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Reduced Differential Transform Method for Solving Multi-Dimensional Time-Fractional Heat Equation with Variable Co-efficient

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Abstract :
In this work, we adapted the new method for solving multi-dimensional time fractional Heat equation by using the reduced differential transform method. Hence we can obtain an analytical solution of the problem is calculated in the form of series with easily computable components. Also this method gives the effectiveness and power of the newly proposed algorithm.

Keywords : Time fractional heat equation, reduced differential transform method.

1. INTRODUCTION

In the past decades the non-linear partial differential equations were presented, which described the motion of an isolated waves in many fields such as plasma physics, hydro dynamics, localized in a small part of space, non-linear optics and so on. Thus the Investigations of exact solutions of certain non-linear PDE's are interesting and they are very important too. upto now, many authors had paid an attention to study the solutions of non-linear partial differential equations by various methods such as Darboux transformation (Wadati et al, 1975), Backlund transformation (Ablowitx and Clarkson, 1991; Coely, 2001), the tanh method (Malfeit, 1992), an inverse scattering method (Gardener et al, 1967), the Sine-Cosine Method (Yan, 1996; Yan and Zhang 2000), Hirota's bilinear method (Hirota, 1971), the Homogeneous Balance method (Wang, 1996; Yan and Zhang, 2001), extended tanh function (Fan , 2001) and the Riccati Expansion method with constant Co-efficients (Yan, 2001).

In contrast with tanh-function method, This method has many merits. This method not only yields a simpler algorithm to produce an algebraic system but this method also can provide a singular solitan solutions with no extra effort, (Fan and Zhang, 1998; Malflict, 1992; Wu et al, 1999; Satusma and Hirota, 1982 and Hirota and Satusma 1981).

Thus the numerical solution of Burger's Equation is such a great importance due to the applications in turbulence modelling (Burger's 1948) and an approximate theory of flow via a shock wave travelling in a viscous fluid (Cole, 1951). This is solved for an analytical solution for arbitrary initial condition (Hopf, 1950). Also, the finite element methods have been applied to Galerkin and Petrov-Galerkin finite element methods which are involved with time dependent grid (Herbst et al, 1982; Caldwell et al, 1981) and to the fluid dynamics problems.

A numerical Sonlutions of cubic Spline global functions were developed by (Grakes & Rubin in 1975) for obtaining the two systems or diagonally dominant equations which were sol;ved to determine the evolution of the system.

Hence the fractional differential equations are almost applied in seveal areas such as Physics, Engineering Sciences and so on. Also, many authors handled various kinds of problems or methodologies to find an exact solution of differential equations, they are Homotopy Analysis method, Pertubation method and variation of Parameter method.

In this present work, we use the reduced differential transform method RDTM, for Constructing an appropriate solution to the multi-dimentional with time fractional order. This technique is an iterative procedure for obtaining a Taylor series solutions of differential equations. It reduces the size of its computational Work and very easily applicable to many physical problems.

2. METHODOLOGY

Definition: 1

Here we discuss the basic definitions of reduced differential transform method for time-fractional. $u(x,t)$ be an analytic function $u(x,t)$ is differentiated continuously with respect to $'t'$, then

$$U_K(x) = \frac{1}{\Gamma(K\alpha+1)}\Big(\frac{\partial^{K\alpha}}{\partial t^{K\alpha}}u(x,t)\Big)_{t=0} \tag{1}$$

where $'\alpha'$ is the parameter describing an order of the time fractional derivative and the t-dimensional spectrum function $U_K(x)$ is called the transformed function.

In this research work $U_K(x)$ denotes the transformed function and $u(x,t)$ represents an original function.

Table : Time fractional reduced differential transform method.

S.No	Functional form (or) Original Form	Transformed Form
1.	$u(x,t)$	$U_K(x) = \frac{1}{\Gamma(K\alpha+1)}(\frac{\partial^{K\alpha}}{\partial t^{K\alpha}}u(x,t))_{t=0}$
2.	$u(x,t) = V(x,t) \pm W(x,t)$	$U_K(x) = V_K(x) \pm W_K(x)$
3.	$u(x,t) = V(x,t)W(x,t)$	$U_K(x) = \sum_{n=0}^{\infty} V_n W_{k-n}(x) = \sum_{n=0}^{\infty} V_n W_{k-n}$
4.	$u(x,t) = \alpha u(x,t)$	$U_K(x) = \alpha U_K(x)$
5.	$u(x,t) = (\frac{\partial^n}{\partial y^n}w(x,t))$	$U_K(x) = (K+1)....(K+n)W_{k+n}(x)$
6.	$u(x,t) = x^m y^n w(x,t)$	$U_K(x) = x^m W_{k-n}(x)$
7.	$u(x,t) = (\frac{\partial^{W\alpha}}{\partial t^W}w(x,t))$	$U_K(x) = \frac{\Gamma(K\alpha+1,N\alpha+1)}{\Gamma(K\alpha+1)}U_{K+N}(x)$

Definition: 2

The differential inverse transform of $U_K(x)$ is defined as follows.

$$u(x,t) = \sum_{k=0}^{\infty} U_K(x)t^{k\alpha} \tag{2}$$

From the equation (1) and equation (2) we obtained that,

$$u(x,t) = \sum_{k=0}^{\infty} \frac{1}{\Gamma(K\alpha+1)}[\frac{\partial^{K\alpha}}{\partial t^{K\alpha}}u(x,t)]_{t=0} t^{k\alpha} \tag{3}$$

Thus by the definition 2.1 and definition 2.2 we can understand the concepts of RDTM for time fractional, from which the derivations are obtained in the form of the power series expansions of a function.

3. NUMERICAL APPLICATIONS OF RDTM

In this section, we apply the RDTM for solving the time fractional heat equations to prove the efficiency and reliability of the proposed RDTM, some examples are presented below.

Example : 3.1

We consider the one-dimensional heat equation with variable co-efficient and time fractional order is of the form,

$$u_t^{\alpha}(x,t) - \frac{x^2}{2}u_{xx}(x,t) - u_x(x,t) = 0 \tag{3.1}$$

and the initial condition is

$$u(x,0) = x^2 \tag{3.2}$$

to find the exact solution of the equation 3.1

we apply the time fractional reduced differential transform method, from which we can find the transformed form of equation 3.1 as,

$$\frac{\Gamma(K\alpha+\alpha+1)}{\Gamma(k\alpha+1)}u_{K+1}(x) = \frac{x^2}{2}\frac{\partial^2}{\partial x^2}U_K(x) + \frac{\partial}{\partial x}U_K(x) \tag{3.3}$$

by using the initial condition, equ 3.2 we have

$$U_K(x) = x^2 \tag{3.4}$$

On substitution of eqn 3.4 onto 3.3 we can obtain the following $U_K(x)$ values successfully,

$$U_1(x) = \frac{(x^2+2x+2)}{\Gamma(\alpha+1)}$$

$$U_2(x) = \frac{1}{\Gamma(2\alpha+1)}\frac{(x^2+2x+2)}{2}$$

$$U_3(x) = \frac{1}{\Gamma(3\alpha+1)}\frac{(x^2+2x+2)}{6}$$

$$U_4(x) = \frac{1}{\Gamma(4\alpha+1)}\frac{(x^2+2x+2)}{24} \text{and so on.}$$

Therefore

$$U_K(x) = \frac{1}{\Gamma(K\alpha+1)}\frac{(x^2+2x+2)}{K!}.$$

Finally, the differential inverse transform of $U_K(x)$ gives that

$$u(x,t) = \sum_{k=0}^{\infty} U_K(x)t^{k\alpha}$$

$$\Rightarrow u(x,t) = \sum_{k=0}^{\infty} \frac{1}{\Gamma(K\alpha+1)}\frac{(x^2+2x+2)}{K!}t^{k\alpha}$$

$$\Rightarrow u(x,t) = (x^2+2x+2)\sum_{k=0}^{\infty} \frac{1}{\Gamma(K\alpha+1)}\frac{t^{k\alpha}}{K!}$$

is an exact solution of eqn 3.1

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Example : 3.2

Let us consider the two dimensional heat equation with variable co-efficients and time fractional order be

$$U_t(x,y,t) - \frac{y^2}{2}u_{xx}(x,y,t) - \frac{x^2}{2}u_{yy}(x,y,t) - u_x(x,y,t) - u_y(x,y,t) \tag{3.5}$$

with an initial condition is

$$u(x,y,0) = y^2 \tag{3.6}$$

using RDTM of equation 3.5 and its initial condition equation 3.6 is

$$\frac{\Gamma(K\alpha+\alpha+1)}{\Gamma(K\alpha+1)}U_{k+1}(x,y) = \frac{y^2}{2}\frac{\partial^2}{\partial x^2}U_K(x,y) + \frac{x^2}{2}\frac{\partial^2}{\partial y^2}U_K(x,y) + \frac{\partial}{\partial x}U_K(x,y) + \frac{\partial}{\partial y}U_K(x,y) \tag{3.7}$$

and the initial condition is

$$u_0(x,y) = y^2 \tag{3.8}$$

of $K = 0$, in 3.7 then we obtain the following values successively. they are also follows.

$$U_1(x,y) = \frac{1}{\Gamma(\alpha+1)}(x^2+2y+2)$$

$$U_2(x,y) = \frac{1}{\Gamma(2\alpha+1)}\,\frac{(y^2+2x+2)}{2}$$

$$U_3(x,y) = \frac{1}{\Gamma(3\alpha+1)}\,\frac{(x^2+2y+2)}{6}$$

$$U_4(x,y) = \frac{1}{\Gamma(4\alpha+1)}\,\frac{(y^2+2x+2)}{24}$$

$$U_5(x,y) = \frac{1}{\Gamma(5\alpha+1)}\,\frac{(x^2+2y+2)}{120}$$

$$U_6(x,y) = \frac{1}{\Gamma(6\alpha+1)}\,\frac{(y^2+2x+2)}{720}\text{and so on.}$$

Thus

$$U_K(x,y) = \begin{cases} \frac{(x^2+2y+2)}{\Gamma(K\alpha+1)K!}, & \text{K is odd} \\ \frac{(y^2+2x+2)}{\Gamma(K\alpha+1)K!}, & \text{K is even} \end{cases}$$

Finally, the time fractional reduced differential transform of $U_k(x,y)$ yields that

$$U(x,y,t) = \sum_{k=0}^{\infty}\frac{U_k(x,y)t^{\alpha K}}{\Gamma(K\alpha+1)K!} = (x^2+2y+2)\left(\frac{t^\alpha}{\Gamma(\alpha+1)} + \frac{t^{3\alpha}}{3!\Gamma(3\alpha+1)} + \frac{t^{5\alpha}}{5!\Gamma(5\alpha+1)} + ...\right)$$

$$+(y^2+2x+2)\left(1 + \frac{t^{2\alpha}}{2!\Gamma(2\alpha+1)} + \frac{t^{4\alpha}}{4!\Gamma(4\alpha+1)} + \frac{t^{6\alpha}}{6!\Gamma(6\alpha+1)} + ...\right)$$

is the required exact solution of eqn 3.5

4. CONCLUSION

Time fractional reduced differential transform method is used to find an exact solutions of one and two dimensional heat equation with variable co-efficients.

This technique is very easier to find an efficient and powerful indetermining an analytical solutions of a linear partial differential equations with time fractional order. Hence this method reveals that the complete reliability and efficiency of RDTM

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