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# Vibration of Non-Homogeneous Parallelogram Plate (SSSS) Having Conflicting Bi-illustrative Thickness and Bi-direct Temperature Variation

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**Abstract**

In this paper authors represented a simple model to study the free vibration of orthotropic parallelogram (SSSS) plate under the effect of bi- parabolic thickness variation and linear temperature distribution in both directions. Due to non-homogeneity of the plate, density is assumed to be linear . The principal differential equation has been solved with the help of variable separable method. The approximated frequency equation is derived with the help of Rayleigh-Ritz technique by two term deflection function. The frequency values for the first and second modes of vibration have been calculated for a parallelogram (SSSS) plate for various values of aspect ratio, thermal gradient, skew angle and taper constants with the help of MAPPLE (latest computational programming tool).

**Keywords**- Vibration, bi-parabolic thickness, orthotropic, non-homogeneity, thermal gradient.

## INTRODUCTION

In engineering, the entire machines and planning structures experiences vibrations so we can't move further without contemplating the effect of vibration. The essential to know the effect of temperature on visco-versatile plates of variable thickness has ended up being basic with the advancement of technology. Tapered plates with uniform and non-uniform thickness and temperature are commonly used in vehicle division, aeronautical field, control plants and marine structure, etc. Distinctive researchers examined the vibration of different plates homogeneous or non-homogeneous having variable thickness and considering or not, the temperature effect.

An extensive review on linear vibration of simply supported elliptical and circular plates has been given by Leissa and Narita [1] , [2]. Singh and Chakraverty [3] studied flexural vibration of skew plate using boundary characterstics orthogonal polynomials in two variables.Tomar and Gupta [4] analyzed the effect of taper gradient in two dimensions on elastic plates, but not on visco-elastic plates. Nair and Durvasula [5] studied vibration of skew plates. Tomar and Gupta[6] studied temperature effect on frequency of an rectangular orthotropic plate with variable thickness in one direction. Gupta, Kumar and Gupta [7] studied vibration of visco-elastic parallelogram plate with parabolic thickness variation. Gupta, Kumar and Gupta [8] considered the effect of parabolic thickness variations on vibration of visco-elastic orthotropic parallelogram plate. Lam , Liew and Chow [9] studied free vibration analysis of isotropic and orthotropic triangular plates.

Jain and Soni [10] analyzed free vibration of rectangular plates of parabolically varying thickness. Khanna and Sharma [11] studied the vibration of visco-elastic square plate with variable thickness and thermal gradient. Sharma & Sharma [12] considered free vibration of visco-elastic orthotropic rectangular plate with bi-parabolic thermal effect and bi-linear thickness variation. Khanna & Sharma [13] studied Analysis of free vibrations of visco-elastic square plate of variable thickness with temperature effect. Khanna & Sharma [14] analyzed mechanical vibration of visco-elastic plate with thickness variation. Sharma and Sharma [15] represented the effect of bi-parabolic thermal and thickness variation on vibration of visco-elastic orthotropic rectangular plate. Sharma and Sharma [16] presented the mathematical study on vibration of visco-elastic parallelogram plate. Khanna, Sharma, Singh and Mangotra [17] considered the bi-parabolic thermal effect on vibration of visco-elastic square plate. Gupta and Lalit [18] studied effect of thermal gradient on vibration of non-homogeneous visco-elastic elliptic plate of variable thickness.

In this paper the researchers analyzed the effect of bi- linear temperature deviation and inconsistent parabolic thickness in two dimensions on the vibrations of non-homogeneous simply supported parallelogram plate (SSSS). We assume that non homogeneity happens in modulus of versatility due to temperature variation. Frequency values for the first and second mode of vibration is obtained for different numerical values of tapering constant, non-homogeneity, thermal gradient and angle proportion. Results are shown in form of diagrams and tables.

## ANALYSIS OF THE MODEL AND SOLUTION

### A. Material

The parallelogram plate R with skew angle θ and sides a,b be shown in figure 1. Since it is a special case of rectangular plate, we take θ = 0º. The plate is taken to be orthotropic and non-uniform. Here

$$\xi = x - y\tan\theta,\ \eta = y\sec\theta \tag{1.1}$$

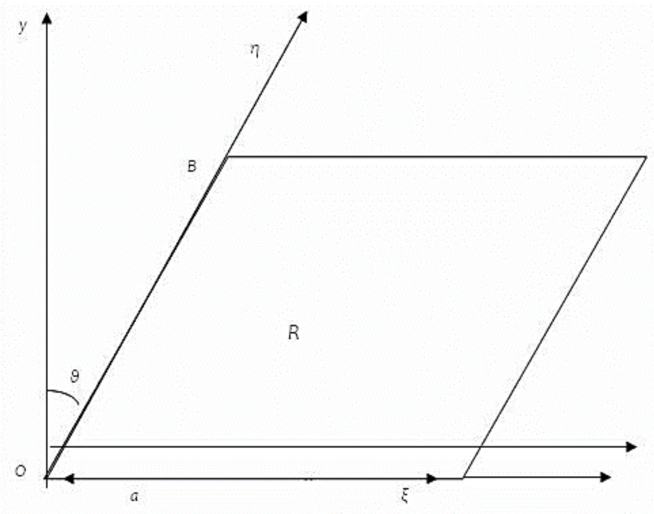


Figure (The parallelogram plate R)

Displacement $W(\xi,\eta,t)$ for free vibration of the parallelogram plate is given by

$$W(\xi,\eta,t) = W(\xi,\eta)T(t) \quad (1.2)$$

Here $W(\xi,\eta)$ is the maximum displacement at time t and T(t) is the time function.

The plate considered here is subjected to linear temperature distribution along $\xi$- and $\eta$- directions, then

$$\tau = \tau_0(1 - \frac{\xi}{a})(1 - \frac{\eta}{b}) \quad (1.3)$$

where 'a' represents length, ' b' represents breadth and $\tau_0$ is temperature at origin of the plate.

For orthotropic material, the temperature dependent modulus of elasticity is taken as

$$E_\xi(\tau) = E_1\left(1 - \gamma\,\tau\right),\ E_\eta(\tau) = E_2\left(1 - \gamma\,\tau\right),\ G_{\xi\eta}(\tau) = G_0\left(1 - \gamma\,\tau\right) \quad (1.4)$$

where $E_\xi$ and $E_\eta$ are Young's moduli in $\xi$- and $\eta$- directions respectively, $G_{\xi\eta}$ is shear modulus and $\gamma$ is taken as slope variation of moduli with temperature. Using eqn. (1.3) in eqn. (1.4) one has

$$E_\xi(\tau) = E_1\left[1 - \alpha(1 - \frac{\xi}{a})(1 - \frac{\eta}{b})\right], \quad E_\eta(\tau) = E_2\left[1 - \alpha(1 - \frac{\xi}{a})(1 - \frac{\eta}{b})\right]$$

$$G_{\xi\eta}(\tau) = G_0\left[1 - \alpha(1 - \frac{\xi}{a})(1 - \frac{\eta}{b})\right] \quad (1.5)$$

Where $\alpha = \gamma\,\tau_0$, ( $0 \le \alpha < 1$ ) is thermal gradient.

The plate's thickness variation for the present study is to be assumed parabolic in $\xi$- and $\eta$- directions which is represented by

$$h = h_0\left[\left(1 + \beta_1\,\frac{\xi^2}{a^2}\right)\left(1 + \beta_2\,\frac{\eta^2}{b^2}\right)\right] \quad (1.6)$$

Here $\beta_1$ and $\beta_2$ are known as tapering constants in $\xi$- and $\eta$- directions respectively and

$h = h_0$ at $\xi, \eta = 0$.

The flexural rigidities ($D_\xi$ , $D_\eta$) and torsional rigidity ($D_{\xi\eta}$ ) of the plate are taken as

$$D_\xi = \frac{E_\xi h^3}{12(1-v_\xi\, v_\eta)},\ D_\eta = \frac{E_\eta h^3}{12(1-v_\xi\, v_\eta)},\ D_{\xi\eta} = \frac{G_{\xi\eta} h^3}{12},$$

$$D_1 = v_\xi\, D_\eta = v_\eta\, D_\xi,\ H = D_1 + 2\,D_{\xi\eta} \quad (1.7)$$

Where $v_\xi$ , $v_\eta$ are Poisson's ratio.

Using eqns. (1.5) and (1.6) in eqn. (1.7) , we have

$$D_\xi = \frac{E_1 h_0^3}{12(1-v_\xi v_\eta)}\left[\left\{1 - \alpha\left(1 - \frac{\xi^2}{a^2}\right)\left(1 - \frac{\eta^2}{b^2}\right)\right\}\left\{\left(1 + \beta_1\,\frac{\xi^2}{a^2}\right)\left(1 + \beta_2\frac{\eta^2}{b^2}\right)\right\}^3\right],$$

$$D_\eta = \frac{E_2 h_0^3}{12(1-v_\xi v_\eta)}\left[\left\{1 - \alpha\left(1 - \frac{\xi^2}{a^2}\right)\left(1 - \frac{\eta^2}{b^2}\right)\right\}\left\{\left(1 + \beta_1\,\frac{\xi^2}{a^2}\right)\left(1 + \beta_2\frac{\eta^2}{b^2}\right)\right\}^3\right],$$

$$D_{\xi\eta} = \frac{G_0 h_0^3}{12}\left[\left\{1 - \alpha\left(1 - \frac{\xi^2}{a^2}\right)\left(1 - \frac{\eta^2}{b^2}\right)\right\}\left\{\left(1 + \beta_1\,\frac{\xi^2}{a^2}\right)\left(1 + \beta_2\frac{\eta^2}{b^2}\right)\right\}^3\right]$$
(1.8)

For non-homogeneous material, linear variation taken in density is

$$\rho = \rho_0\,(1 - c_1\frac{\xi}{a}) \quad (1.9)$$

Where $c_1$ ( $0 \le c_1 < 1$ ) is non − homogeneity constant.

**B. Frequency equation and boundary condition**

Boundary conditions for a non-homogeneous orthotropic (SSSS) parallelogram plate

are taken as

$$\left.\begin{aligned} W = W_{,\xi\xi} = 0 \text{ at } \xi = 0, a \\ W = W_{,\eta\eta} = 0 \text{ at } \eta = 0, b \end{aligned}\right\} \quad (2.1)$$

Two-term deflection function, satisfying the boundary conditions, can be taken as

$$W = [\left(\frac{\xi}{a}\right)\left(\frac{\eta}{b}\right)\left(1 - \frac{\xi}{a}\right)\left(1 - \frac{\eta}{b}\right)][A_1 + A_2\left(\frac{\xi}{a}\right)\left(\frac{\eta}{b}\right)\left(1 - \frac{\xi}{a}\right)\left(1 - \frac{\eta}{b}\right)] \quad (2.2)$$

where $A_1$, $A_2$ are constants to satisfy boundary conditions.

Now, unit less variables having no dimension are using for our convince as

$$X = \frac{\xi}{a}, Y = \frac{\eta}{b}, \hat{W} = \frac{W}{a}, H = \frac{h}{a} \quad (2.3)$$

$$E_1^* = \frac{E_1}{1-v_\xi v_\eta}, \; E_2^* = \frac{E_2}{1-v_\xi v_\eta}, \; E^* = v_\xi E_2^* = v_\eta E_1^* \quad (2.4)$$

Components of $E_1^*$, $E_2^*$, $E^*$ and $G_0$ are $E_1^*$, $E_2^*\sec\theta$, $E^*\sec\theta$ and $G_0\sec\theta$ respectively in $\xi$- and $\eta$- directions.

The expressions for strain energy ($V_E$) and kinetic energy ($T_E$) are taken as

$$V_E = \frac{1}{2}\int_0^a\int_0^b \Big[D_\xi\left(W_{,\xi\xi}\right)^2 + D_\eta\left(W_{,\xi\xi}tan^2\theta - 2W_{,\xi\eta}sec\theta tan\theta + W_{,\eta\eta}sec^2\theta\right)^2 + 2D_1 W_{,\xi\xi}(W_{,\xi\xi}tan^2\theta + 2W_{,\xi\eta}sec\theta tan\theta + W_{,\eta\eta}sec^2\theta) + 4D_{\xi\eta}\left(-W_{,\xi\xi}tan\theta + W_{,\xi\eta}sec\theta\right)^2\Big] cos\theta \, d\eta d\xi \quad (2.5)$$

and

$$T_E = \frac{1}{2} p^2 \int_0^a\int_0^b (\rho h W^2 \cos\theta) d\eta d\xi \quad (2.6)$$

### C. Solution by Rayleigh-Ritz Method

Rayleigh – Ritz technique is utilized to locate a suitable vibrational frequency. This technique chips away at the wonders that the maximum strain energy ($V_E$) must equal to maximum kinetic energy ($T_E$). An equation in the accompanying structure is acquired as

$$\delta(V_E - T_E) = 0 \quad (3.1)$$

Using eqns. (1.8), (2.3), (2.4) in eqn. (2.5) and (2.6), then substituting the values of $V_E$ and $T_E$ in

Eqn. (3.1), we obtained

$$\delta\left(V_E^* - \lambda^2 T_E^*\right) = 0 \quad (3.2)$$

Here,

$$V_E^* = \int_0^a\int_0^b \Big[\Big\{1-\alpha\left(1-\frac{\xi^2}{a^2}\right)\left(1-\frac{\eta^2}{b^2}\right)\Big\}\Big\{\left(1+\beta_1\frac{\xi^2}{a^2}\right)\left(1+\beta_2\frac{\eta^2}{b^2}\right)\Big\}^3\Big]\Big[\Big\{cos^4\theta + \frac{E_2^*}{E_1^*}sin^4\theta + 2\frac{E^*}{E_1^*}sin^2\theta cos^2\theta + 4\frac{G_0}{E_1^*}sin^2\theta cos^2\theta\Big\}W^2_{,\xi\xi} + \frac{E_2^*}{E_1^*}W^2_{,\eta\eta} + 4\Big\{\frac{E_2^*}{E_1^*}sin^2\theta + \frac{G_0}{E_1^*}cos^2\theta\Big\}W^2_{,\xi\eta} + 2\Big\{\frac{E_2^*}{E_1^*}sin^2\theta + \frac{E^*}{E_1^*}cos^2\theta\Big\}W_{,\xi\xi}W_{,\eta\eta} - 4\Big\{\frac{E_2^*}{E_1^*}sin^3\theta + 2\frac{E^*}{E_1^*}sin\theta cos^2\theta + 2\frac{G_0}{E_1^*}sin\theta cos^2\theta\Big\}W_{,\xi\xi}W_{,\xi\eta} - 4\Big\{\frac{E_2^*}{E_1^*}sin\theta\Big\}W_{,\eta\eta}W_{,\xi\eta}\Big] d\eta d\xi \quad (3.3)$$

$$T_E^* = \int_0^a\int_0^b \Big[\left(1 - c_1\frac{\xi}{a}\right)\Big\{\left(1+\beta_1\frac{\xi^2}{a^2}\right)\left(1+\beta_2\frac{\eta^2}{b^2}\right)\Big\}\Big] W^2 \, d\eta d\xi \quad (3.4)$$

and frequency $\lambda^2 = \frac{12p^2\rho_0 a^2 cos^5\theta}{E_1^* h_0^2}$

Now, the value of $A_1$ & $A_2$ is to be determined from (3.2) as

$$\frac{\partial(V_E^* - \lambda^2 T_E^*)}{\partial A_s} = 0 \quad , \quad \text{for } s = 1, 2 \quad (3.5)$$

On solving equation (3.5) , we have

$$m_{s1}A_1 + m_{s2}A_2 = 0, \quad \text{for } s = 1, 2 \quad (3.6)$$

Here $m_{s1}$, $m_{s2}$ (s = 1,2) comprises parametric constant and the frequency parameter.

The determinant of the co-efficient of equation (3.6) must be zero, for non-trivial solution,

we get the equation of frequency as follows

$$\begin{vmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{vmatrix} = 0 \quad (3.7)$$

With the help of equation (3.7), we get quadratic equation in $\lambda^2$. We can obtain two roots of $\lambda^2$ from this equation. These roots gives the first ($\lambda_1$) and second ($\lambda_2$) modes of vibration of frequency for various parameters of tapering constants, thermal gradient and aspect ratio for a simply supported plate.

## RESULT AND DISCUSSION

The frequency ($\lambda$) for 1st and 2nd mode of vibration of an orthotropic (simply supported) parallelogram plate has been calculated for various values of thermal constant ($\alpha$), tapering constant ($\beta_1$ and $\beta_2$), aspect ratio (a/b) and non-homogeneity constant ($c_1$). All the results are obtained by using MATLAB / MAPPLE software. Following parameters are used for these calculations [8]:

$\frac{E_2^*}{E_1^*} = 0.01$, $\frac{E^*}{E_1^*} = 0.3$, $\frac{G_0}{E_1^*} = 0.0333$, $\frac{E_1^*}{\rho} = 3.0 \times 10^5$, $h_0 = 0.01m$ and $\rho_0 = 0.345$

The results are shown in figures [1-5].

**Figure-1** shows thermal gradient ($\alpha$) versus frequency ($\lambda$) with fixed value of aspect ratio (a/b = 1) and different values of taper constants and non-homogeneity constant ($\beta_1 = \beta_2 = c_1$ = 0, 0.4, 0.8). It is evident from Figure-1 that as value of thermal gradient ($\alpha$) increases from 0 to 0.8 corresponding frequency value ($\lambda$) for 1st and 2nd mode of vibration decreases.

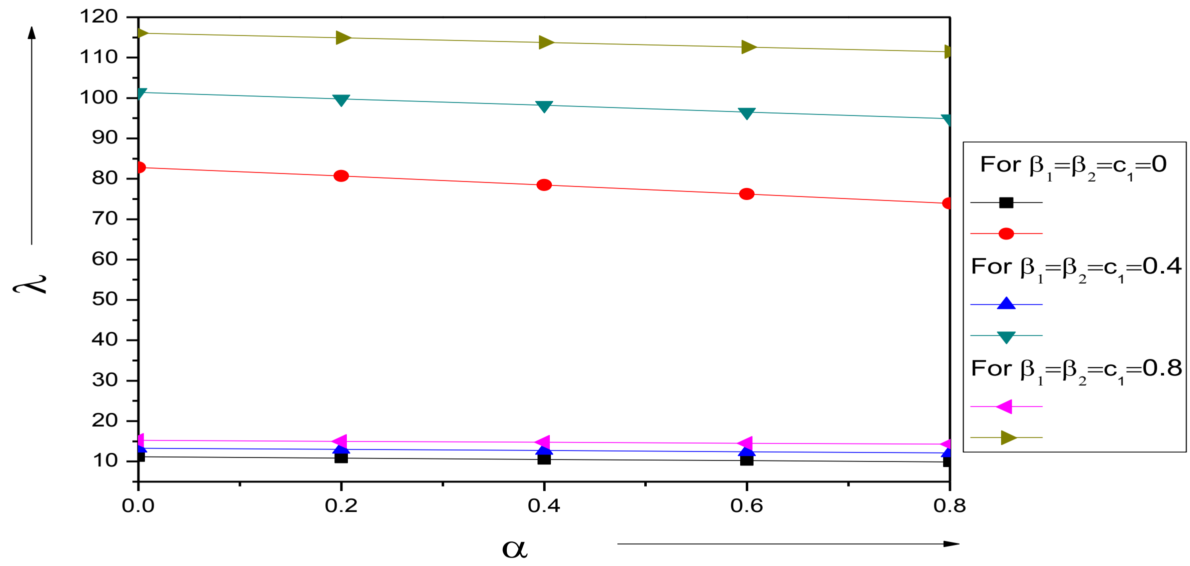


**Figure -1.** Thermal Gradient vs Frequency

**Figure-2** shows taper constant ($\beta_1$) versus Frequency ($\lambda$) with fixed value of aspect ratio (a/b = 1) and different values of thermal gradient, tapering constant and non-homogeneity ($\alpha = \beta_2 = c_1 = 0, 0.4, 0.8$). From Figure-2 it is clear that as value of tapering constant ($\beta_1$) varies from 0 to 0.8 corresponding frequency value ($\lambda$) also increases for 1st and 2nd mode of vibration.

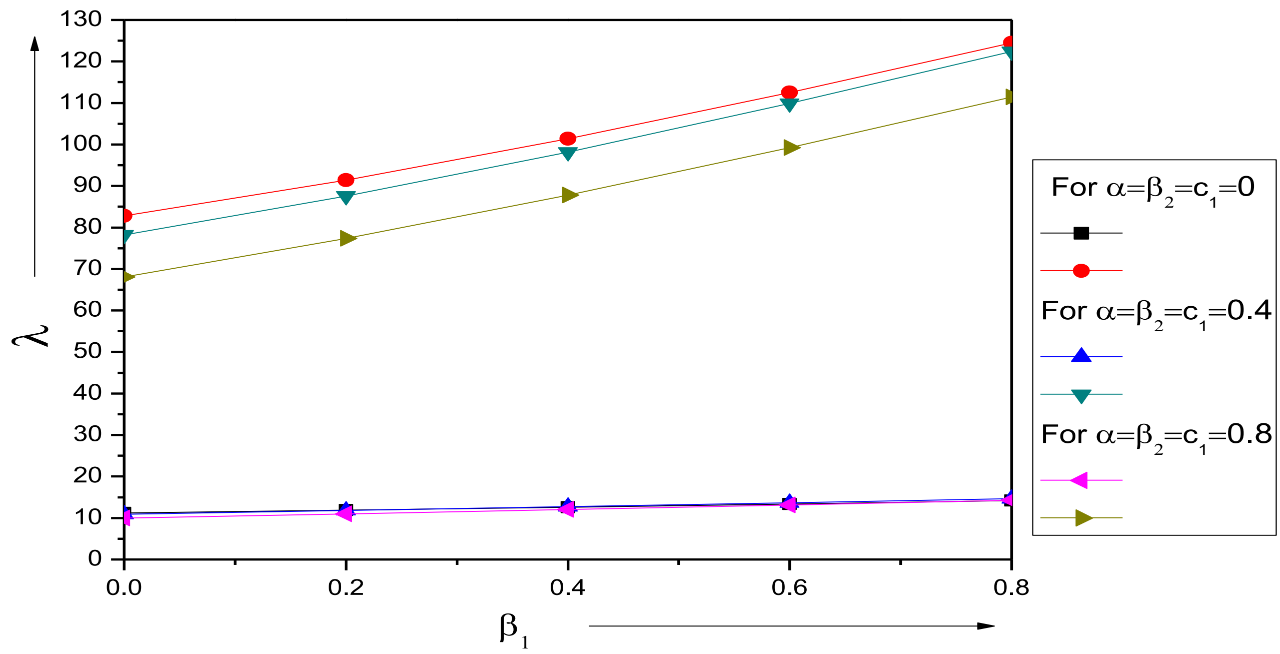


**Figure-2.** Taper Constant vs Frequency

**Figure-3** shows taper constant ($\beta_2$) versus Frequency ($\lambda$) with fixed value of aspect ratio (a/b = 1) and three different values of thermal gradient, tapering constants and non-homogeneity ($\alpha = \beta_1 = c_1 = 0, 0.4, 0.8$). From Figure-3 it is clear that as the value of tapering constant ($\beta_2$) varies from 0 to 0.8 corresponding value of frequency ($\lambda$) also increases for 1st and 2nd mode of vibration.

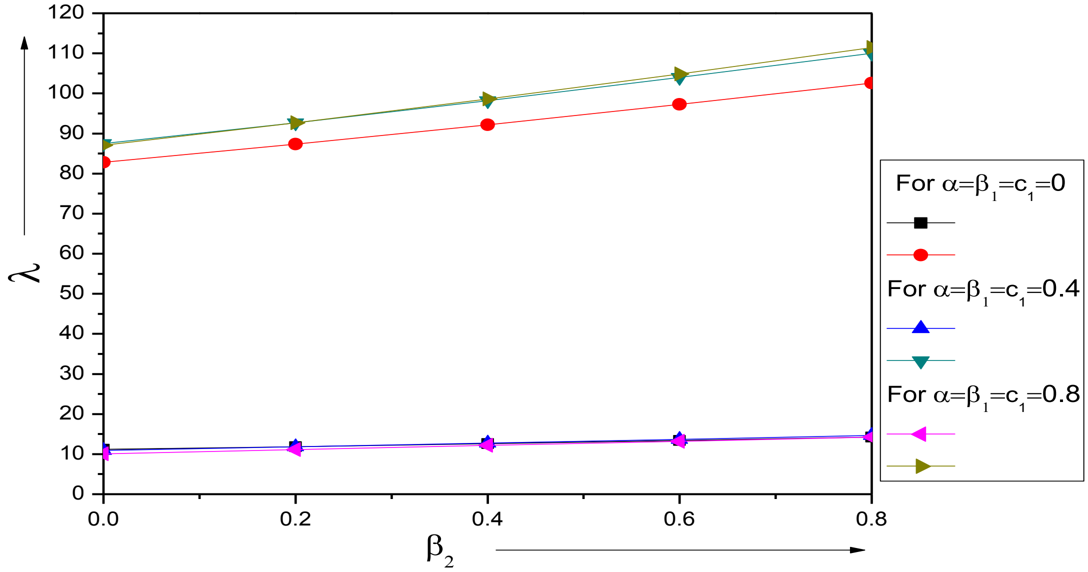


**Figure-3.** Taper Constant vs Frequency

**Figure-4** shows non-homogeneity constant ($c_1$) versus frequency ($\lambda$) with fixed value of aspect ratio (a/b = 1) and different values of tapering constants and thermal constant ($\beta_1 = \beta_2 = \alpha = 0, 0.4, 0.8$). It is evident from Figure-4 that as value of non-homogeneity constant ($c_1$) varies from 0 to 0.8 corresponding value of frequency ($\lambda$) also increases for 1st and 2nd mode of vibration.

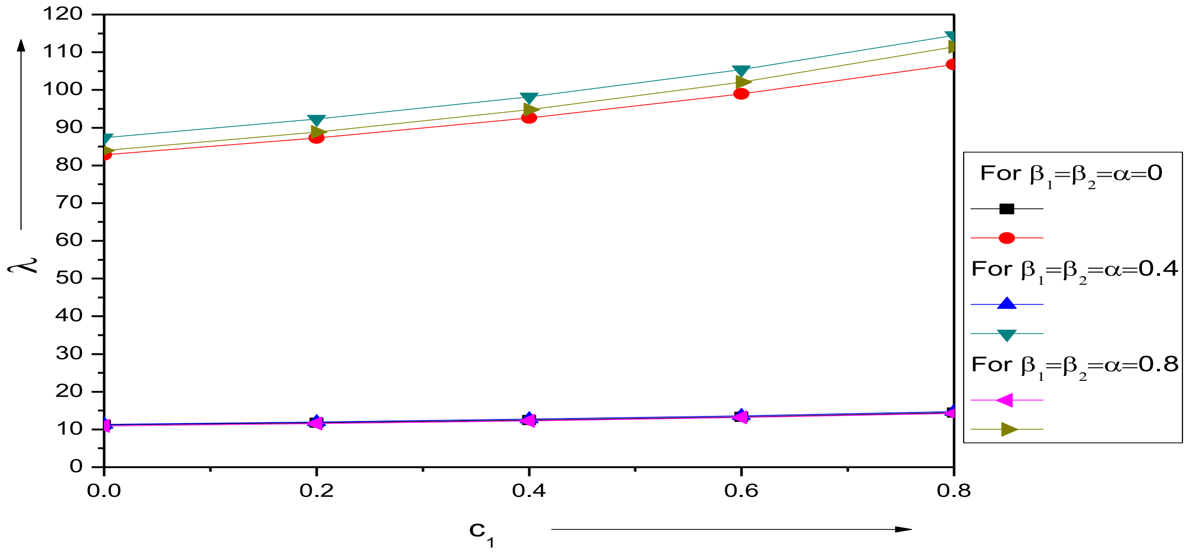


**Figure-4.** Non-homogeneity constant vs Frequency

**Figure-5** shows aspect ratio (a/b) versus frequency (λ) with various values of tapering constants, thermal constant and non-homogeneity ($\beta_1 = \beta_2 = \alpha = c_1 = 0, 0.4, 0.8$). It is evident from Figure-5 that as the value of aspect ratio increases from 0 to 1 corresponding value of frequency (λ) for 1st and 2nd mode of vibration decreases.

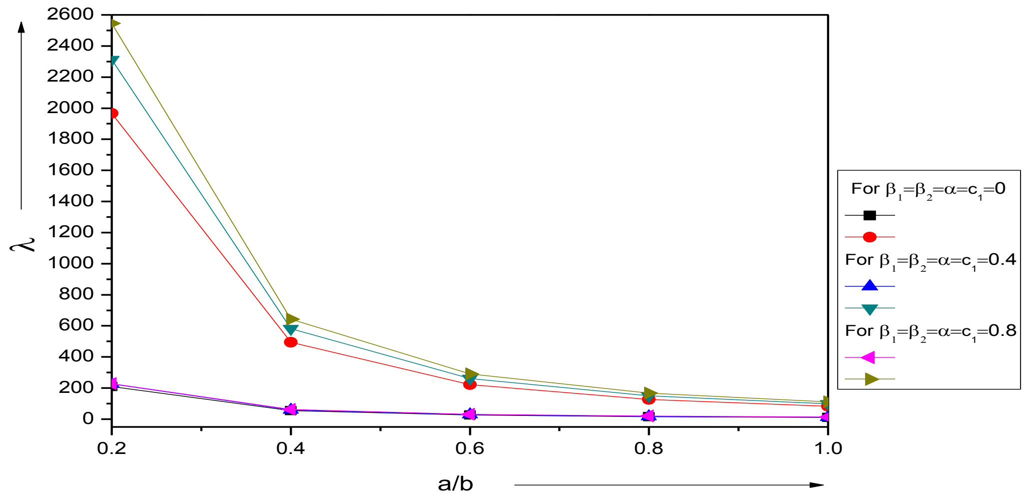


**Figure-5.** Aspect ratio vs Frequency

## CONCLUSION

The present paper describes the behavior of frequencies for first two modes of vibration corresponding to the aspect ratio, tapering constants, thermal gradient and non-homogeneity constant of the material. The authors found that the values for first mode of vibration are comparatively less than the second mode of vibration .On comparison with [16] ,it is concluded that frequency values for non-homogeneous plate (SSSS) is less than the homogeneous plate (CCCC).The frequency can be optimise by taking reasonable variation in parameters. Therefore, engineers and researchers are advised to analyze our findings and build up the plate's structure in such a way to facilitate the fundamental prerequisites.

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