

Closed-form Solution of the Combined Average SNR in General Selection Combiner

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Abstract:

In this paper, we derive a closed-form solution for the average signal-to-noise ratio (SNR) of a generalized selection diversity combiner (GSC) technique. We consider Rayleigh faded channels in which the m greatest direct SNR's diversity branches are selected and combined out of the total number of diversity branches available L . We found that the derived average SNR expression is bounded in the upper limit by the average SNR of maximum ratio combining (MRC), and lower restricted by the average SNR of traditional selection combining.

1. INTRODUCTION

At the point when talking about beamforming we utilized the antenna array of reception to administration numerous clients all the while. The interference elimination plans were detailed free of whether the channels were line-of-sight (LOS) or suffering from multipath fading. Diversity combining or reception techniques give the whole assets of the cluster to administration a single client. Particularly, diversity plans improve unwavering quality by minimizing the channel variances because of fading. The focal thought in assorted qualities is that distinctive antennas get diverse renditions of the same signal. The channels of all these duplicates being in harm fading is little. These plans hence bode well when the fading is autonomous from component to component and are of restricted utilization (past expanding the SNR) if impeccably correlated. Independent fading would emerge in a thick urban environment where the few multipath segments include diversely at every element [1-5].

Diversity combining is an well-organized way to compact the multipath fading since the combined SNR is enhanced as the number of the diversity braches increases. The maximum ratio combiner (MRC) in an optimum combiner since it's SNR is the sum of the SNR's of all single diversity branch. On the other hand, the traditional well-known selection combiner (SC) selects the signal from the diversity path with the greatest immediate SNR. Instead of choosing only the largest received signal, the general selection combiner (GSC) chooses the m highest signals from the total number of the diversity paths L and then added them in coherent way. Hence, GSC represents a compromise between the SC and MRC techniques. In [9], the probability distribution functions for an M-branch GSC system are obtained for independent and identically distributed (i.i.d.) Nakagami fading channels. In [10, 11], expressions for the moment generating function

(MGF) of M-branch GSC system are derived assuming the M branch gains are non-identically distributed. More recent analyses of variants of GSC appear in [12]-[14]. Furthermore, a generalized framework for performance analysis of selection combining diversity in Rayleigh fading channels were derived in [6-8].

It must be underscored that the Rayleigh model is the simplest and generally tractable. Be that as it may, this model is not substantial in all circumstances. The results we introduce here will thusly be just "ballpark" figures, to show the workings of diversity combining method. The physical model expects the fading independent starting with one component then onto the next. Every component, in this manner, goes about as an autonomous sample of the arbitrary fading procedure (here Rayleigh), i.e., every component of the antenna array gets a free duplicate of the transmitted signal. Our objective here is to join these autonomous samples to attain to the craved objective of enhancing the SNR and diminishing the probability of error of the system under investigation.

2. SYSTEM MODEL

Assume we have L diversity branches each of them experiences slow, flat, Rayleigh fading. The received signal from the l th diversity branch in equivalent low-pass form can be written as

$$r_{lk}(t) = \alpha_l e^{j\phi_l} u_{lk}(t) + z_l(t) \\ 0 \leq t \leq T; l = 1, 2, \dots, L; k = 1, 2, \dots, K$$

where $u_{lk}(t)$ is one of the transmitted message signal, T is the symbol period, $z(t)$ is zero-mean complex additive white Gaussian noise with a power spectrum density of N_0 , α_l is a Rayleigh random variable and ϕ_l is a uniformly distributed random variable over $[0, 2\pi]$. A MRC receiver will add coherently the L diversity paths after incrementing them by the complex conjugate of the corresponding fading signals $\alpha_l e^{-j\phi_l}$ and summing them altogether. Therefore, the resulting coherently added signal and noise of the decision signal are, respectively

$$S_c = \sum_{l=0}^L A_k \alpha_l^2$$

$$N_c = \sum_{l=0}^L z_l(t) \alpha_l e^{-j\phi}$$

which means that the instantaneous SNR for MRC is

$$\gamma_{MRC} = \frac{E[|S_c|^2]}{E[|N_c|^2]} = \sum_{l=1}^L \frac{|A_k|^2 \alpha_l^2}{2P_n} \triangleq \sum_{l=1}^L \gamma_l$$

which refers to the sum of the instantaneous SNR's of the L diversity paths (γ_l). Since α_l has a Raleigh distribution, then γ_l is exponentially distributed with a pdf and cdf, respectively, given as

$$f_{\gamma_l}(y_l) = \frac{1}{\bar{\gamma}_l} e^{-y_l/\bar{\gamma}_l} = a_l e^{-a_l y_l} \quad (1)$$

$$F_{\gamma_l}(y_l) = 1 - e^{-a_l y_l} \quad (2)$$

where $\bar{\gamma}_l = \frac{1}{a_l} = E[\alpha_l^2] \times \frac{|A_k|^2}{2P_n}$, and P_n is the power of the noise component in every diversity division. At this moment, instead of adding up signals from all the L diversity paths (as some of the diversity signals may be not strong enough to be combined), the GSC combines m ($m \leq L$) diversity branches. From above analysis, the instantaneous SNR of GSC can be written as

$$\gamma_{GSC} = \sum_{l=1}^m \gamma(l)$$

where

$$\{\gamma(1), \gamma(2), \dots, \gamma(m)\}, \gamma(1) \geq \gamma(2) \geq \dots \geq \gamma(m)$$

is an ordered SNR's set.

3. COMBAINED AVERAGE SNR OF GSC

The joint pdf of the above ordered SNR's was evaluated in [3] and given as:

$$\begin{aligned} f_{\gamma(1)\gamma(2)\dots\gamma(m)}(y_1, y_2, \dots, y_m) \\ = \sum_{i=1}^L f_{\gamma_i}(y_1) \sum_{j=1, j \neq i}^L f_{\gamma_j}(y_2) \dots \\ \sum_{n=1, n \neq i, j, \dots}^L f_{\gamma_n}(y_m) \prod_{n=1}^L F_{\gamma_n}'(y_m) \end{aligned} \quad (3)$$

where the prime denotes the unselected $L - m$ channel outputs.

To simplify the analysis, once the channel fading signals are iid (i.e.; all $f_{\gamma_i}(y_i)$ are independent with the same mean $\bar{\gamma}_i = \bar{\gamma}_j = 1/a$, $\forall i \neq j$), then (1.3) reduced to

$$\begin{aligned} f_{\gamma(1)\gamma(2)\dots\gamma(m)}(y_1, y_2, \dots, y_m) \\ = L(L-1)(L-2)\dots(L-m+1) \\ f_{\gamma}(y_1) f_{\gamma}(y_2) \dots f_{\gamma}(y_m) F_{\gamma}^{L-m}(y_m) \end{aligned} \quad (4)$$

and the average SNR for GSC is Γ_{GSC} is

$$\begin{aligned} \Gamma_{GSC} \\ = \int_0^{\infty} \int_{y_m}^{\infty} \dots \int_{y_2}^{\infty} (y_1 + y_1 + \dots + y_m) f_{\gamma(1)\gamma(2)\dots\gamma(m)} \\ (y_1, y_2, \dots, y_m) dy_1 dy_2 \dots dy_m \end{aligned} \quad (5)$$

by substituting equations (1), (2), and (4) into equation (5), it can be written as

$$\begin{aligned} \Gamma_{GSC} = \\ \frac{L!}{(L-m)!} \int_0^{\infty} \int_{y_m}^{\infty} \dots \int_{y_2}^{\infty} (y_1 + y_1 + \dots + y_m) a^m e^{-a(y_1+y_2+\dots+y_m)} \\ (1 - e^{-a y_m})^{L-m} dy_1 dy_2 \dots dy_m \end{aligned} \quad (6)$$

This integral can be solved after some considerable substitutions, and the results is just given here as

$$\Gamma_{GSC} = \bar{\gamma} \left[m + \frac{m}{m+1} + \frac{m}{m+2} + \dots + \frac{m}{L-1} + \frac{m}{L} \right] \quad (7)$$

where $\bar{\gamma}$ is the SNR of each individual diversity signal path. The lower bound of equation (7) is when $m = 1$, at which $\Gamma_{GSC} = \Gamma_{CSC}$ and

$$\Gamma_{CSC} = \bar{\gamma} \left[1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{L-1} + \frac{1}{L} \right] = \bar{\gamma} \sum_{n=1}^L \frac{1}{n} \quad (8)$$

while the upper bound is when $m = L$, at which $\Gamma_{GSC} = \Gamma_{MRC}$ and

$$\Gamma_{MRC} = \bar{\gamma} L \quad (9)$$

4. RESULTS

To assess the performance of GSC, we plot of the combined average SNR's as a function of the designated diversity paths with a total number of branches $L = 8$ and a SNR of each branch $\bar{\gamma} = 5$ dB in Fig. 1. Similar results for the case in where $\bar{\gamma} = 20$ dB is given in Fig.2. It is readily to note that the combined SNR increased exponentially as the number of the

combined branches m increased. One can also note that the SNR of GSC is upper bounded by the SNR of MRC (40 dB in

Fig. 1 and 160 dB in Fig. 2).

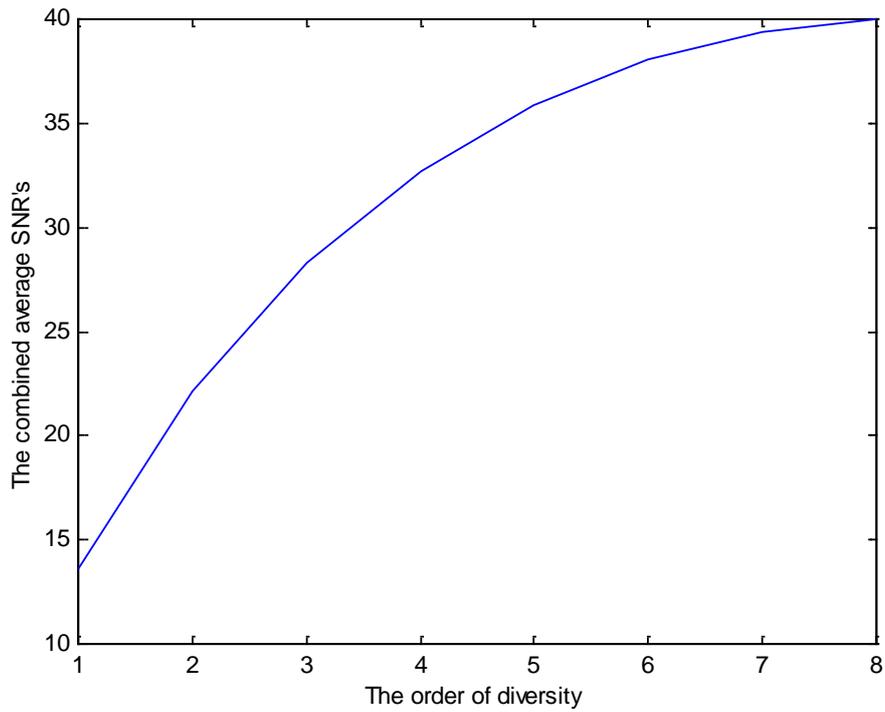


Fig. 1: Combined SNR in GSC as a function of the number of branches selected m with $L = 8$ and $\bar{\gamma} = 5$ dB.

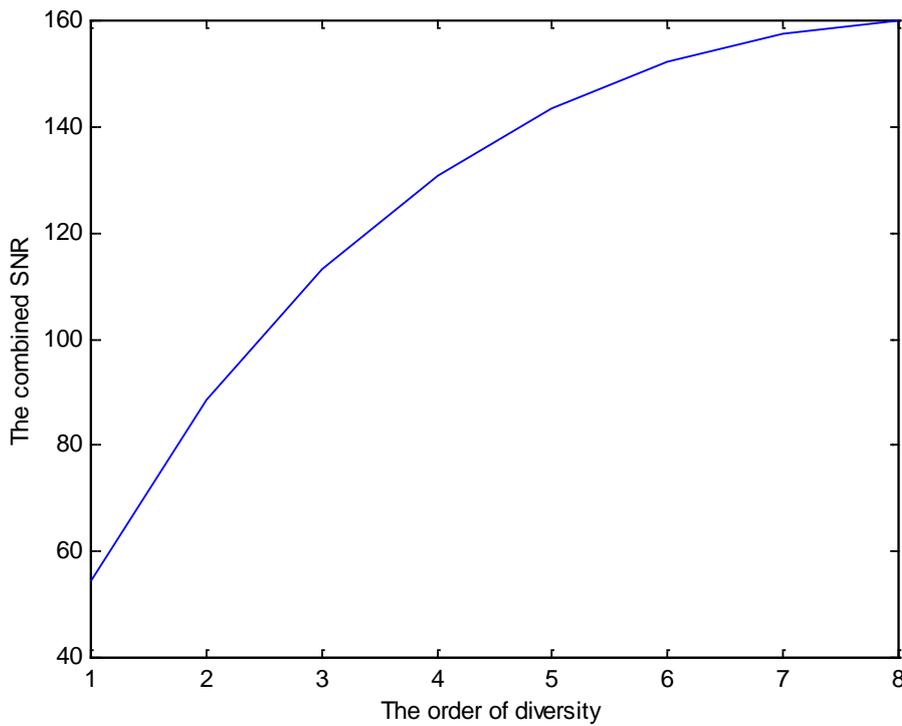


Fig. 2: Combined SNR in GSC as a function of the number of branches selected m with $L = 8$ and $\bar{\gamma} = 20$ dB.

5. CONCLUSION

In this paper, general order selection diversity in i.i.d. Rayleigh fading channels is analyzed. The expected SNR and capacity of m -th order user with transmit beamforming are analyzed with a closed-form relation for efficient computation. This expression is found to be bounded in the upper manner by that of maximum ratio combining and lower bounded by that of the well-known selection combining algorithm. It can be also shown that GSC diversity is still effective in antenna-dominant environments when the spatial channel is fully correlated

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