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Effect of Suction and Injection on Unsteady Solute Transfer in a Micropolar Fluid under the Influence of Magnetic Field

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Abstract
A micropolar fluid is flowing in a channel with suction and injection under the influence of magnetic field. An unsteady mass transfer is considered under these conditions. The governing equations arising are solved analytically and solutions obtained are graphically depicted. It us seen that the effect of micropolar fluid is to slow down convection, hence facilitating retainment of solute at the targeted region.

AMS Subject Classification: 76Rxx

Keywords: Suction, injection, micropolar fluid, magnetic field, solute transfer.

1. Introduction

Blood is the main source of delivering nutrients, drugs and oxygen to the tissue removal of wastes. Blood consists of particulate matter which cause spinning and micro rotation giving rise to an angular velocity. Blood behaves like Newtonian as well as non-Newtonian depending on circumstances. [1] have developed a model presenting theoretical study of slow of Bingham fluid in porous media. A theoretical study of blood flow in micro circulation has been considered by [2] and [3], in which blood is considered as casson fluid.

[4] have considered couple stress fluid through stenostic blood vessels. [5] have studied peristaltic flow of couple stress fluid. [6] and [7] have studied the blood flow through a stenosed catheterised artery.

In case of dialysis, flow through capillaries in tissue region involve permeable wall. [8] have studied pulsatile flow through circular tubes with varying cross section. [9] have analysed the second grade fluid flowing in a channel with effects of suction and injection and side walls. [10] have considered flow past porous boundary and got the exact solution. [11] studied the effect of couple stress on flow in a doubly connected region.

In the present study, a convective diffusive mass transfer of an injected tracer in a micropolar fluid is considered flowing through a rectangular channel with suction and injection under the influence of magnetic field.

2. Mathematical Formulation

The physical configuration shows a rectangular channel with suction and injection under the influence of magnetic field.

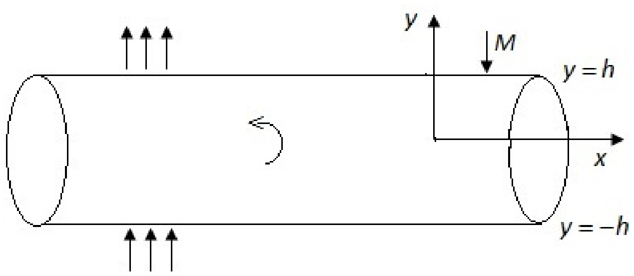

Figure 1: Physical configuration

Under the assumption of a fully developed flow, the governing equation for a micropolar fluid is given by,

$$\rho\left[v\frac{\partial u}{\partial y}\right]=-\frac{\partial p}{\partial x}+(\mu+\kappa)\frac{\partial^2 u}{\partial y^2}+\kappa\frac{\partial N}{\partial y}, \tag{1}$$

$$\left[\mu+\frac{1}{2}\kappa\right]j\frac{\partial^2 N}{\partial y^2}-\kappa\left[2N+\frac{\partial u}{\partial y}\right], \tag{2}$$

where u, N, κ, j and μ are the axial velocity, angular velocity, vortex viscosity, microinertia and dynamic viscosity respectively.

The species equation is given by,

$$\frac{\partial c}{\partial t}+u\frac{\partial c}{\partial x}=D\left[\frac{\partial^2 c}{\partial x^2}+\frac{\partial^2 c}{\partial y^2}\right], \tag{3}$$

subject to boundary conditions,
On velocity:

$$\left.\begin{aligned} u=-u_0 \ \text{at} \quad y=h\\ u=u_0 \ \text{at} \quad y=-h\\ N=0 \ \text{at} \quad y=\pm h\end{aligned}\right\}, \tag{4}$$

On concentration:

$$\left.\begin{aligned} &c(0,x,y)=c_0\\ &D\frac{\partial c}{\partial y}=-kc \ \text{at} \quad y=h\\ &D\frac{\partial c}{\partial y}=kc \ \text{at} \quad y=-h\\ &c(t,\infty,y)=\frac{\partial c}{\partial y}(t,\infty,y)=0\end{aligned}\right\}. \tag{5}$$

Non-dimensionalisation is carried out using the parameters,

$$\begin{aligned}(u^*,v^*,N^*)=\frac{(u,v,Nh)}{u_0},\ y^*=\frac{y}{h},\ t^*=\frac{Dt}{h^2},\\ x^*=\frac{Dx}{h^2u_0},\ \theta=\frac{c}{c_0},\ Pe=\frac{hu_0}{D},\ \beta=\frac{kh}{D}.\end{aligned} \tag{6}$$

and neglecting the asterisks(*) for simplicity we get,

$$Re\frac{\partial u}{\partial y}v=-P+(1+R)\frac{\partial^2 u}{\partial y^2}+R\frac{\partial N}{\partial y}, \tag{7}$$

$$\frac{\partial^2 N}{\partial y^2}-\frac{RB}{1+\frac{1}{2}R}\left[2N+\frac{\partial u}{\partial y}\right]=0, \tag{8}$$

$$\frac{\partial \theta}{\partial t}+u\frac{\partial \theta}{\partial x}=\frac{1}{Pe^2}\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}, \tag{9}$$

subject to boundary conditions,

$$\left.\begin{aligned} u=-1 \ \text{at} \quad y=1\\ u=1 \ \text{at} \quad y=-1\\ N=0 \ \text{at} \quad y=\pm 1\end{aligned}\right\}, \tag{10}$$

and

$$\left.\begin{aligned} &\theta(0,x,y)=1\\ &\frac{\partial\theta}{\partial y}=-\beta\theta \ \text{at} \quad y=1\\ &\frac{\partial\theta}{\partial y}=\beta\theta \ \text{at} \quad y=-1\\ &\theta(t,\infty,y)=\frac{\partial\theta}{\partial y}(t,\infty,y)=0\end{aligned}\right\}. \tag{11}$$

Solving analytically for velocity and angular velocity, using boundary conditions given in (10) we get,

$$\begin{aligned} N=\frac{P}{c}e^{\left(\frac{a}{3}+\alpha\right)y}&\left[\frac{cosh\left(\frac{a}{3}-\alpha\right)}{cos\beta}cos\beta y+\frac{sinh\left(\frac{a}{3}-\alpha\right)}{sin\beta}sin\beta y\right]\\ &+c_1e^{\frac{a}{3}y}\left[e^{k_1y}-e^{\alpha y}\left\{fraccosh\left(k_1-\alpha\right)cos\beta cos\beta y\right.\right.\\ &\left.\left.+\frac{sinh\left(\frac{a}{3}-\alpha\right)}{sin\beta}sin\beta y\right\}\right],\end{aligned} \tag{12}$$

and

$$\begin{aligned} u=-\frac{P}{c}&\left[\frac{c_1\Gamma_3e^{\left(\frac{a}{3}+k_1\right)y}}{\frac{a}{3}+k_1}-\frac{c_1e^{\left(\frac{a}{3}+\alpha\right)y}}{g^2+\beta^2}\right.\\ &\{\Gamma_4\left(gcos\beta y+\beta sin\beta y\right)-\Gamma_5\left(gsin\beta y-\beta cos\beta y\right)\}\\ &+c_2-\frac{e^{\left(\frac{a}{3}+\alpha\right)y}}{g^2+\beta^2}\\ &\left.\{\Gamma_1\left(gcos\beta y+\beta sin\beta y\right)+\Gamma_2\left(gsin\beta y-\beta cos\beta y\right)\}\right].\end{aligned} \tag{13}$$

The constants are listed in the appendix.

To solve the species equation the method is used by [12] is modified and adopted. Averaging θ in the region beween -1 to 1 and using the dispersion model,

$$\theta=\sum_{k=0}^{\infty}f_k(t,y)\frac{\partial^k\theta_m}{\partial x^k},$$

and

$$\frac{\partial\theta_m}{\partial t}=\sum_{j=0}^{\infty}K_j\frac{\partial^j\theta_m}{\partial x^j}. \tag{14}$$

Using the equation (16) in the equation (9) and simplifying, we get

$$\frac{\partial f_k}{\partial t}=\frac{\delta_{k2}}{Pe^2}f_{k-2}+\frac{\partial^2 f_k}{\partial y^2}-uf_{k-1}-\sum_{k=0}^{\infty}\sum_{j=0}^{k}K_jf_{k-j}. \tag{15}$$

Solving for f_0 and f_1, the resulting equations and using the condition $\int_{-1}^{1}f_0dy=1$, $\int_{-1}^{1}f_kdy=0$ for $k=1,2,3.....$ we obtain

$$f_0=\frac{\sum A_ie^{-\mu_i^2t}cos\mu_iy}{\sum\frac{A_i}{\mu_i}e^{-\mu_i^2t}sin\mu_iy}, \tag{16}$$

and

$$K_0\left(t\right)=-\frac{\sum A_n\mu_n e^{-\mu_n^2 t} sin\mu_n y}{\sum \dfrac{A_n}{\mu_n} e^{-\mu_n^2 t} sin\mu_n y}. \tag{17}$$

For large time as $t \to \infty$ we get,

$$f_0=\frac{\mu_0}{sin\mu_0},$$

and

$$K_0=-\mu_0^2. \tag{18}$$

Similarly for solving for f_1 and f_2 using,

$$\frac{\partial f_k}{\partial y}=\begin{cases}-\beta f_k & \text{at } y=1\\ \beta f_k & \text{at } y=-1\end{cases}, \tag{19}$$

we can find,

$$K_1\left(t\right)=\frac{I_1(0,0)}{1+\dfrac{sin2\mu_0}{2\mu_0}},$$

and

$$K_2\left(t\right)=\frac{1}{Pe^2}-\frac{sin\mu_0}{\mu_0\left(1+\dfrac{sin2\mu_0}{2\mu_0}\right)}\sum B_{j,1}I\left(j,0\right), \tag{20}$$

where $I(j,l) = \int_{-1}^{1} ucos\mu_j y\, cos\mu_l y\, dy$ for $j \neq l$ and $I(j,l)=\int_{-1}^{1} ucos^2\mu_j y\, dy$ for $j=l$.

The mean concentration is obtained by solving,

$$\frac{\partial\theta_m}{\partial t}=K_0(t)\theta_m+K_1(t)\frac{\partial\theta_m}{\partial x}+K_2(t)\frac{\partial^2\theta_m}{\partial x^2}, \tag{21}$$

subject to boundary conditions,

$$\theta_m\left(0,x\right)=\begin{cases}1 & \text{if } \mid x\mid\nleq L\\ 0 & \text{if } \mid x\mid> L\end{cases}, \tag{22}$$

we get

$$\theta_m=\frac{1}{\sqrt{2M}}\left[\zeta-\frac{\xi^2}{4M}\right], \tag{23}$$

where $M=K_0t,\ \ \zeta=x+K_1t$ and $\xi=K_2t.$

3. Numerical Results

In the present study, the species transport equation governing convective diffusive mass transfer of traces introduced in a micropolar fluid flowing in a rectangular channel with effects of suction and injection is studied. Using analytical methods concentration is depicted graphically for different parameters arising in the study corresponding to the values which taken from the availanle literature.

Figure 2 shows the effect of microrotation which is negligible for long time but increases with increasing microrotation parameter. But in axial direction the effect is considerable for a fixed time. At $t=0.3$ we see that initially the concentration is higher for microrotation parameter $R=2$ compared to $R=5$.

In figure 3 we can observe the intersection of curves at $X=0$ and from 0 to 1, the curve of $R=5$ is showing higher value. As the fluid flows from $X=-1$ to $X=1$ due to convection, the effect of microrotation gets reversed.

Figure 4 shows the effect of microrotaion against time. In general, the effect of micropolar fluid is to slow down the convection of drug during the process. As the suction velocity decreases, the convection along axis increases and hence concentration increases axially.

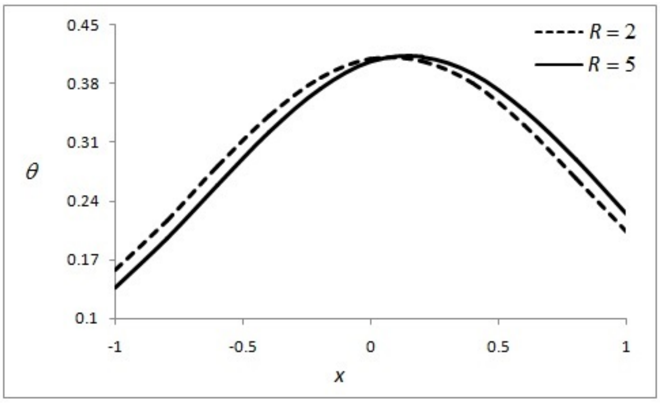

Figure 2: Concentration profile for different values of microrotation parameter at $t=0.3$

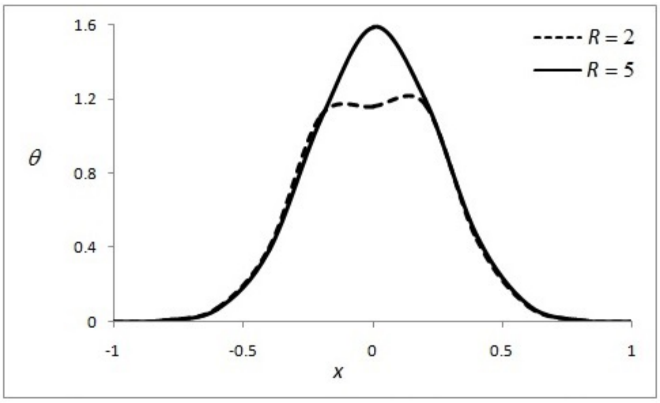

Figure 3: Concentration profile for different values of microrotation parameter at $t=0.03$

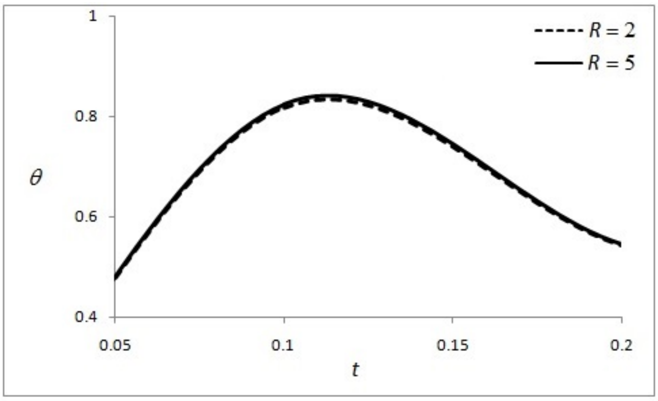

Figure 4: Concentration profile vs. time for different values of microrotation parameter

4. Conclusions

The effect of micropolar fluid under the influence of magnetic field in presence of suction and injection at walls on convective diffusive mass transfer is studied. Effect of micropolar fluid is more visible in axial direction. As time progresses, the effect of microrotation is negligible. Effect of suction is to reduce the convection.

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Appendix

$$c_1 = \frac{1}{g^2+\beta^2}\left[\frac{2c+\dfrac{2l_1}{g^2+\beta^2}}{l_2-l_3}\right],$$

$$c_2 = c - \frac{e^g}{g^2+\beta^2} l_4 + c_1 l_5,$$

$$l_1 = (\Gamma_3 g - \Gamma_2\beta)\, cos\beta sinh(g) + (\Gamma_2 g + \Gamma_3\beta)\, sin\beta cosh(g),$$

$$l_2 = \frac{2\Gamma_1}{\dfrac{a}{3}+k_1} sinh\left(\frac{a}{3}+k_1\right),$$

$$l_3 = \frac{2}{g^2+\beta^2}(\Gamma_4 g - \Gamma_2\beta)\, cos\beta sinh(g) + (\Gamma_2 g + \Gamma_4\beta)\, sin\beta cosh(g),$$

$$l_4 = \Gamma_1(gcos\beta + \beta sin\beta) + \Gamma_2(gsin\beta - \beta cos\beta),$$

$$l_5 = \frac{e^{\frac{a}{3}+k_1}}{\dfrac{a}{3}+k_1} - \frac{e^g}{g^2+\beta^2}\left[\Gamma_4(gcos\beta + \beta sin\beta) - \Gamma_5(gsin\beta - \beta cos\beta)\right],$$

$$\Gamma_1 = \frac{1+\dfrac{R}{2}}{RB}\left[\frac{cosh\left(\dfrac{a}{3}-\alpha\right)}{cos\beta}\left(g^2-\beta^2\right) + \frac{sinh\left(\dfrac{a}{3}-\alpha\right)}{sin\beta}(2g\beta)\right] + \frac{2cosh\left(\dfrac{a}{3}-\alpha\right)}{cos\beta},$$

$$\Gamma_2 = \frac{1+\dfrac{R}{2}}{RB}\left[\frac{cosh\left(\dfrac{a}{3}-\alpha\right)}{cos\beta}(-2g\beta) + \frac{sinh\left(\dfrac{a}{3}-\alpha\right)}{sin\beta}\left(g^2+\beta^2\right)\right] + \frac{sinh\left(\dfrac{a}{3}-\alpha\right)}{sin\beta},$$

$$\Gamma_3 = \frac{1+\dfrac{R}{2}}{RB}\left[\frac{a}{3}+k_1\right]^2 - 2,$$

$$\Gamma_4 = \frac{1+\dfrac{R}{2}}{RB}\left[\frac{cosh(k_1-\alpha)}{cos\beta}\left(g^2-\beta^2\right) + \frac{sinh(k_1-\alpha)}{sin\beta}(2g\beta)\right] + \frac{2cosh(k_1-\alpha)}{cos\beta},$$

$$\Gamma_5 = \frac{1+\dfrac{R}{2}}{RB}\left[\frac{cosh(k_1-\alpha)}{cos\beta}(2g\beta) - \frac{sinh(k_1-\alpha)}{sin\beta}\left(g^2+\beta^2\right)\right] + \frac{2sinh(k_1-\alpha)}{sin\beta},$$

$$g = \frac{a}{3}+\alpha, \quad \beta = -\frac{\sqrt{3}\left[2^{\frac{2}{3}}\left(a^2+3b\right) - \alpha_0^{\frac{2}{3}}\right]}{\left[2^4\alpha_0\right]^{\frac{1}{3}}}, \quad \alpha = -\frac{\left[2^{\frac{2}{3}}\left(a^2+3b\right) + \alpha_0^{\frac{2}{3}}\right]}{\left[2^4\alpha_0\right]^{\frac{1}{3}}},$$

$$k_1 = \frac{\left[2^{\frac{2}{3}}\left(a^2+3b\right) + \alpha_0^{\frac{2}{3}}\right]}{\left[2\alpha_0\right]^{\frac{1}{3}}},$$

$$\alpha_0 = 2a^3 + 9ab - 27c + 3\sqrt{81c^2 - 24abc - 12a^3c - 12b^3 - 3a^2b^2},$$

$$a = \frac{vRe}{1+R}, \quad b = \frac{RB(2+R)}{(1+R)\left(1+\dfrac{R}{2}\right)}, \quad c = \frac{2vReRB}{(1+R)\left(1+\dfrac{R}{2}\right)}.$$