature distribution for various values of chemical reaction parameter

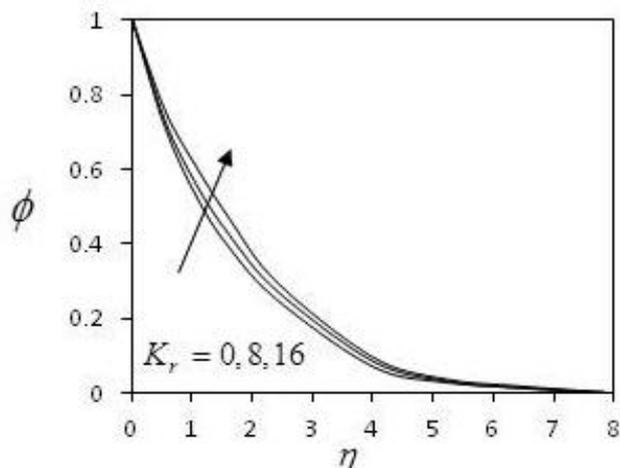


**Figure 9.** Temperature distribution for various values of heat source parameter.

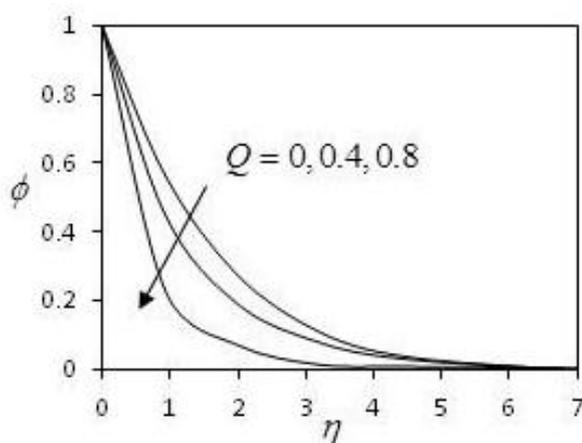
The effects of the Prandtl number ( $Pr$ ), chemical reaction parameter ( $K_r$ ), heat source parameter ( $Q$ ), and Dufour (or Soret) ( $Du$  or  $Sr$ ) number on temperature profiles are presented in Figures 11-14, respectively. From the figures, it is seen that concentration profiles increases with the increasing values of Prandtl number and chemical reaction parameter whereas it decreases with the increasing values of heat source parameter and Dufour number (or decreasing Soret number).



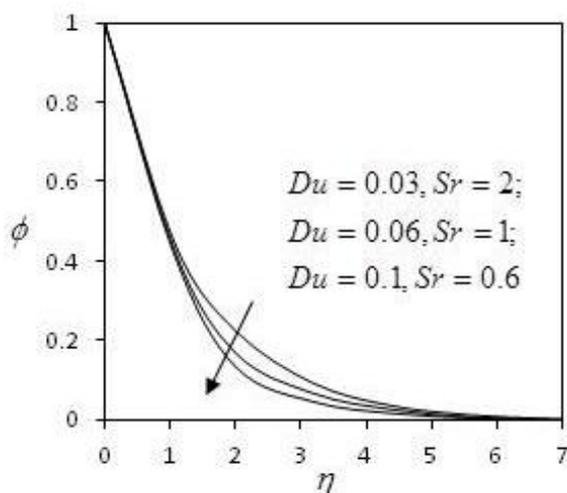
**Figure 11.** Concentration distribution for various values of Prandtl number.



**Figure 12.** Temperature distribution for various values of chemical reaction parameter



**Figure 13.** Concentration distribution for various values of heat source parameter.



**Figure 14.** Concentration dist

**Table 1.** Computations of skin friction ( $-f''(0)$ ), Nusselt number ( $-\theta'(0)$ ), and Sherwood number ( $-\phi'(0)$ ).

Pr	H	K <sub>r</sub>	Q	Du	Sr	$-f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.72	1	0.5	0.2	0.03	2	0.0113	0.3935	0.6465
1.0	1	0.5	0.2	0.03	2	0.0681	0.4734	0.6271
1.5	1	0.5	0.2	0.03	2	0.1334	0.5899	0.5945
0.72	0	0.5	0.2	0.03	2	-0.9795	0.5341	0.6086
0.72	2	0.5	0.2	0.03	2	0.5880	0.2886	0.6924
0.72	1	0	0.2	0.03	2	0.0110	0.3745	0.6436
0.72	1	8	0.2	0.03	2	0.0682	0.4902	0.6272
0.72	1	0.5	0	0.03	2	0.0735	0.5238	0.5905
0.72	1	0.5	0.4	0.03	2	0.0094	0.2926	0.8257
0.72	1	0.5	0.2	0.06	1	0.0467	0.3940	0.7370
0.72	1	0.5	0.2	0.10	0.60	0.0544	0.3943	0.7376

From the Table 1, it is seen that the skin friction increases with the increasing values of Prandtl number, Magnetic field parameter, chemical reaction parameter, and Dufour number (or decreasing Soret number), but reverse effect is seen for heat source parameter. Nusselt number increases with the increasing values of Prandtl number, chemical reaction parameter but it decreases for increasing  $Du$  (or decreasing  $Sr$ ) and increasing heat source parameter. Also, it is seen that the Sherwood number decreases with the increasing values of Prandtl number, chemical reaction parameter,  $Du$  (or decreasing  $Sr$ ), but it increases with the increasing values of magnetic field parameter, heat source parameter.

## CONCLUSION

1. Prandtl number, magnetic field parameter, chemical reaction parameter, Dufour number decreases the fluid velocity, but increase in heat source parameter increases the fluid velocity.
2. Temperature of the fluid increases with the increasing values of magnetic field parameter and heat source parameter, but it decreases with an increasing values of Prandtl number, chemical reaction parameter, and Dufour number (or decreasing Soret number).
3. Concentration profiles increases with the increasing values of Prandtl number and chemical reaction parameter, but it decreases with the increasing values of heat source parameter and Dufour number (or decreasing Soret number).

## ACKNOWLEDGEMENTS

Authors thanks to Prof. Rita Choudhury, Prof. B.R. Sharma, and Dr. D Borgohain of Gauhati University, Dibrugarh University, and Jorhat Institute of Science & Technology, Assam (India), for their help and valuable suggestions.

## REFERENCES

- [1] Sakiadis, B.C., 1961, "Boundary layer behaviour on continuous solid surfaces: I. Boundary layer equations for two dimensional and axisymmetric flow," *AICHE J.*, 7, pp. 26-28.
- [2] Raptis, A., 1998, "Radiation and free convection flow through a porous medium," *Int. Commun. in Heat and Mass Transfer*, 25, pp. 289-295.
- [3] Chamkha, A.J., 1999, "Hydromagnetic three dimensional free convection on a vertical stretching surface with heat generation or absorption," *International Journal of Heat and Fluid Flow*, 20(1), pp. 84-92.
- [4] Jaiswal, B.S., and Soundalgekar, V.M., 2001, "Oscillating plate temperature effects on a flow past an infinite vertical porous plate with constant suction and embedded in a porous medium," *Heat and Mass Transfer*, 37, pp. 125-131.
- [5] Bakier, A.Y., 2001, "Thermal radiation effect of mixed convection from vertical surfaces in saturated porous media," *Int. commun. of Heat and Mass Transfer*, 28 (1), pp. 119-126.
- [6] Raptis, A., and Perdikis, C., 2003, "Thermal radiation of an optically thin gray gas," *Int. J. Applied Mechanical Engg.*, 8(1), pp. 131-134.
- [7] Makinde, O.D., 2005, "Free convection flow with thermal radiation and mass transfer past a moving vertical porous plate," *Int. Commun. in Heat and Mass Transfer*, 32, pp. 1411-1419.
- [8] Cortell, R., 2007, "MHD flow and mass transfer of an electrically conducting fluid of second grade in a porous medium over a stretching sheet with chemically reactive species," *Chem. Eng. and Process*, 46, pp. 721-728.
- [9] Choudhury, R., and Das, U. J., 2010, "Hydromagnetic flow and heat transfer of a visco-elastic fluid on a continuously moving vertical surface," *Int. J. of Appl. Math and Mech.*, 6 (13), pp. 1-10.
- [10] Das, U.J., 2014, "Viscoelastic effects on unsteady MHD free convection and mass transfer for viscoelastic fluid flow past a hot vertical porous plate with heat generation/absorption through porous medium," *ANNALS of Faculty Engg. Hunedoara- Int. J. of Engg.*, 2, pp.165-172.
- [11] Das, U.J., 2016, "Visco-elastic effects on free convective MHD heat and mass transfer flow past a semi-infinite vertical moving plate with time dependent suction in presence of radiation and chemical reaction," 46, pp.67-72.