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Consideration on Vibration of Automobile In Spatial Model With The Loss Of Contact Taken Into Account

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Abstract

This paper establishes the physical - mathematical models for considering vertical vibrations of two-axles automobiles with dependent suspensions where the loss of contact between the wheels and the road surface is taken into account. The vehicle under consideration is modelled as a spatial vibration system with seven degrees of freedom while the road is assumed undeformed and has the profile predeterministic. The wheel-road contact state is taken into account by introducing four coefficients which are called as the contact state parameters. Two typical types of road profile are described in analytical expressions and some results on wheel-road reaction forces coming from numerical computations are also presented in the paper.

Keywords: Vibration, automobile, spatial model, loss of contact, wheel separartion.

1. INTRODUCTION.

When an automobile (vehicles in general) is moving, the unevenness of the road surface and the inherent imbalance of the moving parts make it consecutively lie in vibration state, where vertical vibration has gained much attention. Vertical vibration makes changes in pressure at the contact areas, causes dynamic loads which can threaten to damage both the vehicle and the road. When the level of vibration is large enough, the wheels may separate from the road surface and cause the phenomenon of losing contact which can be also called as the loss of contact, the constraint loss, the wheel separation, or the hopping phenomenon. This will lose the control features of the vehicle in speed and direction, those reduce the safety of motion.

In the references [2, 3, 4, 5, 6, 7, 8], vibrations of automobiles according to the 7-DOF spatial model are considered, but the loss of contact between the wheels and the road surface is ignored, i.e. the assumption that the wheels always lie in the contact state with the road surface is accepted. This is not true for some practical situations. Therefore, formulation of the problem on vibrations of automobiles where the loss of contact is taken into account is needed.

This paper presents an approach to the problem mentioned above. Automobiles under consideration are two-axles ones with dependent suspensions and modelled as a 7-DOF spatial vibration system. For the first step of the approach, the road deformation is ignored and the road profile is assumed predeterministic.

2. VIBRATION MODEL OF AUTOMOBILE AND DIFFERENTIAL EQUATION OF MOTION.

2.1. Assumptions.

The following assumptions are applied to the dynamic model of the automobile:

1) the body and the two axles of the automobile are considered as absolutely rigid objects (masses);
2) the road deformation is neglected;
3) the automobile under consideration has a longitudinal symmetrical plane and the mass centers of the three masses are located in this plane;
4) the behaviors of all spring - damper couples in the model are linear;
5) the effect lines of spring and damping forces in the same couple coincide in a line perpendicular to the road nominal plane;
6) the automobile moves straight with a constant speed;
7) the profile of the road surface is predeterministic.

2.2. Vibration model of automobile.

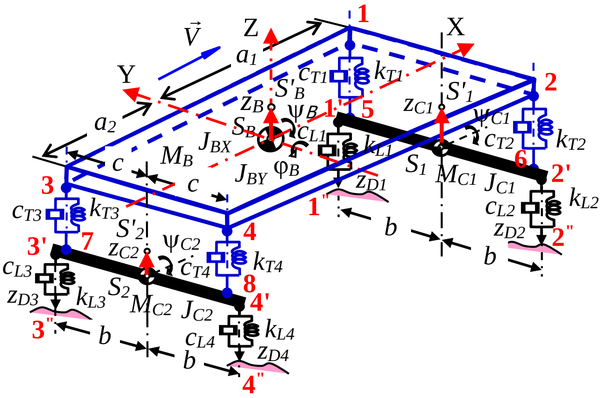

Figure1: Dynamic model of a two-axles automobile with dependent suspensions.

Based on the construction of the automobile and the assumptions mentioned above, we can make the dynamic model of the considered automobile as shown in figure 1 where the four wheels are numbered as 1, 2, 3, 4 corresponding to the front-left, the front-right, the rear-left and the rear-right ones. In the figure, vector $\vec{V}$ shows the direction of movement and the points S_B, S_1, S_2 are the mass centers of the body, the front and the rear axles, respectively. The four upper spring - damper couples represent the vehicle suspensions while the four under ones describe the elasticity and damping of the four wheels. Sixteen points numbered as 1÷8, 1'÷4' and 1"÷ 4" are the points where the spring - damping couples connect to the body, the two axles and the road surface.

In addition, M_B, M_{C1}, M_{C2}, J_{BX}, J_{BY}, J_{C1}, J_{C2} are the inertial characteristics of the vehicle body and the two axles; $\{k_{Tj}, c_{Tj}\}$, $\{k_{Lj}, c_{Lj}\}$, j=1÷4, are stiffness and damping coefficients of the suspensions and the wheels, respectively; a_1, a_2, b, c are the typical geometrical parameters; z_{Dj}, j=1÷4, is the height of the road surface at the contact point (1"÷4") of the j-th wheel and measured from the nominal plane of the road. The reference system OXYZ is mounted at the mass center of the vehicle body to describe its three vibration components.

The generalized coordinate vector describing vibration of the mechanical system with three masses (the body, the front and the rear axles) can be written as:

$$\vec{q} = [z_B, \varphi_B, \psi_B, z_{C1}, \psi_{C1}, z_{C2}, \psi_{C2}]^T \quad (1)$$

The displacement components in the generalized coordinate vector are taken from the so called natural position of the systems where all the springs lie in the natural state (not stretched or compressed) while the four wheels contact the road nominal plane.

2.3. Wheel-road interaction forces and contact state parameters.

The vertical displacements of connecting points of the spring - damper couples are expressed through the generalized coordinates as follows:

$$\begin{aligned} & z_1 = z_B + a_1\phi_B + c\psi_B,\ z_2 = z_B + a_1\phi_B - c\psi_B, \\ & z_3 = z_B - a_2\phi_B + c\psi_B,\ z_4 = z_B - a_2\phi_B - c\psi_B, \\ & z_5 = z_{C1} + c\psi_{C1},\ z_6 = z_{C1} - c\psi_{C1}, \\ & z_7 = z_{C2} + c\psi_{C2},\ z_8 = z_{C2} - c\psi_{C2}, \\ & z'_1 = z_{C1} + b\psi_{C1},\ z'_2 = z_{C1} - b\psi_{C1}, \\ & z'_3 = z_{C2} + b\psi_{C2},\ z'_4 = z_{C2} - b\psi_{C2}. \end{aligned} \quad (2)$$

Let R_{DJ} (j=1÷4) be the reaction force from the road upwards to the j-th wheel. When the wheel separates from the road surface or lies in the limit position of contact, R_{DJ} =0. When the wheel really lies in the contact state with the road surface, the reaction force R_{DJ} has the value that equals to the force acting in the spring - damper couple which represents the corresponding wheel:

$$R_{Dj} = k_{Lj}(z_{Dj} - z'_j) + c_{Lj}(\dot{z}_{Dj} - \dot{z}'_j) \quad (3)$$

The above argument hints at using the following quantity for testing the loss of contact at the j-th wheel happening or not:

$$\bar{R}_{Dj} = k_{Lj}(z_{Dj} - z'_j) + c_{Lj}(\dot{z}_{Dj} - \dot{z}'_j) \quad (4)$$

and, moreover, we get the formula for the road-wheel reaction force at the j-th wheel:

$$R_{Dj} = \begin{cases} k_{Lj}(z_{Dj} - z'_j) + c_{Lj}(\dot{z}_{Dj} - \dot{z}'_j) : \bar{R}_{Dj} \geq 0 \\ 0 : \bar{R}_{Dj} < 0 \end{cases} \quad (5)$$

Mathematically, equation (5) can also be expressed as:

$$R_{Dj} = s_j\bar{R}_{Dj} = s_j[k_{Lj}(z_{Dj} - z'_j) + c_{Lj}(\dot{z}_{Dj} - \dot{z}'_j)] \quad (6)$$

where s_j=1 if $\bar{R}_{Dj} \geq 0$ and s_j=0 if $\bar{R}_{Dj} < 0$. The parameter s_j is called as the contact state parameter at the j-th wheel, because its value (1 or 0) reflects the state of contact or separation of the wheel-road couple.

2.4. Differential equations of motion of the automobile.

The differential equations of motion of the automobile can be established by applying the Newton's second law to the three masses after releasing them from their connectors. Besides the gravity forces, the connecting forces at the four wheel spring - damper couples are calculated according to equation (6) and the connecting forces at the four suspension spring - damper couples are determined as follows (F_{Tj} is the resultant force of the suspension spring - damper couple at the j-th wheel):

$$\begin{aligned} & F_{T1} = k_{T1}(z_5 - z_1) + c_{T1}(\dot{z}_5 - \dot{z}_1) \\ & F_{T2} = k_{T2}(z_6 - z_2) + c_{T2}(\dot{z}_6 - \dot{z}_2) \\ & F_{T3} = k_{T3}(z_7 - z_3) + c_{T3}(\dot{z}_7 - \dot{z}_3) \\ & F_{T4} = k_{T4}(z_8 - z_4) + c_{T4}(\dot{z}_8 - \dot{z}_4) \end{aligned} \quad (7)$$

It is noted that the direction of the suspension spring - damper forces is upward to the body and downward to the two axles. Besides, the stiffness and damping coefficients of the suspension spring - damper couples on the same axle are equal, respectively, and so are the wheel spring - damper couples, i.e:

$$\begin{aligned} & k_{T1}=k_{T2}=k_{Tf},\ k_{T3}=k_{T4}=k_{Tr},\ k_{L1}=k_{L2}=k_{Lf},\ k_{L3}=k_{L4}=k_{Lr}, \\ & c_{T1}=c_{T2}=c_{Tf},\ c_{T3}=c_{T4}=c_{Tr},\ c_{L1}=c_{L2}=c_{Lf},\ c_{L3}=c_{L4}=c_{Lr}. \end{aligned} \quad (8)$$

The differential equations of motion of the considered automobile obtained after applying Newton's second law to the three masses and doing some arrangements are as follows:

$$\begin{aligned} & M_B\ddot{z}_B + 2(c_{Tf} + c_{Tr})\dot{z}_B + 2(c_{Tf}a_1 - c_{Tr}a_2)\dot{\varphi}_B - 2c_{Tf}\dot{z}_{C1} \\ & \quad - 2c_{Tr}\dot{z}_{C2} + 2(k_{Tf} + k_{Tr})z_B + 2(k_{Tf}a_1 - k_{Tr}a_2)\varphi_B \\ & \quad - 2k_{Tf}z_{C1} - 2k_{Tr}z_{C2} = -M_B g \end{aligned} \quad (9)$$

$$\begin{aligned} & J_{BY}\ddot{\varphi}_B + 2(c_{Tf}a_1 - c_{Tr}a_2)\dot{z}_B + 2(c_{Tf}a_1^2 + c_{Tr}a_2^2)\dot{\varphi}_B \\ & \quad - 2c_{Tf}a_1\dot{z}_{C1} + 2c_{Tr}a_2\dot{z}_{C2} + 2(k_{Tf}a_1 - k_{Tr}a_2)z_B \\ & \quad + 2(k_{Tf}a_1^2 + k_{Tr}a_2^2)\varphi_B - 2k_{Tf}a_1z_{C1} + 2k_{Tr}a_2z_{C2} = 0 \end{aligned} \quad (10)$$

$$J_{BX}\ddot{\psi}_B + 2(c_{Tf} + c_{Tr})c^2\dot{\psi}_B - 2c_{Tf}c^2\dot{\psi}_{C1} - 2c_{Tr}c^2\dot{\psi}_{C2} + 2(k_{Tf} + k_{Tr})c^2\psi_B - 2k_{Tf}c^2\psi_{C1} - 2k_{Tr}c^2\psi_{C2} = 0 \quad (11)$$

$$M_{C1}\ddot{z}_{C1} - 2c_{Tf}\dot{z}_B - 2c_{Tf}a_1\dot{\varphi}_B + [2c_{Tf} + (s_1+s_2)c_{Lf}]\dot{z}_{C1} + (s_1-s_2)c_{Lf}b\dot{\psi}_{C1} - 2k_{Tf}z_B - 2k_{Tf}a_1\varphi_B + [2k_{Tf} + (s_1+s_2)k_{Lf}]z_{C1} + (s_1-s_2)k_{Lf}b\psi_{C1} = -M_{C1}g + s_1(k_{Lf}z_{D1} + c_{Lf}\dot{z}_{D1}) + s_2(k_{Lf}z_{D2} + c_{Lf}\dot{z}_{D2}) \quad (12)$$

$$J_{C1}\ddot{\psi}_{C1} - 2c_{Tf}c^2\dot{\psi}_B + (s_1-s_2)c_{Lf}b\dot{z}_{C1} + [2c_{Tf}c^2 + (s_1+s_2)c_{Lf}b^2]\dot{\psi}_{C1} - 2k_{Tf}c^2\psi_B + (s_1-s_2)k_{Lf}bz_{C1} + [2k_{Tf}c^2 + (s_1+s_2)k_{Lf}b^2]\psi_{C1} = [s_1(k_{Lf}z_{D1} + c_{Lf}\dot{z}_{D1}) - s_2(k_{Lf}z_{D2} + c_{Lf}\dot{z}_{D2})]b \quad (13)$$

$$M_{C2}\ddot{z}_{C2} - 2c_{Tr}\dot{z}_B + 2c_{Tr}a_2\dot{\varphi}_B + [2c_{Tr} + (s_3+s_4)c_{Lr}]\dot{z}_{C2} + (s_3-s_4)c_{Lr}b\dot{\psi}_{C2} - 2k_{Tr}z_B + 2k_{Tr}a_2\varphi_B + [2k_{Tr} + (s_3+s_4)k_{Lr}]z_{C2} + (s_3-s_4)k_{Lr}b\psi_{C2} = -M_{C2}g + s_3(k_{Lr}z_{D3} + c_{Lr}\dot{z}_{D3}) + s_4(k_{Lr}z_{D4} + c_{Lr}\dot{z}_{D4}) \quad (14)$$

$$J_{C2}\ddot{\psi}_{C2} - 2c_{Tr}c^2\dot{\psi}_B + (s_3-s_4)c_{Lr}b\dot{z}_{C2} + [2c_{Tr}c^2 + (s_3+s_4)c_{Lr}b^2]\dot{\psi}_{C2} - 2k_{Tr}c^2\psi_B + (s_3-s_4)k_{Lr}bz_{C2} + [2k_{Tr}c^2 + (s_3+s_4)k_{Lr}b^2]\psi_{C2} = [s_3(k_{Lr}z_{D3} + c_{Lr}\dot{z}_{D3}) - s_4(k_{Lr}z_{D4} + c_{Lr}\dot{z}_{D4})]b \quad (15)$$

If the loss of contact is not taken, the differential equations of motion of the vehicle are obtained from the equations (9) ÷ (15) by setting $s_j = 1$ ($j = 1 \div 4$).

2.5. Matrix form of the differential equations of motion.
The differential equations of motion of the vehicle can be written in matrix form:

$$[M]\ddot{\vec{q}} + [C]\dot{\vec{q}} + [K]\vec{q} = \vec{F} \quad (16)$$

where $[M]$, $[C]$, $[K]$ are, respectively, the mass, damping and stiffness matrices of size 7×7, $\vec{q}$ is the generalized coordinate vector which is determined according to equation (1) and $\vec{F}$ is the excitation vector.
These matrices and vectors can be written as follows:
- The mass matrix $[M]$ is a diagonal one:

$$[M] = \mathrm{diag}\,[M_B,\ J_{BY},\ J_{BX},\ M_{C1},\ J_{C1},\ M_{C2},\ J_{C2}] \quad (17)$$

- The stiffness matrix $[K]$ with the rows as:

$$\begin{aligned}
[K_{1i}] &= [2(k_{Tf} + k_{Tr}),\ 2(a_1k_{Tf} - a_2k_{Tr}),\ 0,\ -2k_{Tf},\ 0,\ -2k_{Tr},\ 0] \\
[K_{2i}] &= [2(a_1k_{Tf} - a_2k_{Tr}),\ 2(a_1^2k_{Tf} + a_2^2k_{Tr}),\ 0,\ -2a_1k_{Tf},\ 0,\ 2a_2k_{Tr},\ 0] \\
[K_{3i}] &= [0,\ 0,\ 2c^2(k_{Tf} + k_{Tr}),\ 0,\ -2c^2k_{Tf},\ 0,\ -2c^2k_{Tr}] \\
[K_{4i}] &= [-2k_{Tf},\ -2a_1k_{Tf},\ 0,\ 2k_{Tf} + (s_1+s_2)k_{Lf},\ b(s_1-s_2)k_{Lf},\ 0,\ 0] \\
[K_{5i}] &= [0,\ 0,\ -2c^2k_{Tf},\ b(s_1-s_2)k_{Lf},\ 2c^2k_{Tf} + b^2(s_1+s_2)k_{Lf},\ 0,\ 0] \\
[K_{6i}] &= [-2k_{Tr},\ 2a_2k_{Tf},\ 0,\ 0,\ 0,\ 2k_{Tr} + (s_3+s_4)k_{Lr},\ b(s_3-s_4)k_{Lr}] \\
[K_{7i}] &= [0,\ 0,\ -2c^2k_{Tr},\ 0,\ 0,\ b(s_3-s_4)k_{Lr},\ 2c^2k_{Tr} + b^2(s_3+s_4)k_{Lr}]
\end{aligned} \quad (18)$$

- The damping matrix $[C]$ is determined similarly to the stiffness matrix. It can be obtained from the stiffness matrix by replacing the symbols K or k (stiffness) by the corresponding symbols C or c (damping coefficient).

- The excitation vector:

$$\vec{F} = \begin{bmatrix} -M_Bg \\ 0 \\ 0 \\ -M_{C1}g + s_1(k_{L1}z_{D1} + c_{L1}\dot{z}_{D1}) + s_2(k_{L2}z_{D2} + c_{L2}\dot{z}_{D2}) \\ b\left[s_1(k_{L1}z_{D1} + c_{L1}\dot{z}_{D1}) - s_2(k_{L2}z_{D2} + c_{L2}\dot{z}_{D2})\right] \\ -M_{C2}g + s_3(k_{L3}z_{D3} + c_{L3}\dot{z}_{D3}) + s_4(k_{L4}z_{D4} + c_{L4}\dot{z}_{D4}) \\ b\left[s_3(k_{L3}z_{D3} + c_{L3}\dot{z}_{D3}) - s_4(k_{L4}z_{D4} + c_{L4}\dot{z}_{D4})\right] \end{bmatrix} \quad (19)$$

2.6. Method for solving the differential equations of motion

Because of the presence of the contact state parameters s_j ($j = 1 \div 4$), matrices $[K]$, $[C]$ and excitation vector $\vec{F}$ depend on time. Therefore, the differential equations of motion of the automobile in the case of taking the loss of contact should be solved numerically.
An important thing in the process of solving this equations is determining the value of the contact state parameters at all steps of computation according to condition (5), corresponding to testing that the loss of contact occurs or not.
Two things directly related to solving differential equations are describing the functions $z_{Dj} = z_{Dj}(t)$ ($j = 1 \div 4$) and setting up the initial condition of computation.
The functions $z_{Dj} = z_{Dj}(t)$ and $\dot{z}_{Dj} = \dot{z}_{Dj}(t)$ regarding both the road surface profile and the speed of movement will be introduced in the examples illustrating numerical computation. For the initial conditions, it is reasonable to consider the situation where the vehicle is moving on a completely smooth road surface then entering a uneven one.

The initial condition in this case can be expressed as:

$$\begin{cases} s_1\|_{t=0} = s_2\|_{t=0} = s_3\|_{t=0} = s_4\|_{t=0} = 1 \\ \vec{q}\|_{t=0} = \vec{q}_0,\ \dot{\vec{q}}\|_{t=0} = \vec{0},\ \ddot{\vec{q}}\|_{t=0} = \vec{0} \\ z_{D1}\|_{t=0} = z_{D2}\|_{t=0} = z_{D3}\|_{t=0} = z_{D4}\|_{t=0} = 0 \\ \dot{z}_{D1}\|_{t=0} = \dot{z}_{D2}\|_{t=0} = \dot{z}_{D3}\|_{t=0} = \dot{z}_{D4}\|_{t=0} = 0 \\ \vec{F}\|_{t=0} = \vec{F}_0 = [-M_{\mathrm{B}}g,\ 0,\ 0,\ -M_{\mathrm{C1}}g,\ 0,\ -M_{\mathrm{C2}}g,\ 0]^{\mathrm{T}} \end{cases} \tag{20}$$

Substituting (20) into (16) allows one to get the formula for the initial values of the generalized coordinates:

$$\vec{q}_0 = [K]_0^{-1}\vec{F}_0 \tag{21}$$

where $[K]_0$ is the value of matrix $[K]$ at the initial time point, $[K]_0 = [K]\|_{t=0}$.

The procedure for numerically solving the differential equations of motion where the loss of contact is taken into account can be presented as follows:

Step 1. Assigning values to input quantities consisting of the typical geometry and dynamic parameters, the acceleration of gravity, the speed of movement, etc.

Step 2. Establishing the expressions of the functions $z_{Dj} = z_{Dj}(t)$ and $\dot{z}_{Dj} = \dot{z}_{Dj}(t)$ corresponding to the type of road surface profile that is selected for consideration.

Step 3. Setting initial conditions according to equations (20) and (21).

Step 4. Choosing the time interval $[0,\ t_{\max}]$ and the time step Δt for computation.

Step 5. Initiating the zero value of the counting variable i, i=0.

Step 6. Assigning the values at the i-th computation point to those variables: t_i=0, s_1=s_2=s_3=s_4=1, $\vec{q}_i = \vec{q}_0$, $\dot{\vec{q}}_i = 0$, $\ddot{\vec{q}}_i = 0$.

Step 7. Computing the values of the stiffness and damping matrices and excitation vector force at the i-th computation point, $[K]_i$, $[C]_i$, $\vec{F}_i$.

Step 8. Computing the values of the generalized coordinate vector together with its first and second derivatives at the time point t_{i+1}:=t_i+Δt corresponding to the (i+1)-th computation point ($\vec{q}_{i+1}$, $\dot{\vec{q}}_{i+1}$, $\ddot{\vec{q}}_{i+1}$) according to the numerical method chosen.

Step 9. Calculating the values of the quantities $z_{\mathrm{D}j}$, $\dot{z}_{\mathrm{D}j}$ at the time point t_{i+1} and deducing the values of the quantities $\bar{R}_{Dj}$, s_j and the reaction forces R_{Dj} at that time point.

Step 10. Assigning i:=i+1, t_i:=t_i+Δt and repeating all the process of computation from step 7.

The process of computation stops until the condition t_i>$t_{\max}$ is reached.

The obtained results are the time-depend functions those reflect the dynamic response of the vehicle as follows:

$$\vec{q} = \vec{q}(t),\ \dot{\vec{q}} = \dot{\vec{q}}(t),\ \ddot{\vec{q}} = \ddot{\vec{q}}(t),\ R_{Dj} = R_{Dj}(t) \tag{22}$$

By programming we can get the connecting forces acting in the suspension spring - damper couples and the amount of time when the loss of contact occurs and consider the influence of factors related to the automobile, the road and the speed of movement on the vehicle dynamic response.

3. SOME RESULTS FROM NUMERICAL COMPUTATION.

This section will introduce some typical results from numerical computation. The values of the parameters related to the automobile are taken according to the one labeled GAZ-66 [1] as follows:

M_{B} = 2200 kg, M_{C1} = 660 kg, M_{C2} = 580 kg, J_{BX} = 756 kg.m^2, J_{BY} = 2750 kg.m^2, k_{Tf} = 246000 N/m, k_{Tr} = 196000 N/m, c_{Tf} = c_{Tr} = 1500 N.s/m, k_{Lf} = k_{Lr} = 800000 N/m, c_{Lf} = c_{Lr} = 62000 N.s/m, a_1 = 1.563 m, a_2 = 1.737 m, b = 0.90 m, c = 0.60 m.

Applying formula (21), we can get the initial values of the generalized coordinates as:

$$\vec{q}_0 = [-0.0351\ \mathrm{m};\ 0.0005\ \mathrm{rad};\ 0\ \mathrm{rad};\ -0{,}0111\ \mathrm{m};\ 0\ \mathrm{rad};\ -0{,}0099\ \mathrm{m};\ 0\ \mathrm{rad}]^{\mathrm{T}}. \tag{23}$$

The calculation results are presented in two types of the road profile: *a*) the sinusoidal wave with many consecutive cycles and, *b*) the style of a single parabolic pulse.

3.1. The road profile typed the sinusoidal wave with many consecutive cycles

This type of road profile and its typical geometrical parameters as shown in Figure 2 where the origin point O is chosen corresponding to the initial moment t = 0 and x_0 is the distance that a front wheel needs to pass from the initial moment before entering the uneven road.

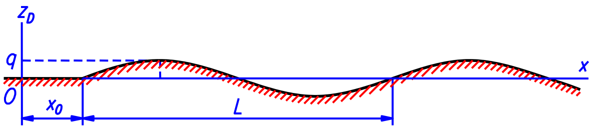

Figure 2: The road profile typed sinusoidal wave with many consecutive cycles.

For distinguishing the left and the right tracks, the letters "*L*" and "*R*" will be added to the corresponding subscripts. The expressions for the functions z_{Dj} = $z_{Dj}(x)$ (j=1÷4) can be written (based on Figure 2) as follows:

z_{D1} = 0 for $x < x_{0\mathrm{L}}$,

$z_{D1} = q_{\mathrm{L}} \sin\left[\frac{2\pi}{L_{\mathrm{L}}}(x - x_{0\mathrm{L}})\right]$ for $x \geq x_{0\mathrm{L}}$,

z_{D2} = 0 for $x < x_{0\mathrm{R}}$,

$z_{D2} = q_{\mathrm{R}} \sin\left[\frac{2\pi}{L_{\mathrm{R}}}(x - x_{0\mathrm{R}})\right]$ for $x \geq x_{0\mathrm{R}}$,

z_{D3} = 0 for $x < x_{0\mathrm{L}} + d_a$, (24)

$z_{D3} = q_L \sin\left[\frac{2\pi}{L_L}(x - x_{0L} - d_a)\right]$ for $x \ge x_{0L}+d_a$,

$z_{D4} = 0$ for $x < x_{0R}+d_a$,

$z_{D4} = q_R \sin\left[\frac{2\pi}{L_R}(x - x_{0R} - d_a)\right]$ for $x \ge x_{0R}+d_a$

where $d_a = (a_1+a_2)$ is the distance between the two axles in the horizontal direction.

Because the quantities x, x_0, d_a can be expressed through the speed of movement V (V=const) and the time variable t as $x = Vt$, $x_0 = Vt_0$ and $d_a = Vt_a$, so that the expressions of the functions z_{Dj} (j=1÷4) can be written as functions of time, $z_{Dj} = z_{Dj}(t)$. Besides, from the expressions (24), one can get the expressions of functions $\dot{z}_{Dj} = \dot{z}_{Dj}(t)$, j=1÷4, simply by taking their derivatives with respect to time.

The values of the parameters in equations (24) which are used in the following computations are taken as:

$$q_L = q_R = 0.065\text{m},\ L_L = L_R = 7.0\text{m},$$
$$t_{0L}=t_{0R} = 0.5\text{s},\ t_{max} = 5\text{s} \qquad (25)$$

The change in the reaction forces at wheel 1 and wheel 4 with respect to time in case V=60 km/h is presented in Figure 3. The plots with many flat pieces lying in abscissa show that the loss of contact appears clearly in this situation. In addition, the reaction forces rapidly increase at the moments of entering the uneven road because the quantities $\dot{z}_{Dj}$ (j=1÷4) makes steps at these moments.

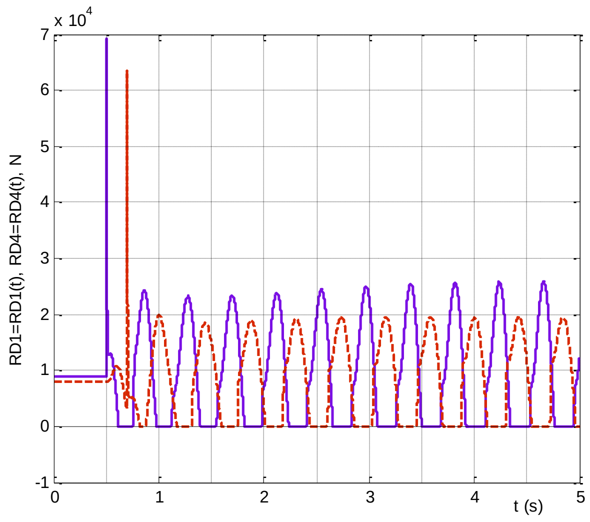

Figure 3: The changing of reaction forces R_{D1} and R_{D4} with respect to time in case the road profile typed sinusoidal wave with many consecutive cycles. (R_{D1} - blue continuous, R_{D4} - red dash)

Table 1: The total times of losing contact (T_{LCj}) vs. the speed of movement (V) in case the road profile typed sinusoidal wave with many consecutive cycles.

V (km/h)		40	50	60	70	80	90
T_{LCj} (s)	wheel 1	0	0.1227	1.8661	1.9940	2.0715	2.0522
	wheel 2	0	0.1227	1.8661	1.9940	2.0715	2.0522
	wheel 3	0	0	1.6051	2.0024	2.1795	2.1509
	wheel 4	0	0	1.6051	2.0024	2.1795	2.1509

Table 1 presents the total times of losing contact (denoted as T_{LCj}) at the four wheels depending on the speed of movement (V) in case the data set (25) are used.

The data in Table 1 show that: *a*) there exists a limit in speed of the automobile so that the loss of contact does not appear if the vehicle speed is less than this limit value; *b*) beyond this limit, the total times of losing contact change with the increase in speed of the vehicle. The law of fluctuation is still not certain but the values of quantities T_{LCj} are quite great in comparison with the total time of consideration t_{max}.

3.2. The road profile typed a single parabolic pulse.

The road profile typed a single parabolic pulse is showed in Figure 4. The letters "L" and "R" are also added to the corresponding subscripts for distinguishing the left and the right tracks.

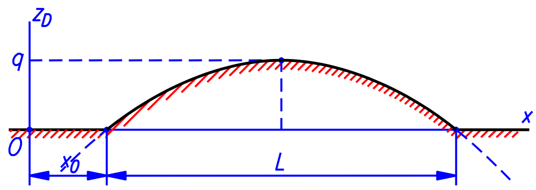

Figure 4: The road profile typed a single parabolic pulse.

The expressions for the functions $z_{Dj} = z_{Dj}(x)$, j=1÷4, in this case are written as:

$$z_{D1} = -\frac{4q_L}{L_L^2}(x - x_{0L})(x - x_{0L} - L_L) \text{ for } x_{0L} \le x \le (x_{0L}+L_L),$$

$$z_{D2} = -\frac{4q_R}{L_R^2}(x - x_{0R})(x - x_{0R} - L_R) \text{ for } x_{0R} \le x \le (x_{0R}+L_R), \qquad (26)$$

$$z_{D3} = -\frac{4q_L}{L_L^2}(x - x_{0L} - d_a)(x - x_{0L} - d_a - L_L) \text{ for } (x_{0L}+d_a) \le x \le (x_{0L}+d_a+L_L),$$

$$z_{D4} = -\frac{4q_R}{L_R^2}(x - x_{0R} - d_a)(x - x_{0R} - d_a - L_R) \text{ for } (x_{0R}+d_a) \le x \le (x_{0R}+d_a+L_R).$$

$z_{Dj} = 0$, j=1÷4, in case x is out of the corresponding intervals.

Because the concerned quantities in equations (26) can be expressed through the variables V and t ($x = Vt$, $x_{0L} = Vt_{0L}$, $x_{0R} = Vt_{0R}$, $L_L = Vt_{1L}$, $L_R = Vt_{1R}$, $d_a = Vt_a$) so that the functions z_{Dj} are the ones of time. From these expressions, the corresponding expressions of quantities $\dot{z}_{Dj}(t)$ by taking their derivatives with respect to time.

The values of the parameters in equations (26) which are applied in the numerical computations are taken as follows:

$$q_L = q_R = 0.05\text{m},\ L_L = L_R = 0.60\text{m},\ t_{0L} = t_{0R} = 0.5\text{s} \qquad (27)$$

The change in the reaction forces at wheel 1 and wheel 3 with respect to time in case V=20 km/h is presented in Figure 5. The plots show that the loss of contact appears just at a quite small value in speed of movement.

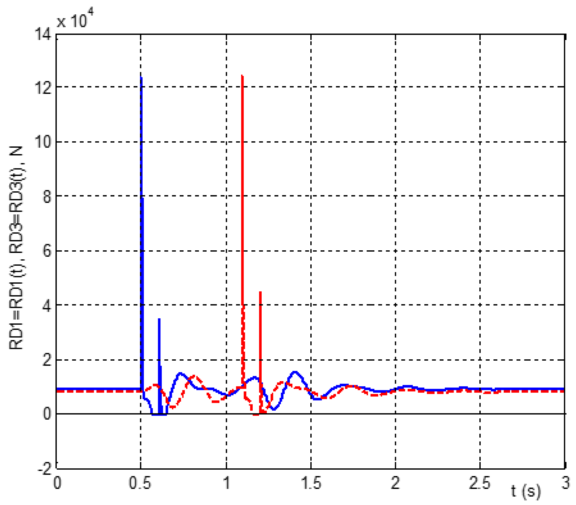

Figure 5: The change in the reaction forces R_{D1} and R_{D3} with respect to time in case the road profile typed a single parabolic pulse (R_{D1} - blue continuous, R_{D3} - red dash).

The dependence of the total times of losing contact T_{LCj} (j=1÷4) at the four wheels on the speed of movement (V) is presented in Table 2.

Table 2: The total times of losing contact (T_{LCj}) vs. the speed of movement (V) in case the road profile typed a single parabolic pulse.

V (km/h)		5	10	20
T_{LCj} (s)	wheels 1 & 2	0	0.0791	0.0647
	wheels 3 & 4	0	0.0686	0.0393
V (km/h)		30	40	50
T_{LCj} (s)	wheels 1 & 2	0.1013	0.1341	0.1596
	wheels 3 & 4	0.1639	0.1168	0.0885

The results in Table 2 show that: *a*) there is a limit in speed of movement so that the loss of contact does not appear if the speed does not exceed this limit value; *b*) the loss of contact appears at all the four wheels of the vehicle even when the speed of movement decreases down to 10 km/h; *c*) beyond the speed limit, the total times of losing contact change together with the increase in speed of movement but the law of fluctuation is not certain.

4. CONCLUSION.

The paper has made a spatial vibration model of a two-axles automobile with dependent suspensions. The differential equations of motion of the vehicles where the loss of contact is taken into account have been established with introducing the so-called contact state parameters. A procedure for numerically solving the differential equations of motion has been proposed where the contact state of the four wheels to the road surface is controlled at every step of computation. Two typical types of road profile have been mathematically described and also applied in some cases of numerical computations. The obtained results show that it is quite easy for the loss of contact to appear in case the road profile belongs to the type of a single pulse, and that the total times of losing contact at the four wheels clearly change with respect to the speed of movement but the laws of their fluctuation may not be certain unless a process of thorough consideration has been carried out.

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