

Optimizing the Problem Solving in Ordinary Differential Equations using MATLAB

Mamta Kumari

Department of mathematics, M.D.U., Rohtak, Haryana, India.

Abstract

The purpose of this paper is to represent the solutions of ordinary differential equation (ODE) using 'dsolve command' in MATLAB and discuss about graphical representation, traditional paper pencil method of solution for ordinary differential equation (ODE)

Keywords: - ODE, MATLAB, Dsolve command

1. INTRODUCTION

In mathematics, an ordinary differential equation is an equation in which involves only one independent variable and one or more derivatives of dependent variable with respect to the independent variable. The order of the ODE is the order of the highest derivative. The degree of highest derivative which appears in it is called the degree of differential equation and the linear ordinary differential equation is one in which function and all its higher order derivatives are of first degree, the non-linear ODE is the equation in which function and/or at least one of its higher order derivative of any order are multiplied together, the ODE is said to be non-linear. The homogenous equation in which the function and its higher order derivatives are equal to zero otherwise called non-homogenous ODE.

It is important to note that our focus here is on the solution of typical ordinary differential equation problems using 'dsolve command' in MATLAB. Also, give the graphical representation of solution for ODE using MATLAB and give the difference between both methods traditional paper pencil method, 'dsolve command'.

1.1 MATLAB:

"MATLAB" comprises of two words "MAT" and "LAB" where "MAT" stands for 'MATRIX' and "LAB" stands for 'LABORATORY'. Actually, it is developing by a mathematician and computer science professor at university of Michigan, university of Mexico and Stanford University started developing MATLAB in late 1970s. It is user-friendly, multipurpose software used friendly in institutions, research centers and industries due to its build in editing, debugging tools and object oriented programming support capabilities. Cleve Moler also founded Math-works, an American private corporation; with co-founder Jack little for the commercialization of MATLAB in 1984. MATLAB was written in C language and now it is available with C++.

MATLAB is case sensitive (i.e., "uppercase and lowercase. In MATLAB code, use an exact match with regard to case for variables, files and functions. If you have a variable, you cannot refer to that variable as A. It is a best practice to use lowercase only when naming function. Another example also $A + B \neq a + b$ ") high level array based language where variables can be matrix / vector / n-dimensional array with central flow statements, functions, data structures, input-output and object oriented programming features.

Actually, MATLAB was written to provide easy way of access to matrix software developed by the LINPACK (Linear system package) and EISPACK (Eigen system package) projects and matrix computation: solving system of linear equation, matrix algebra, computing eigenvalue and eigenvector, etc.

1.2 Solution of ordinary differential equation using MATLAB:

In MATLAB, the differential equation is solved in many ways but differential equation solving in a very easy manner normally by using the 'dsolve' command.

The complete syntax for this command is as follows:

$y = \text{dsolve}$ ('differential equation in the form of D operator', 'initial condition', 'variable with respect to which differentiation is being done')

Note: When initial condition is given in numerical form then we use that condition otherwise we avoid it. When the variable is not shown then MATLAB differentiate it with respect to the variable 't' by default.

In MATLAB, for write the differential equation (whose solution is required) is two ways:

1. Direct Method: In this method, the required text for the solution of differential equation (as per required format) is written in the command window step by step. The previous step is executed, we write the next step.

2. Indirect Method: In this method, the required text for the differential equation (complete program of differential equation) is written in the text editor window and saved this program as m -file (MATLAB file). This m -file is called in the command window by its name and is implemented there. There are many ordinary differential equation (ODE) solver in MATLAB, e.g., ODE 45, ODE 23 and ODE 113 etc. ODE 45 is the first choice among of all types given before. This ODE is based on explicit Runge-Kutta formula.

Example: Determine the homogeneous solution of the following differential equations: with initial condition or without initial condition. Refer MATLAB command window for the solution.

```
Command Window
>> %%Homogenous solution of Differential Equation
>> y=dsolve('D2y+y=0','t')

y =

C1*sin(t)+C2*cos(t)

>>
```

Determine the non-homogeneous solution of the following differential equations with initial conditions or without initial conditions.

Refer MATLAB command window for the solution.

```
Command Window
>> %%Non-Homogenous solution of differential equation with intial condition
>> y=dsolve('D2y+y=cos(t)','y(pi)=-1,y(0)=1','t')

y =

sin(t)*C2+cos(t)+1/2*sin(t)*t

>> |
```

Problem: The given differential equation is

$$\frac{d^2y}{dt^2} + 144y = \cos(11t) \quad \dots\dots\dots(1)$$

with initial condition $y(0) = 0, Dy(0) = 0$

The symbolic form of the equation (1) is

$$D^2y + 144y = \cos(11t)$$

$$(D^2 + 144)y = \cos(11t)$$

Therefore, A.E. is $m^2 + 144 = 0$

$$\text{or } m^2 = -144 \quad \Rightarrow \quad m = \pm i12$$

$$\therefore \text{C.F.} = c_1 \cos 12t + c_2 \sin 12t$$

Now we find P.I. of the differential equation (1).

$$\text{P.I.} = \frac{1}{D^2 + 144} \cos 11t$$

$$= \frac{1}{-121 + 144} \cos 11t$$

$$\left(\because \frac{1}{f(D^2)} \cos(ax+b); D^2 = -a^2 \right)$$

$$= \frac{1}{23} \cos 11t$$

Complete solution of the differential equation (1)

$$y(t) = c_1 \cos 12t + c_2 \sin 12t + \frac{1}{23} \cos 11t \quad \dots\dots\dots (2)$$

The initial conditions are $y(0) = 0; Dy(0) = 0$

$$-\frac{1}{23} = c_1;$$

$$y'(t) = -12c_1 \sin 12t + 12c_2 \cos 12t - \frac{11}{23} \sin 11t$$

$$0 = 12c_2 \quad \Rightarrow \quad c_2 = 0$$

Put the values of arbitrary constants c_1 and c_2 in equation (2)

$$y(t) = \frac{1}{23} \cos 11t - \frac{1}{23} \cos 12t$$

Now we solve this problem in MATLAB.

```
Command Window
>> %%Homogenous solution of Differential Equation
>> y=dsolve('D2y+144*y=cos(11*t)','y(0)=0,Dy(0)=0','t')

y =

-1/23*cos(12*t)+1/23*cos(11*t)

>> |
```

2. GRAPHICAL REPRESENTATION OF SOLUTION i.e.,

$$y = \frac{-1}{23} \left(\cos 12t + \frac{1}{23} \cos 11t \right) \text{ for ODE}$$

$$\frac{d^2y}{dt^2} + 144y = \cos(11t); y(0) = 0, y'(0) = 0 \quad \dots\dots\dots (1)$$

The actual screenshot of this equation coding in MATLAB is given below

```

Command Window
>> syms t
>> y=-1/23*cos(12*t)+1/23*cos(11*t);
>> ezplot(y, [0 2*pi])
>> xlabel('t-axis');
>> ylabel('y-axis');
>> title('y=-1/23*cos(12*t)+1/23*cos(11*t)');
>>
    
```

- Firstly, we write syms command in command window. Here main purpose of using 'syms' command for creating symbolic, numbers, variables and expression i.e., sym $\left(\frac{1}{3}\right)$, sym('y') etc. Here we have syms't' command.

- Secondly, we write the solution of ODE (1)

$$\gg y = -\frac{1}{23} * \cos(12 * t) + \frac{1}{23} * \cos(11 * t);$$

for ez plotting we maintain the expression and the range of independent variable 't'.

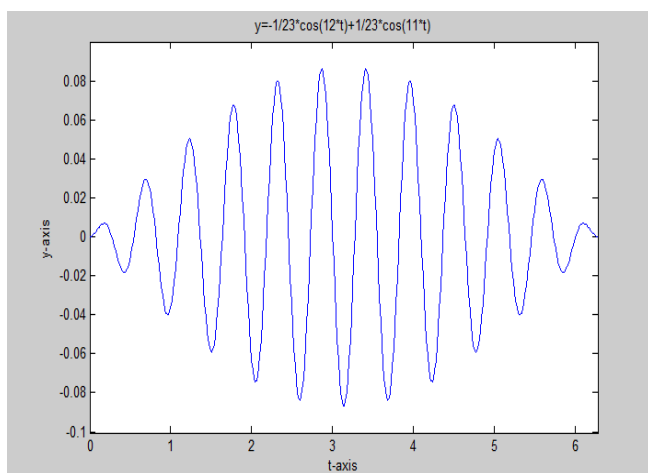
- After this, we give the name of axis and title by applying the commands.

```

>> xlabel('t-axis');
>> ylabel('y-axis');
title('y = -1/23*cos(12*t) + 1/23*cos(11*t)');
    
```

All these command are given above in command window:

Plot of ODE



Now we try to explain the graphical format of the solution of ODE. The above graph of the solution of ODE is depends upon the expression and range when we take different range in same expression, we get everywhere in new result. In the above graph we take the range of independent variable 't' is from '0 to 2π ' this means that we are plotting the graph between the range '0 to 2π ' and also the range of dependent variable 'y' is automatically explained by MATLAB corresponding to 't-axis' i.e., we get the range of 'y' from '0.1 to 0.8' corresponding the range of 't'.

Now we see that our graph is started from '0' and after that also have the range of 'y' lie between '- 0.1 to 0.8' corresponding to t-axis. When we focus on this graph, we see that graph is like a wave. Our graph is started from '0' y-axis and between the range of 't' from '0 to 6.2' i.e., '0 to 2π ' we have some peaks upward to y-axis from '0 to 0.08' and downward to y-axis from '0 to -0.08'.

3. BENEFITS OF THIS GRAPH IN MATHEMATICS

By using pattern of graph, we get easily accurate answer in less time. i.e., we discuss about the topics boundedness, continuity, differentiability, maximum value and minimum value, monotonicity, connected and convex etc.

Now we verify these terms in solution of ODE (1).

- It is a bounded graph because bounds of this graph is $t = 0$ and $t = 6.2$.
- It is a continuous function because there is no break in the given graph.
- It is also differentiable everywhere, because there is no sharp corners.
- Minimum value is $y = -0.09$ at $t = 3.14$.
- Y is not monotonic because it is piecewise monotonically increasing and piecewise monotonically decreasing.
- It is connected but not convex.

4. CONCLUSION

MATLAB is a software development environment that offers high-performance mathematical computation, visualization capabilities, application development tools, data analysis. We solve the ordinary differential equation by using 'dsolve command' in MATLAB. We see that when we solve any ordinary differential equation in MATLAB, it provides an immediate solution to even difficult lengthy differential equations. We can write statements in MATLAB and give the command according to the respective models. MATLAB then calculates the results of the problem immediately and we get the solution of the problem very soon with greater accuracy and in fraction of time. When we solve the ordinary differential equation by using traditional paper pencil method, we see that in some difficult and lengthy problems it take more time and not sure about accuracy. With the help of

MATLAB we get the graphical representation of solutions for different ordinary differential equation. This helps interpretation of solutions more easily and it's helpful, for quicker decision making. We can plot 2-D and 3-D graphs easily with the help of MATLAB. However, we cannot plot some graphs by hand like 3-D graphs and with the help of graphical representation we solve some mathematical terms, e.g., continuity and differentiability of function etc. When we solve these terms by using traditional method of continuity and differentiability of any function, we see that it take more time and not sure about accuracy. Sometime this method is not reliable for us. We take benefits of these graphical representation for solving mathematical problems like: continuity, differentiability, compactness, connectedness, convex, boundedness and monotonic functions etc.

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