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A Modified Class of Log-Type Estimators for Population Mean Using Auxiliary Information on Variables

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Abstract

In this paper, a generalised class of estimators is proposed for the estimation of population mean $\bar{y}$ using the auxiliary variable. Bias and MSE of the proposed class of estimator is obtained upto the first order of approximation. It has been proven that the proposed class of estimators are more efficient than the usual regression, ratio, and product type estimators. This fact is also supported through an empirical study which is given at the end as an illustration.

Key words: study variable, auxiliary variable, bias, mean square error, efficiency.

1. INTRODUCTION

Using auxiliary information is a famous trend in survey sampling as it increases the efficiency of the estimators. Sampling literature deals with variety of such type of estimators using the auxiliary information. When the correlation coefficient between the study variable and auxiliary variable is positive , we use ratio estimator whereas if the correlation is negative, product estimator is highly preferable for better estimation of the population mean. Dwivedi (1981), Pandey and Dubey (1988), Upadhyaya and Singh (1999), Singh and Tailor (2003), Singh (2003), Kalidar and Cingi (2006), Singh et al. (2004, 2008), Bhushan et al. (2017a, 2017b, 2017c), Bhushan and Misra (2017d), Bhushan and Gupta(2019a,2019b, 2019c, 2019d, 2020) and Singh and Agnihotri (2008) have also made use of various parameters of the auxiliary variable such as standard deviation, coefficient of variation, correlation coefficient, coefficient of skewness, coefficient of kurtosis, etc. The efficiency of the estimators proposed by the above authors is approximately equal to the usual regression estimator. Recently Bhushan et al (2015) have proposed a generalised class of logarithmic type estimators, probably for the first time, for estimating population mean. They have also given a better class of log type estimators using the Searls approach. In this paper, we have proposed an alternative class of estimators which includes the estimators / classes of estimators given by, Khoshnevisan et al (2007), Koyuncu and Kalidar (2009a), Bhushan et al (2015) and Bhushan et al (2015a). It has been shown that the suggested class of estimator is more efficient among all the estimators discussed within, in the sense of lesser mean square error, both theoretically and numerically.

Consider a finite population $U=(U_1,U_2\ldots,U_N)$ of size N from which a sample of size n is drawn according to simple random sampling without replacement (SRSWOR). Let y_i and x_i denotes the values of the study and auxiliary variables for the i th unit, $(i=1,2,...,N)$ of the population. Further let $\bar{y}$ and $\bar{x}$ be the sample means of the study and auxiliary variables, respectively. Let us define $e_0=\left(\bar{y}-\bar{Y}\right)/\bar{Y}$ and $e_1=\left(\bar{x}-\bar{X}\right)/\bar{X}$.

Using these notations,

$$E(e_0)=E(e_1)=0\,,\quad E\left(e_0^2\right)=\lambda C_x^2\,,\quad E\left(e_1^2\right)=\lambda C_y^2\,,$$

$$E(e_0e_1)=\lambda C_{xy}=\lambda\rho C_xC_y=\lambda kC_x^2$$

where

$$C_x^2=S_x^2\Big/\bar{X}^2\,,\ C_y^2=S_y^2\Big/\bar{Y}^2\,,\ C_{xy}=S_{xy}\Big/\bar{X}\bar{Y}\,,$$

$$S_x^2=\sum_{i=1}^{N}\left(xi-\bar{X}\right)^2\Big/(N-1)\,,\ S_y^2=\sum_{i=1}^{N}\left(yi-\bar{Y}\right)^2\Big/(N-1)\,,$$

$$S_{xy}=\sum_{i=1}^{N}\left(xi-\bar{X}\right)\left(yi-\bar{Y}\right)\Big/(N-1)\,,\ k=\rho\left(C_y/C_x\right)\quad\text{and}$$

$$\lambda=(N-n)/Nn$$

2. THE SUGGESTED CLASS OF ESTIMATORS

We define a class of estimators for the population mean $\overline{Y}$ as

$$T = w_1\bar{y}\left(\frac{\overline{X}^*}{\alpha\bar{x}^* + (1-\alpha)\overline{X}^*}\right)^g + w_2\bar{y}\left[1+\log\left(\frac{\bar{x}^*}{\overline{X}^*}\right)^\beta\right] \quad (2.1)$$

where

$$\overline{X}^* = a\overline{X} + b$$
$$\bar{x}^* = a\bar{x} + b \quad (2.2)$$

And $a(\neq 0), b$ are either real numbers or functions of the known parameters of the auxiliary variables x such as the standard deviations S_x, coefficient of variation C_x, coefficient of kurtosis $\beta_2(x)$ and correlation coefficient ρ of the population, α is a suitable constant, (g,β) being constants which take values $(0,1,-1)$ for designing the different estimators, and (w_1, w_2) are suitably chosen constants to be determined such that mean square error (MSE)of 'T' is minimum. It is to be mentioned that:

(i) For $(w_1, w_2) = (1,0)$, the class of estimators T can be reduced to the class of estimators due to Khoshnevisan et al. (2007)

$$T_K = \bar{y}\left(\frac{\overline{X}^*}{\alpha\bar{x}^* + (1-\alpha)\overline{X}^*}\right)^g \quad (2.3)$$

(ii) For $(w_1, w_2) = (w_1, 0)$, the class of estimators T can be transformed to the class of estimators due to Koyuncu and Kalidar (2009a)

$$\eta_K = w_1\bar{y}\left(\frac{\overline{X}^*}{\alpha\bar{x}^* + (1-\alpha)\overline{X}^*}\right)^g \quad (2.4)$$

(iii) For $(w_1, w_2) = (0,1)$, the class of estimators T can be reduced to the class of estimators given by Bhushan et al (2015)

$$\xi = \bar{y}\left[1+\log\left(\frac{\bar{x}^*}{\overline{X}^*}\right)^\beta\right] \quad (2.5)$$

(iv) For $(w_1, w_2) = (0, w_2)$, the class of estimators T transforms to the class of estimators given by Bhushan et al (2015a)

$$\psi = w_2\bar{y}\left[1+\log\left(\frac{\bar{x}^*}{\overline{X}^*}\right)^\beta\right] \quad (2.6)$$

Expressing T at (2.1) in terms of $e's$, we have

$$T = w_1\overline{Y}(1+e_0)(1+\alpha v e_1)^{-g} + w_2\overline{Y}(1+e_0)\left[1+\beta\log(1+ve_1)\right] \quad (2.7)$$

where $v = a\overline{X}/(a\overline{X}+b)$

Expanding the right hand side of (2.7), neglecting terms of $e's$ having higher power greater than two and subtracting $\overline{Y}$ from both sides we have

$$(T-\overline{Y}) = \overline{Y}\begin{bmatrix} w_1\left\{1+e_0-\alpha vg(e_1+e_0e_1)+\dfrac{g(g+1)}{2}\alpha^2v^2e_1^2\right\} \\ +w_2\left\{1+e_0+\beta v(e_1+e_0e_1)-\beta\dfrac{v^2}{2}e_1^2\right\}-1\end{bmatrix} \quad (2.8)$$

Taking expectation on both sides of (2.8), we get the bias of the class of estimators T to the first order of approximation as

$$Bias(T) = \overline{Y}\begin{bmatrix} w_1\left\{1+\left(\dfrac{\lambda\alpha vg}{2}\right)C_x^2\left(\alpha v(g+1)-2k\right)\right\} \\ +w_2\left\{1+\left(\dfrac{\lambda\beta v}{2}\right)C_x^2(2k-v)\right\}-1\end{bmatrix} \quad (2.9)$$

Squaring both sides of (2.8) we have

$$(T-\overline{Y})^2 = \overline{Y}^2\begin{bmatrix} 1+w_1^2\left\{1+2e_0+e_0^2-2\alpha vge_1-4\alpha vge_0e_1+\alpha^2v^2g(2g+1)e_1^2\right\} \\ +w_2^2\left\{1+2e_0+e_0^2+2\beta ve_1+\beta v^2(\beta-1)e_1^2+4\beta ve_0e_1\right\} \\ +2w_1w_2\left\{1+2e_0+e_0^2+\beta ve_1-\alpha vge_1+2\beta ve_0e_1-2\alpha vge_0e_1+\dfrac{v^2e_1^2}{2}\left(g(g+1)\alpha^2-\beta-2\alpha\beta g\right)\right\} \\ -2w_1\left\{1+e_0-\alpha vg(e_1+e_0e_1)+\dfrac{g(g+1)}{2}\alpha^2v^2e_1^2\right\} \\ -2w_2\left\{1+e_0+\beta\upsilon(e_1+e_0e_1)-\beta\dfrac{\upsilon^2}{2}e_1^2\right\}\end{bmatrix} \quad (2.10)$$

Taking expectation of both sides of (2.10), we get the MSE of the class of estimators $'T'$ to the first order of approximation as

$$MSE(T)=\bar{Y}^2\left[1+w_1^2A+w_2^2C+2w_1w_2D-2w_1B-2w_2E\right] \quad (2.11)$$

where

$$A=\left[1+\lambda\left\{C_y^2+\left(\alpha^2v^2\left(2g^2+g\right)-4\alpha vgk\right)C_x^2\right\}\right]$$

$$B=\left[1+\lambda\left\{\frac{\alpha^2v^2g\left(g+1\right)}{2}-\alpha vgk\right\}C_x^2\right]$$

$$C=\left[1+\lambda\left\{C_y^2+\left(\beta v^2\left(\beta-1\right)+4\beta vk\right)C_x^2\right\}\right]$$

$$D=\left[1+\lambda\left\{C_y^2+\left(2vk\left(\beta-\alpha g\right)+\frac{v^2}{2}\left(g\left(g+1\right)\alpha^2-\beta-2\alpha\beta g\right)\right)C_x^2\right\}\right]$$

$$E=\left[1+\lambda\left\{\frac{\beta v}{2}\left(2k-v\right)\right\}C_x^2\right]$$

The *MSE* of the class of estimators T at (2.11) is minimised for

$$w_1=\frac{\left(BC-DE\right)}{\left(AC-D^2\right)}=w_{1(opt)} \quad \text{(say)} \quad (2.12)$$

and $$w_2=\frac{\left(AE-BD\right)}{\left(AC-D^2\right)}=w_{2(opt)} \quad \text{(say)} \quad (2.13)$$

Substituting (2.12) and (2.13) in (2.11) we get the minimum *MSE* of the class of estimators T as

$$MSE_{\min}(T)=\bar{Y}^2\left[1-\frac{\left(B^2C-2BDE+AE^2\right)}{\left(AC-D^2\right)}\right] \quad (2.14)$$

Theorem 2.1. *To the first order of approximation*

$$MSE_{\min}(T)\geq\bar{Y}^2\left[1-\frac{\left(B^2C-2BDE+AE^2\right)}{\left(AC-D^2\right)}\right]$$

With equality holding if

$$w_1=w_{1(opt)}$$

$$w_2=w_{2(opt)}$$

Putting $(w_1,w_2)=(1,0),(w_1,0),(0,1),(0,w_2)$ in equation (2.10) we will get the MSEs of the class of estimators T_K (Khoshnevisan et al. (2007)), η_K (Koyuncu and Kalidar (2009a)), ξ (Bhushan et al 2015) and ψ (Bhushan et al 2015a)respectively to the first order of approximation and will be given as:

$$MSE(T_K)=\bar{Y}^2\left(1+A-2B\right)$$

$$MSE(T_K)=\lambda\bar{Y}^2\left(C_y^2+\alpha^2v^2g^2C_x^2-2\alpha vgkC_x^2\right) \quad (2.15)$$

$$MSE(\eta_K)=\bar{Y}^2\left(1+w_1^2A-2w_1B\right) \quad (2.16)$$

$$MSE(\xi)=\bar{Y}^2\left(1+C-2E\right)$$

$$MSE(\xi)=\lambda\bar{Y}^2\left(C_y^2+\beta^2v^2C_x^2-2\beta vkC_x^2\right) \quad (2.17)$$

$$MSE(\psi)=\bar{Y}^2\left(1+w_2^2C-2w_2E\right) \quad (2.18)$$

The $MSE(T_K)$, $MSE(\eta_K)$, $MSE(\xi)$ and $MSE(\psi)$ are minimised for the optimum values

$$\alpha_{(opt)}=\frac{k}{vg}$$

$$\beta_{(opt)}=\frac{-k}{v}$$

$$w_{1(opt)}^*=\frac{B}{A}$$

$$w_{2(opt)}^*=\frac{E}{C} \quad \text{respectively}$$

Thus, the $MSE_{\min}(T_K)$ and $MSE_{\min}(\eta_K)$ are respectively given as

$$MSE_{\min}(T_K)=MSE_{\min}(\xi)=\lambda\bar{Y}^2C_y^2\left(1-\rho^2\right) \quad (2.19)$$

$$MSE_{\min}(\eta_K)=\bar{Y}^2\left(1-\frac{B^2}{A}\right) \quad (2.20)$$

$$MSE_{\min}(\psi)=\bar{Y}^2\left(1-\frac{E^2}{C}\right) \quad (2.21)$$

The $MSE_{\min}(T_K)$, $MSE_{\min}(\xi)$ given by (2.19) is same as that of the approximate variance/MSE of the known regression estimator,

$$\bar{y}_{lr}=\bar{y}+\beta\left(\bar{X}-\bar{x}\right)$$

3. PARTICULAR CASE $\left(w_1 + w_2 = 1\right)$

For $\left(w_1 + w_2 = 1\right)$, the proposed class of estimators transforms to

$$T^* = w_1 \bar{y}\left(\frac{\bar{X}^*}{\alpha \bar{x}^* + (1-\alpha)\bar{X}^*}\right)^g + (1-w_1)\bar{y}\left[1+\log\left(\frac{\bar{x}^*}{\bar{X}^*}\right)^\beta\right] \tag{3.1}$$

Theorem 3.1

The bias and MSE of the proposed class of estimators, to the first order of approximation is given by

$$Bias\left(T^*\right) = \bar{Y}\left[\begin{array}{l} w_1\left\{\frac{g(g+1)}{2}\lambda\alpha^2 v^2 C_x^2 - \lambda\alpha v g k C_x^2 + \frac{\lambda\beta v^2 C_x^2}{2} - \lambda\beta v k C_x^2\right\} \\ -\left\{\frac{\lambda\beta v^2 C_x^2}{2} - \lambda\beta v k C_x^2\right\}\end{array}\right] \tag{3.2}$$

$$MSE\left(T^*\right) = \bar{Y}^2\left[\begin{array}{c} 1+C-2E+w_1^2\left(A+C-2D\right) \\ -2w_1\left(C-D+B-E\right)\end{array}\right] \tag{3.3}$$

Corollary 3.2

The resultant $MSE\left(T^*\right)$ is minimised for the optimum value

$$w_1 = \frac{\left(C-D+B-E\right)}{\left(A+C-2D\right)} = w_{1(opt)}^{**} \tag{3.4}$$

And the minimum $MSE\left(T^*\right)$ is obtained as

$$MSE_{\min}\left(T^*\right) = \bar{Y}^2\left[1+C-2E-\frac{\left(C-D+B-E\right)^2}{\left(A+C-2D\right)}\right] \tag{3.5}$$

$$= \lambda\bar{Y}^2 C_y^2\left(1-\rho^2\right) = MSE\left(\bar{y}_{lr}\right) \tag{3.6}$$

Therefore, the following theorem is established.

Theorem 3.3. *To the first order of approximation*

$$MSE_{\min}\left(T^*\right) \geq \bar{Y}^2\left[1+C-2E-\frac{\left(C-D+B-E\right)^2}{\left(A+C-2D\right)}\right]$$

with equality holding if

$$w_1 = w_{1(opt)}^{**}$$

4. EFFICIENCY COMPARISON

For comparison of the efficiency of the proposed class of estimators T with the known unbiased mean per unit estimator $\bar{y}$, usual ratio estimator $\bar{y}_R$, usual product estimator $\bar{y}_P$, we have used their respective MSEs to the first order of approximation,

$$Var\left(\bar{y}\right) = MSE\left(\bar{y}\right) = \lambda\bar{Y}^2 C_y^2 \tag{4.1}$$

$$MSE\left(\bar{y}_R\right) = \lambda\bar{Y}^2\left[C_y^2 + C_x^2 - 2\rho_{yx}C_y C_x\right] \tag{4.2}$$

$$MSE\left(\bar{y}_P\right) = \lambda\bar{Y}^2\left[C_y^2 + C_x^2 + 2\rho_{yx}C_y C_x\right] \tag{4.3}$$

$$\begin{aligned} &Var\left(\bar{y}\right) - \left[MSE_{\min}\left(T_K\right) = MSE_{\min}\left(\xi\right) = MSE\left(\bar{y}_{lr}\right)\right] \\ &= \lambda\bar{Y}^2 C_y^2\rho_{yx}^2 \geq 0 \end{aligned} \tag{4.4}$$

$$MSE\left(\bar{y}_R\right) - MSE\left(\bar{y}_{lr}\right) = \lambda\bar{Y}^2 C_x^2\left(1-k\right)^2 \geq 0 \tag{4.5}$$

$$MSE\left(\bar{y}_P\right) - MSE\left(\bar{y}_{lr}\right) = \lambda\bar{Y}^2 C_x^2\left(1+k\right)^2 \geq 0 \tag{4.6}$$

$$\begin{aligned} &MSE\left(T_K\right) - \left[MSE_{\min}\left(T_K\right) = MSE\left(\bar{y}_{lr}\right)\right] \\ &= \lambda\bar{Y}^2 C_x^2\left(\alpha v g - k\right)^2 \geq 0 \end{aligned} \tag{4.7}$$

$$\begin{aligned} &MSE\left(\xi\right) - \left[MSE_{\min}\left(\xi\right) = MSE\left(\bar{y}_{lr}\right)\right] \\ &= \lambda\bar{Y}^2 C_x^2\left(\beta v - k\right)^2 \geq 0 \end{aligned} \tag{4.8}$$

$$MSE\left(T_K\right) - MSE_{\min}\left(\eta_K\right) = \bar{Y}^2\frac{\left(A-B\right)^2}{A} \geq 0 \tag{4.9}$$

$$MSE\left(\xi\right) - MSE_{\min}\left(\psi\right) = \bar{Y}^2\frac{\left(C-E\right)^2}{E} \geq 0 \tag{4.10}$$

$$MSE_{\min}\left(\eta_K\right) - MSE_{\min}\left(T\right) = \bar{Y}^2\frac{\left(AE-BD\right)^2}{A\left(AC-D^2\right)} \geq 0 \tag{4.11}$$

$$MSE_{\min}\left(\psi\right) - MSE_{\min}\left(T\right) = \bar{Y}^2\frac{\left(BC-DE\right)^2}{C\left(AC-D^2\right)} \geq 0 \tag{4.12}$$

$$\begin{aligned} &MSE_{\min}\left(T^*\right) - MSE_{\min}\left(T\right) = \\ &\bar{Y}^2\frac{\left[C\left(B-A\right)+D\left(D-B\right)+E\left(A-D\right)\right]^2}{\left(AC-D^2\right)\left(A+C-2D\right)} \geq 0 \end{aligned} \tag{4.13}$$

From (4.1)-(4.13), we get the following inequalities:

$$MSE_{\min}(T) \le MSE_{\min}(T^*) \le MSE(\bar{y}_R) \le Var(\bar{y}) \quad (4.14)$$

$$k \ge \frac{1}{2}$$

$$MSE_{\min}(T) \le MSE_{\min}(T^*) \le MSE(\bar{y}_P) \le Var(\bar{y}) \quad (4.15)$$

$$k \le -\frac{1}{2}$$

$$MSE_{\min}(T) \le MSE_{\min}(\eta_K) \le MSE(T_K) \quad (4.16)$$

$$MSE_{\min}(T) \le MSE_{\min}(\xi) \le MSE(\psi) \quad (4.17)$$

Table 1: Some members of the class of estimators $'T'$

Ratio-type logarithmic estimator $(g,\beta)=(0,-1)$	a	b
$T_{rl1} = w_1\bar{y} + w_2\bar{y}\left[1+\log\left(\frac{\bar{X}}{\bar{x}}\right)\right]$	1	0
$T_{rl2} = w_1\bar{y} + w_2\bar{y}\left[1+\log\left(\frac{\bar{X}+C_x}{\bar{x}+C_x}\right)\right]$	1	C_x
$T_{rl3} = w_1\bar{y} + w_2\bar{y}\left[1+\log\left(\frac{\beta_2(x).\bar{X}+C_x}{\beta_2(x).\bar{x}+C_x}\right)\right]$	$\beta_2(x)$	C_x
$T_{rl4} = w_1\bar{y} + w_2\bar{y}\left[1+\log\left(\frac{C_x.\bar{X}+\beta_2(x)}{C_x.\bar{x}+\beta_2(x)}\right)\right]$	C_x	$\beta_2(x)$
$T_{rl5} = w_1\bar{y} + w_2\bar{y}\left[1+\log\left(\frac{\bar{X}+S_x}{\bar{x}+S_x}\right)\right]$	1	S_x
$T_{rl6} = w_1\bar{y} + w_2\bar{y}\left[1+\log\left(\frac{\beta_1(x)\bar{X}+S_x}{\beta_1(x)\bar{x}+S_x}\right)\right]$	$\beta_1(x)$	S_x
$T_{rl7} = w_1\bar{y} + w_2\bar{y}\left[1+\log\left(\frac{\beta_2(x)\bar{X}+S_x}{\beta_2(x)\bar{x}+S_x}\right)\right]$	$\beta_2(x)$	S_x
$T_{rl8} = w_1\bar{y} + w_2\bar{y}\left[1+\log\left(\frac{\bar{X}+\rho}{\bar{x}+\rho}\right)\right]$	1	ρ
$T_{rl9} = w_1\bar{y} + w_2\bar{y}\left[1+\log\left(\frac{\bar{X}+\beta_2(x)}{\bar{x}+\beta_2(x)}\right)\right]$	1	$\beta_2(x)$
$T_{rl10} = w_1\bar{y} + w_2\bar{y}\left[1+\log\left(\frac{C_x\bar{X}+\rho}{C_x\bar{x}+\rho}\right)\right]$	C_x	ρ
$T_{rl11} = w_1\bar{y} + w_2\bar{y}\left[1+\log\left(\frac{\rho\bar{X}+C_x}{\rho\bar{x}+C_x}\right)\right]$	ρ	C_x
$T_{rl12} = w_1\bar{y} + w_2\bar{y}\left[1+\log\left(\frac{\beta_2(x)\bar{X}+\rho}{\beta_2(x)\bar{x}+\rho}\right)\right]$	$\beta_2(x)$	ρ
$T_{rl13} = w_1\bar{y} + w_2\bar{y}\left[1+\log\left(\frac{\rho\bar{X}+\beta_2(x)}{\rho\bar{x}+\beta_2(x)}\right)\right]$	ρ	$\beta_2(x)$

Table 2: Some members of the class of estimators $'T'$

Product-type logarithmic estimator $(g,\beta)=(0,1)$	a	b
$T_{pl1}=w_1\bar{y}+w_2\bar{y}\left[1+\log\left(\frac{\bar{x}}{\bar{X}}\right)\right]$	1	0
$T_{pl2}=w_1\bar{y}+w_2\bar{y}\left[1+\log\left(\frac{\bar{x}+C_x}{\bar{X}+C_x}\right)\right]$	1	C_x
$T_{pl3}=w_1\bar{y}+w_2\bar{y}\left[1+\log\left(\frac{\beta_2(x).\bar{x}+C_x}{\beta_2(x).\bar{X}+C_x}\right)\right]$	$\beta_2(x)$	C_x
$T_{pl4}=w_1\bar{y}+w_2\bar{y}\left[1+\log\left(\frac{C_x.\bar{x}+\beta_2(x)}{C_x.\bar{X}+\beta_2(x)}\right)\right]$	C_x	$\beta_2(x)$
$T_{pl5}=w_1\bar{y}+w_2\bar{y}\left[1+\log\left(\frac{\bar{x}+S_x}{\bar{X}+S_x}\right)\right]$	1	S_x
$T_{pl6}=w_1\bar{y}+w_2\bar{y}\left[1+\log\left(\frac{\beta_1(x).\bar{x}+S_x}{\beta_1(x).\bar{X}+S_x}\right)\right]$	$\beta_1(x)$	S_x
$T_{pl7}=w_1\bar{y}+w_2\bar{y}\left[1+\log\left(\frac{\beta_2(x).\bar{x}+S_x}{\beta_2(x).\bar{X}+S_x}\right)\right]$	$\beta_2(x)$	S_x
$T_{pl8}=w_1\bar{y}+w_2\bar{y}\left[1+\log\left(\frac{\bar{x}+\rho}{\bar{X}+\rho}\right)\right]$	1	ρ
$T_{pl9}=w_1\bar{y}+w_2\bar{y}\left[1+\log\left(\frac{\bar{x}+\beta_2(x)}{\bar{X}+\beta_2(x)}\right)\right]$	1	$\beta_2(x)$
$T_{pl10}=w_1\bar{y}+w_2\bar{y}\left[1+\log\left(\frac{C_x.\bar{x}+\rho}{C_x.\bar{X}+\rho}\right)\right]$	C_x	ρ
$T_{pl11}=w_1\bar{y}+w_2\bar{y}\left[1+\log\left(\frac{\rho\bar{x}+C_x}{\rho\bar{X}+C_x}\right)\right]$	ρ	C_x
$T_{pl12}=w_1\bar{y}+w_2\bar{y}\left[1+\log\left(\frac{\beta_2(x).\bar{x}+\rho}{\beta_2(x).\bar{X}+\rho}\right)\right]$	$\beta_2(x)$	ρ
$T_{pl13}=w_1\bar{y}+w_2\bar{y}\left[1+\log\left(\frac{\rho\bar{x}+\beta_2(x)}{\rho\bar{X}+\beta_2(x)}\right)\right]$	ρ	$\beta_2(x)$

Table 3: Some members of the class of estimators $'T'$

Ratio-ratio type logarithmic estimator $(\alpha, g, \beta) = (1,1,-1)$	a	b
$T_{rrl1} = w_1 \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right) + w_2 \bar{y}\left[1+\log\left(\frac{\bar{X}}{\bar{x}}\right)\right]$	1	0
$T_{rrl2} = w_1 \bar{y}\left(\frac{\bar{X}+C_x}{\bar{x}+C_x}\right) + w_2 \bar{y}\left[1+\log\left(\frac{\bar{X}+C_x}{\bar{x}+C_x}\right)\right]$	1	C_x
$T_{rrl3} = w_1 \bar{y}\left(\frac{\beta_2(x)\bar{X}+C_x}{\beta_2(x)\bar{x}+C_x}\right) + w_2 \bar{y}\left[1+\log\left(\frac{\beta_2(x).\bar{X}+C_x}{\beta_2(x).\bar{x}+C_x}\right)\right]$	$\beta_2(x)$	C_x
$T_{rrl4} = w_1 \bar{y}\left(\frac{C_x\bar{X}+\beta_2(x)}{C_x\bar{x}+\beta_2(x)}\right) + w_2 \bar{y}\left[1+\log\left(\frac{C_x.\bar{X}+\beta_2(x)}{C_x.\bar{x}+\beta_2(x)}\right)\right]$	C_x	$\beta_2(x)$
$T_{rrl5} = w_1 \bar{y}\left(\frac{\bar{X}+S_x}{\bar{x}+S_x}\right) + w_2 \bar{y}\left[1+\log\left(\frac{\bar{X}+S_x}{\bar{x}+S_x}\right)\right]$	1	S_x
$T_{rrl6} = w_1 \bar{y}\left(\frac{\beta_1(x)\bar{X}+S_x}{\beta_1(x)\bar{x}+S_x}\right) + w_2 \bar{y}\left[1+\log\left(\frac{\beta_1(x).\bar{X}+S_x}{\beta_1(x).\bar{x}+S_x}\right)\right]$	$\beta_1(x)$	S_x
$T_{rrl7} = w_1 \bar{y}\left(\frac{\beta_2(x)\bar{X}+S_x}{\beta_2(x)\bar{x}+S_x}\right) + w_2 \bar{y}\left[1+\log\left(\frac{\beta_2(x).\bar{X}+S_x}{\beta_2(x).\bar{x}+S_x}\right)\right]$	$\beta_2(x)$	S_x
$T_{rrl8} = w_1 \bar{y}\left(\frac{\bar{X}+\rho}{\bar{x}+\rho}\right) + w_2 \bar{y}\left[1+\log\left(\frac{\bar{X}+\rho}{\bar{x}+\rho}\right)\right]$	1	ρ
$T_{rrl9} = w_1 \bar{y}\left(\frac{\bar{X}+\beta_2(x)}{\bar{x}+\beta_2(x)}\right) + w_2 \bar{y}\left[1+\log\left(\frac{\bar{X}+\beta_2(x)}{\bar{x}+\beta_2(x)}\right)\right]$	1	$\beta_2(x)$
$T_{rrl10} = w_1 \bar{y}\left(\frac{C_x\bar{X}+\rho}{C_x\bar{x}+\rho}\right) + w_2 \bar{y}\left[1+\log\left(\frac{C_x.\bar{X}+\rho}{C_x.\bar{x}+\rho}\right)\right]$	C_x	ρ
$T_{rrl11} = w_1 \bar{y}\left(\frac{\rho\bar{X}+C_x}{\rho\bar{x}+C_x}\right) + w_2 \bar{y}\left[1+\log\left(\frac{\rho\bar{X}+C_x}{\rho\bar{x}+C_x}\right)\right]$	ρ	C_x
$T_{rrl12} = w_1 \bar{y}\left(\frac{\beta_2(x)\bar{X}+\rho}{\beta_2(x)\bar{x}+\rho}\right) + w_2 \bar{y}\left[1+\log\left(\frac{\beta_2(x).\bar{X}+\rho}{\beta_2(x).\bar{x}+\rho}\right)\right]$	$\beta_2(x)$	ρ
$T_{rrl13} = w_1 \bar{y}\left(\frac{\rho\bar{X}+\beta_2(x)}{\rho\bar{x}+\beta_2(x)}\right) + w_2 \bar{y}\left[1+\log\left(\frac{\rho\bar{X}+\beta_2(x)}{\rho\bar{x}+\beta_2(x)}\right)\right]$	ρ	$\beta_2(x)$

Table 4: Some members of the class of estimators $'T'$

ratio-product type logarithmic estimators $(\alpha, g, \beta) = (1,1,1)$	a	b
$T_{rpl1} = w_1\bar{y}\left(\frac{\bar{X}}{\bar{x}}\right) + w_2\bar{y}\left[1+\log\left(\frac{\bar{x}}{\bar{X}}\right)\right]$	1	0
$T_{rpl2} = w_1\bar{y}\left(\frac{\bar{X}+C_x}{\bar{x}+C_x}\right) + w_2\bar{y}\left[1+\log\left(\frac{\bar{x}+C_x}{\bar{X}+C_x}\right)\right]$	1	C_x
$T_{rpl3} = w_1\bar{y}\left(\frac{\beta_2(x)\bar{X}+C_x}{\beta_2(x)\bar{x}+C_x}\right) + w_2\bar{y}\left[1+\log\left(\frac{\beta_2(x).\bar{x}+C_x}{\beta_2(x).\bar{X}+C_x}\right)\right]$	$\beta_2(x)$	C_x
$T_{rpl4} = w_1\bar{y}\left(\frac{C_x\bar{X}+\beta_2(x)}{C_x\bar{x}+\beta_2(x)}\right) + w_2\bar{y}\left[1+\log\left(\frac{C_x.\bar{x}+\beta_2(x)}{C_x.\bar{X}+\beta_2(x)}\right)\right]$	C_x	$\beta_2(x)$
$T_{rpl5} = w_1\bar{y}\left(\frac{\bar{X}+S_x}{\bar{x}+S_x}\right) + w_2\bar{y}\left[1+\log\left(\frac{\bar{x}+S_x}{\bar{X}+S_x}\right)\right]$	1	S_x
$T_{rpl6} = w_1\bar{y}\left(\frac{\beta_1(x)\bar{X}+S_x}{\beta_1(x)\bar{x}+S_x}\right) + w_2\bar{y}\left[1+\log\left(\frac{\beta_1(x).\bar{x}+S_x}{\beta_1(x).\bar{X}+S_x}\right)\right]$	$\beta_1(x)$	S_x
$T_{rpl7} = w_1\bar{y}\left(\frac{\beta_2(x)\bar{X}+S_x}{\beta_2(x)\bar{x}+S_x}\right) + w_2\bar{y}\left[1+\log\left(\frac{\beta_2(x).\bar{x}+S_x}{\beta_2(x).\bar{X}+S_x}\right)\right]$	$\beta_2(x)$	S_x
$T_{rpl8} = w_1\bar{y}\left(\frac{\bar{X}+\rho}{\bar{x}+\rho}\right) + w_2\bar{y}\left[1+\log\left(\frac{\bar{x}+\rho}{\bar{X}+\rho}\right)\right]$	1	ρ
$T_{rpl9} = w_1\bar{y}\left(\frac{\bar{X}+\beta_2(x)}{\bar{x}+\beta_2(x)}\right) + w_2\bar{y}\left[1+\log\left(\frac{\bar{x}+\beta_2(x)}{\bar{X}+\beta_2(x)}\right)\right]$	1	$\beta_2(x)$
$T_{rpl10} = w_1\bar{y}\left(\frac{C_x\bar{X}+\rho}{C_x\bar{x}+\rho}\right) + w_2\bar{y}\left[1+\log\left(\frac{C_x.\bar{x}+\rho}{C_x.\bar{X}+\rho}\right)\right]$	C_x	ρ
$T_{rpl11} = w_1\bar{y}\left(\frac{\rho\bar{X}+C_x}{\rho\bar{x}+C_x}\right) + w_2\bar{y}\left[1+\log\left(\frac{\rho\bar{x}+C_x}{\rho\bar{X}+C_x}\right)\right]$	ρ	C_x
$T_{rpl12} = w_1\bar{y}\left(\frac{\beta_2(x)\bar{X}+\rho}{\beta_2(x)\bar{x}+\rho}\right) + w_2\bar{y}\left[1+\log\left(\frac{\beta_2(x).\bar{x}+\rho}{\beta_2(x).\bar{X}+\rho}\right)\right]$	$\beta_2(x)$	ρ
$T_{rpl13} = w_1\bar{y}\left(\frac{\rho\bar{X}+\beta_2(x)}{\rho\bar{x}+\beta_2(x)}\right) + w_2\bar{y}\left[1+\log\left(\frac{\rho\bar{x}+\beta_2(x)}{\rho\bar{X}+\beta_2(x)}\right)\right]$	ρ	$\beta_2(x)$

Table 5: Some members of the class of estimators $'T'$

Product-product type logarithmic estimators $(\alpha, g, \beta) = (1, -1, 1)$	a	b
$T_{ppl1} = w_1\bar{y}\left(\frac{\bar{x}}{\bar{X}}\right) + w_2\bar{y}\left[1 + \log\left(\frac{\bar{x}}{\bar{X}}\right)\right]$	1	0
$T_{ppl2} = w_1\bar{y}\left(\frac{\bar{x}+C_x}{\bar{X}+C_x}\right) + w_2\bar{y}\left[1 + \log\left(\frac{\bar{x}+C_x}{\bar{X}+C_x}\right)\right]$	1	C_x
$T_{ppl3} = w_1\bar{y}\left(\frac{\beta_2(x)\bar{x}+C_x}{\beta_2(x)\bar{X}+C_x}\right) + w_2\bar{y}\left[1 + \log\left(\frac{\beta_2(x).\bar{x}+C_x}{\beta_2(x).\bar{X}+C_x}\right)\right]$	$\beta_2(x)$	C_x
$T_{ppl4} = w_1\bar{y}\left(\frac{C_x\bar{x}+\beta_2(x)}{C_x\bar{X}+\beta_2(x)}\right) + w_2\bar{y}\left[1 + \log\left(\frac{C_x\bar{x}+\beta_2(x)}{C_x\bar{X}+\beta_2(x)}\right)\right]$	C_x	$\beta_2(x)$
$T_{ppl5} = w_1\bar{y}\left(\frac{\bar{x}+S_x}{\bar{X}+S_x}\right) + w_2\bar{y}\left[1 + \log\left(\frac{\bar{x}+S_x}{\bar{X}+S_x}\right)\right]$	1	S_x
$T_{ppl6} = w_1\bar{y}\left(\frac{\beta_1(x)\bar{x}+S_x}{\beta_1(x)\bar{X}+S_x}\right) + w_2\bar{y}\left[1 + \log\left(\frac{\beta_1(x).\bar{x}+S_x}{\beta_1(x).\bar{X}+S_x}\right)\right]$	$\beta_1(x)$	S_x
$T_{ppl7} = w_1\bar{y}\left(\frac{\beta_2(x)\bar{x}+S_x}{\beta_2(x)\bar{X}+S_x}\right) + w_2\bar{y}\left[1 + \log\left(\frac{\beta_2(x).\bar{x}+S_x}{\beta_2(x).\bar{X}+S_x}\right)\right]$	$\beta_2(x)$	S_x
$T_{ppl8} = w_1\bar{y}\left(\frac{\bar{x}+\rho}{\bar{X}+\rho}\right) + w_2\bar{y}\left[1 + \log\left(\frac{\bar{x}+\rho}{\bar{X}+\rho}\right)\right]$	1	ρ
$T_{ppl9} = w_1\bar{y}\left(\frac{\bar{x}+\beta_2(x)}{\bar{X}+\beta_2(x)}\right) + w_2\bar{y}\left[1 + \log\left(\frac{\bar{x}+\beta_2(x)}{\bar{X}+\beta_2(x)}\right)\right]$	1	$\beta_2(x)$
$T_{ppl10} = w_1\bar{y}\left(\frac{C_x\bar{x}+\rho}{C_x\bar{X}+\rho}\right) + w_2\bar{y}\left[1 + \log\left(\frac{C_x.\bar{x}+\rho}{C_x.\bar{X}+\rho}\right)\right]$	C_x	ρ
$T_{ppl11} = w_1\bar{y}\left(\frac{\rho\bar{x}+C_x}{\rho\bar{X}+C_x}\right) + w_2\bar{y}\left[1 + \log\left(\frac{\rho\bar{x}+C_x}{\rho\bar{X}+C_x}\right)\right]$	ρ	C_x
$T_{ppl12} = w_1\bar{y}\left(\frac{\beta_2(x)\bar{x}+\rho}{\beta_2(x)\bar{X}+\rho}\right) + w_2\bar{y}\left[1 + \log\left(\frac{\beta_2(x).\bar{x}+\rho}{\beta_2(x).\bar{X}+\rho}\right)\right]$	$\beta_2(x)$	ρ
$T_{ppl13} = w_1\bar{y}\left(\frac{\rho\bar{x}+\beta_2(x)}{\rho\bar{X}+\beta_2(x)}\right) + w_2\bar{y}\left[1 + \log\left(\frac{\rho\bar{x}+\beta_2(x)}{\rho\bar{X}+\beta_2(x)}\right)\right]$	ρ	$\beta_2(x)$

Table 6: Some members of the class of estimators $'T'$

Product-ratio type logarithmic estimators $(\alpha, g, \delta) = (1, -1, 1)$	a	b
$T_{prl1} = w_1\bar{y}\left(\frac{\bar{x}}{\bar{X}}\right) + w_2\bar{y}\left[1+\log\left(\frac{\bar{X}}{\bar{x}}\right)\right]$	1	0
$T_{prl2} = w_1\bar{y}\left(\frac{\bar{x}+C_x}{\bar{X}+C_x}\right) + w_2\bar{y}\left[1+\log\left(\frac{\bar{X}+C_x}{\bar{x}+C_x}\right)\right]$	1	C_x
$T_{prl3} = w_1\bar{y}\left(\frac{\beta_2(x)\bar{x}+C_x}{\beta_2(x)\bar{X}+C_x}\right) + w_2\bar{y}\left[1+\log\left(\frac{\beta_2(x).\bar{X}+C_x}{\beta_2(x).\bar{x}+C_x}\right)\right]$	$\beta_2(x)$	C_x
$T_{prl4} = w_1\bar{y}\left(\frac{C_x\bar{x}+\beta_2(x)}{C_x\bar{X}+\beta_2(x)}\right) + w_2\bar{y}\left[1+\log\left(\frac{C_x.\bar{X}+\beta_2(x)}{C_x.\bar{x}+\beta_2(x)}\right)\right]$	C_x	$\beta_2(x)$
$T_{prl5} = w_1\bar{y}\left(\frac{\bar{x}+S_x}{\bar{X}+S_x}\right) + w_2\bar{y}\left[1+\log\left(\frac{\bar{X}+S_x}{\bar{x}+S_x}\right)\right]$	1	S_x
$T_{prl6} = w_1\bar{y}\left(\frac{\beta_1(x)\bar{x}+S_x}{\beta_1(x)\bar{X}+S_x}\right) + w_2\bar{y}\left[1+\log\left(\frac{\beta_1(x).\bar{X}+S_x}{\beta_1(x).\bar{x}+S_x}\right)\right]$	$\beta_1(x)$	S_x
$T_{prl7} = w_1\bar{y}\left(\frac{\beta_2(x)\bar{x}+S_x}{\beta_2(x)\bar{X}+S_x}\right) + w_2\bar{y}\left[1+\log\left(\frac{\beta_2(x).\bar{X}+S_x}{\beta_2(x).\bar{x}+S_x}\right)\right]$	$\beta_2(x)$	S_x
$T_{prl8} = w_1\bar{y}\left(\frac{\bar{x}+\rho}{\bar{X}+\rho}\right) + w_2\bar{y}\left[1+\log\left(\frac{\bar{X}+\rho}{\bar{x}+\rho}\right)\right]$	1	ρ
$T_{prl9} = w_1\bar{y}\left(\frac{\bar{x}+\beta_2(x)}{\bar{X}+\beta_2(x)}\right) + w_2\bar{y}\left[1+\log\left(\frac{\bar{X}+\beta_2(x)}{\bar{x}+\beta_2(x)}\right)\right]$	1	$\beta_2(x)$
$T_{prl10} = w_1\bar{y}\left(\frac{C_x\bar{x}+\rho}{C_x\bar{X}+\rho}\right) + w_2\bar{y}\left[1+\log\left(\frac{C_x.\bar{X}+\rho}{C_x.\bar{x}+\rho}\right)\right]$	C_x	ρ
$T_{prl11} = w_1\bar{y}\left(\frac{\rho\bar{x}+C_x}{\rho\bar{X}+C_x}\right) + w_2\bar{y}\left[1+\log\left(\frac{\rho\bar{X}+C_x}{\rho\bar{x}+C_x}\right)\right]$	ρ	C_x
$T_{prl12} = w_1\bar{y}\left(\frac{\beta_2(x)\bar{x}+\rho}{\beta_2(x)\bar{X}+\rho}\right) + w_2\bar{y}\left[1+\log\left(\frac{\beta_2(x).\bar{X}+\rho}{\beta_2(x).\bar{x}+\rho}\right)\right]$	$\beta_2(x)$	ρ
$T_{prl13} = w_1\bar{y}\left(\frac{\rho\bar{x}+\beta_2(x)}{\rho\bar{X}+\beta_2(x)}\right) + w_2\bar{y}\left[1+\log\left(\frac{\rho\bar{X}+\beta_2(x)}{\rho\bar{x}+\beta_2(x)}\right)\right]$	ρ	$\beta_2(x)$

Table 7: Some members of the class of estimators T_K

Ratio type estimators $(\alpha, g) = (1,1)$	Product type estimators $(\alpha, g) = (1,-1)$	a	b
$T_{Kr1} = w_1\bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)$	$T_{Kp1} = w_1\bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)$	1	0
$T_{Kr2} = w_1\bar{y}\left(\frac{\bar{X}+C_x}{\bar{x}+C_x}\right)$	$T_{Kp2} = w_1\bar{y}\left(\frac{\bar{x}+C_x}{\bar{X}+C_x}\right)$	1	C_x
$T_{Kr3} = w_1\bar{y}\left(\frac{\beta_2(x)\bar{X}+C_x}{\beta_2(x)\bar{x}+C_x}\right)$	$T_{Kp3} = w_1\bar{y}\left(\frac{\beta_2(x)\bar{x}+C_x}{\beta_2(x)\bar{X}+C_x}\right)$	$\beta_2(x)$	C_x
$T_{Kr4} = w_1\bar{y}\left(\frac{C_x\bar{X}+\beta_2(x)}{C_x\bar{x}+\beta_2(x)}\right)$	$T_{Kp4} = w_1\bar{y}\left(\frac{C_x\bar{x}+\beta_2(x)}{C_x\bar{X}+\beta_2(x)}\right)$	C_x	$\beta_2(x)$
$T_{Kr5} = w_1\bar{y}\left(\frac{\bar{X}+S_x}{\bar{x}+S_x}\right)$	$T_{Kp5} = w_1\bar{y}\left(\frac{\bar{x}+S_x}{\bar{X}+S_x}\right)$	1	S_x
$T_{Kr6} = w_1\bar{y}\left(\frac{\beta_1(x)\bar{X}+S_x}{\beta_1(x)\bar{x}+S_x}\right)$	$T_{Kp6} = w_1\bar{y}\left(\frac{\beta_1(x)\bar{x}+S_x}{\beta_1(x)\bar{X}+S_x}\right)$	$\beta_1(x)$	S_x
$T_{Kr7} = w_1\bar{y}\left(\frac{\beta_2(x)\bar{X}+S_x}{\beta_2(x)\bar{x}+S_x}\right)$	$T_{Kp7} = w_1\bar{y}\left(\frac{\beta_2(x)\bar{x}+S_x}{\beta_2(x)\bar{X}+S_x}\right)$	$\beta_2(x)$	S_x
$T_{Kr8} = w_1\bar{y}\left(\frac{\bar{X}+\rho}{\bar{x}+\rho}\right)$	$T_{Kp8} = w_1\bar{y}\left(\frac{\bar{x}+\rho}{\bar{X}+\rho}\right)$	1	ρ
$T_{Kr9} = w_1\bar{y}\left(\frac{\bar{X}+\beta_2(x)}{\bar{x}+\beta_2(x)}\right)$	$T_{Kp9} = w_1\bar{y}\left(\frac{\bar{x}+\beta_2(x)}{\bar{X}+\beta_2(x)}\right)$	1	$\beta_2(x)$
$T_{Kr10} = w_1\bar{y}\left(\frac{C_x\bar{X}+\rho}{C_x\bar{x}+\rho}\right)$	$T_{Kp10} = w_1\bar{y}\left(\frac{C_x\bar{x}+\rho}{C_x\bar{X}+\rho}\right)$	C_x	ρ
$T_{Kr11} = w_1\bar{y}\left(\frac{\rho\bar{X}+C_x}{\rho\bar{x}+C_x}\right)$	$T_{Kp11} = w_1\bar{y}\left(\frac{\rho\bar{x}+C_x}{\rho\bar{X}+C_x}\right)$	ρ	C_x
$T_{Kr12} = w_1\bar{y}\left(\frac{\beta_2(x)\bar{X}+\rho}{\beta_2(x)\bar{x}+\rho}\right)$	$T_{Kp12} = w_1\bar{y}\left(\frac{\beta_2(x)\bar{x}+\rho}{\beta_2(x)\bar{X}+\rho}\right)$	$\beta_2(x)$	ρ
$T_{Kr13} = w_1\bar{y}\left(\frac{\rho\bar{X}+\beta_2(x)}{\rho\bar{x}+\beta_2(x)}\right)$	$T_{Kp13} = w_1\bar{y}\left(\frac{\rho\bar{x}+\beta_2(x)}{\rho\bar{X}+\beta_2(x)}\right)$	ρ	$\beta_2(x)$

Table 8: Some members of the class of estimators η_K

Ratio type estimators $(\alpha,g)=(1,1)$	Product type estimators $(\alpha,g)=(1,-1)$	a	b
$\eta_{Kr1}=w_1\bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)$	$\eta_{Kp1}=w_1\bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)$	**1**	**0**
$\eta_{Kr2}=w_1\bar{y}\left(\frac{\bar{X}+C_x}{\bar{x}+C_x}\right)$	$\eta_{Kp2}=w_1\bar{y}\left(\frac{\bar{x}+C_x}{\bar{X}+C_x}\right)$	**1**	C_x
$\eta_{Kr3}=w_1\bar{y}\left(\frac{\beta_2(x)\bar{X}+C_x}{\beta_2(x)\bar{x}+C_x}\right)$	$\eta_{Kp3}=w_1\bar{y}\left(\frac{\beta_2(x)\bar{x}+C_x}{\beta_2(x)\bar{X}+C_x}\right)$	$\beta_2(x)$	C_x
$\eta_{Kr4}=w_1\bar{y}\left(\frac{C_x\bar{X}+\beta_2(x)}{C_x\bar{x}+\beta_2(x)}\right)$	$\eta_{Kp4}=w_1\bar{y}\left(\frac{C_x\bar{x}+\beta_2(x)}{C_x\bar{X}+\beta_2(x)}\right)$	C_x	$\beta_2(x)$
$\eta_{Kr5}=w_1\bar{y}\left(\frac{\bar{X}+S_x}{\bar{x}+S_x}\right)$	$\eta_{Kp5}=w_1\bar{y}\left(\frac{\bar{x}+S_x}{\bar{X}+S_x}\right)$	**1**	S_x
$\eta_{Kr6}=w_1\bar{y}\left(\frac{\beta_1(x)\bar{X}+S_x}{\beta_1(x)\bar{x}+S_x}\right)$	$\eta_{Kp6}=w_1\bar{y}\left(\frac{\beta_1(x)\bar{x}+S_x}{\beta_1(x)\bar{X}+S_x}\right)$	$\beta_1(x)$	S_x
$\eta_{Kr7}=w_1\bar{y}\left(\frac{\beta_2(x)\bar{X}+S_x}{\beta_2(x)\bar{x}+S_x}\right)$	$\eta_{Kp7}=w_1\bar{y}\left(\frac{\beta_2(x)\bar{x}+S_x}{\beta_2(x)\bar{X}+S_x}\right)$	$\beta_2(x)$	S_x
$\eta_{Kr8}=w_1\bar{y}\left(\frac{\bar{X}+\rho}{\bar{x}+\rho}\right)$	$\eta_{Kp8}=w_1\bar{y}\left(\frac{\bar{x}+\rho}{\bar{X}+\rho}\right)$	**1**	ρ
$\eta_{Kr9}=w_1\bar{y}\left(\frac{\bar{X}+\beta_2(x)}{\bar{x}+\beta_2(x)}\right)$	$\eta_{Kp9}=w_1\bar{y}\left(\frac{\bar{x}+\beta_2(x)}{\bar{X}+\beta_2(x)}\right)$	**1**	$\beta_2(x)$
$\eta_{Kr10}=w_1\bar{y}\left(\frac{C_x\bar{X}+\rho}{C_x\bar{x}+\rho}\right)$	$\eta_{Kp10}=w_1\bar{y}\left(\frac{C_x\bar{x}+\rho}{C_x\bar{X}+\rho}\right)$	C_x	ρ
$\eta_{Kr11}=w_1\bar{y}\left(\frac{\rho\bar{X}+C_x}{\rho\bar{x}+C_x}\right)$	$\eta_{Kp11}=w_1\bar{y}\left(\frac{\rho\bar{x}+C_x}{\rho\bar{X}+C_x}\right)$	ρ	C_x
$\eta_{Kr12}=w_1\bar{y}\left(\frac{\beta_2(x)\bar{X}+\rho}{\beta_2(x)\bar{x}+\rho}\right)$	$\eta_{Kp12}=w_1\bar{y}\left(\frac{\beta_2(x)\bar{x}+\rho}{\beta_2(x)\bar{X}+\rho}\right)$	$\beta_2(x)$	ρ
$\eta_{Kr13}=w_1\bar{y}\left(\frac{\rho\bar{X}+\beta_2(x)}{\rho\bar{x}+\beta_2(x)}\right)$	$\eta_{Kp13}=w_1\bar{y}\left(\frac{\rho\bar{x}+\beta_2(x)}{\rho\bar{X}+\beta_2(x)}\right)$	ρ	$\beta_2(x)$

Table 9: Some members of the class of estimators ξ

Ratio type estimators $(\beta=-1)$	Product type estimators $(\beta=1)$	a	b
$\xi_{r1}=\bar{y}\left[1+\log\left(\frac{\bar{X}}{\bar{x}}\right)\right]$	$\xi_{p1}=\bar{y}\left[1+\log\left(\frac{\bar{x}}{\bar{X}}\right)\right]$	1	0
$\xi_{r2}=\left[1+\log\left(\frac{\bar{X}+C_x}{\bar{x}+C_x}\right)\right]$	$\xi_{p2}=\bar{y}\left[1+\log\left(\frac{\bar{x}+C_x}{\bar{X}+C_x}\right)\right]$	1	C_x
$\xi_{r3}=\left[1+\log\left(\frac{\beta_2(x).\bar{X}+C_x}{\beta_2(x).\bar{x}+C_x}\right)\right]$	$\xi_{p3}=\bar{y}\left[1+\log\left(\frac{\beta_2(x).\bar{x}+C_x}{\beta_2(x).\bar{X}+C_x}\right)\right]$	$\beta_2(x)$	C_x
$\xi_{r4}=\left[1+\log\left(\frac{C_x.\bar{X}+\beta_2(x)}{C_x.\bar{x}+\beta_2(x)}\right)\right]$	$\xi_{p4}=\bar{y}\left[1+\log\left(\frac{C_x.\bar{x}+\beta_2(x)}{C_x.\bar{X}+\beta_2(x)}\right)\right]$	C_x	$\beta_2(x)$
$\xi_{r5}=\left[1+\log\left(\frac{\bar{X}+S_x}{\bar{x}+S_x}\right)\right]$	$\xi_{p5}=\bar{y}\left[1+\log\left(\frac{\bar{x}+S_x}{\bar{X}+S_x}\right)\right]$	1	S_x
$\xi_{r6}=\left[1+\log\left(\frac{\beta_1(x).\bar{X}+S_x}{\beta_1(x).\bar{x}+S_x}\right)\right]$	$\xi_{p6}=\bar{y}\left[1+\log\left(\frac{\beta_1(x).\bar{x}+S_x}{\beta_1(x).\bar{X}+S_x}\right)\right]$	$\beta_1(x)$	S_x
$\xi_{r7}=\left[1+\log\left(\frac{\beta_2(x).\bar{X}+S_x}{\beta_2(x).\bar{x}+S_x}\right)\right]$	$\xi_{p7}=\bar{y}\left[1+\log\left(\frac{\beta_2(x).\bar{x}+S_x}{\beta_2(x).\bar{X}+S_x}\right)\right]$	$\beta_2(x)$	S_x
$\xi_{r8}=\left[1+\log\left(\frac{\bar{X}+\rho}{\bar{x}+\rho}\right)\right]$	$\xi_{p8}=\bar{y}\left[1+\log\left(\frac{\bar{x}+\rho}{\bar{X}+\rho}\right)\right]$	1	ρ
$\xi_{r9}=\left[1+\log\left(\frac{\bar{X}+\beta_2(x)}{\bar{x}+\beta_2(x)}\right)\right]$	$\xi_{p9}=\bar{y}\left[1+\log\left(\frac{\bar{x}+\beta_2(x)}{\bar{X}+\beta_2(x)}\right)\right]$	1	$\beta_2(x)$
$\xi_{r10}=\left[1+\log\left(\frac{C_x.\bar{X}+\rho}{C_x.\bar{x}+\rho}\right)\right]$	$\xi_{p10}=\bar{y}\left[1+\log\left(\frac{C_x\bar{x}+\rho}{C_x\bar{X}+\rho}\right)\right]$	C_x	ρ
$\xi_{r11}=\left[1+\log\left(\frac{\rho\bar{X}+C_x}{\rho\bar{x}+C_x}\right)\right]$	$\xi_{p11}=\bar{y}\left[1+\log\left(\frac{\rho\bar{x}+C_x}{\rho\bar{X}+C_x}\right)\right]$	ρ	C_x
$\xi_{r12}=\left[1+\log\left(\frac{\beta_2(x).\bar{X}+\rho}{\beta_2(x).\bar{x}+\rho}\right)\right]$	$\xi_{p12}=\bar{y}\left[1+\log\left(\frac{\beta_2(x).\bar{x}+\rho}{\beta_2(x).\bar{X}+\rho}\right)\right]$	$\beta_2(x)$	ρ
$\xi_{r13}=\left[1+\log\left(\frac{\rho\bar{X}+\beta_2(x)}{\rho\bar{x}+\beta_2(x)}\right)\right]$	$\xi_{p13}=\bar{y}\left[1+\log\left(\frac{\rho\bar{x}+\beta_2(x)}{\rho\bar{X}+\beta_2(x)}\right)\right]$	ρ	$\beta_2(x)$

Table 10: Some members of the class of estimators ψ

Ratio type estimators $(\beta=-1)$	Product type estimators $(\beta=1)$	a	b
$\psi_{r1}=w_2\bar{y}\left[1+\log\left(\frac{\bar{X}}{\bar{x}}\right)\right]$	$\psi_{p1}=w_2\bar{y}\left[1+\log\left(\frac{\bar{x}}{\bar{X}}\right)\right]$	1	0
$\psi_{r2}=w_2\bar{y}\left[1+\log\left(\frac{\bar{X}+C_x}{\bar{x}+C_x}\right)\right]$	$\psi_{p2}=w_2\bar{y}\left[1+\log\left(\frac{\bar{x}+C_x}{\bar{X}+C_x}\right)\right]$	1	C_x
$\psi_{r3}=w_2\bar{y}\left[1+\log\left(\frac{\beta_2(x).\bar{X}+C_x}{\beta_2(x).\bar{x}+C_x}\right)\right]$	$\psi_{p3}=w_2\bar{y}\left[1+\log\left(\frac{\beta_2(x).\bar{x}+C_x}{\beta_2(x).\bar{X}+C_x}\right)\right]$	$\beta_2(x)$	C_x
$\psi_{r4}=w_2\bar{y}\left[1+\log\left(\frac{C_x.\bar{X}+\beta_2(x)}{C_x.\bar{x}+\beta_2(x)}\right)\right]$	$\psi_{p4}=w_2\bar{y}\left[1+\log\left(\frac{C_x.\bar{x}+\beta_2(x)}{C_x.\bar{X}+\beta_2(x)}\right)\right]$	C_x	$\beta_2(x)$
$\psi_{r5}=w_2\bar{y}\left[1+\log\left(\frac{\bar{X}+S_x}{\bar{x}+S_x}\right)\right]$	$\psi_{p5}=w_2\bar{y}\left[1+\log\left(\frac{\bar{x}+S_x}{\bar{X}+S_x}\right)\right]$	1	S_x
$\psi_{r6}=w_2\bar{y}\left[1+\log\left(\frac{\beta_1(x).\bar{X}+S_x}{\beta_1(x).\bar{x}+S_x}\right)\right]$	$\psi_{p6}=w_2\bar{y}\left[1+\log\left(\frac{\beta_1(x).\bar{x}+S_x}{\beta_1(x).\bar{X}+S_x}\right)\right]$	$\beta_1(x)$	S_x
$\psi_{r7}=w_2\bar{y}\left[1+\log\left(\frac{\beta_2(x).\bar{X}+S_x}{\beta_2(x).\bar{x}+S_x}\right)\right]$	$\psi_{p7}=w_2\bar{y}\left[1+\log\left(\frac{\beta_2(x).\bar{x}+S_x}{\beta_2(x).\bar{X}+S_x}\right)\right]$	$\beta_2(x)$	S_x
$\psi_{r8}=w_2\bar{y}\left[1+\log\left(\frac{\bar{X}+\rho}{\bar{x}+\rho}\right)\right]$	$\psi_{p8}=w_2\bar{y}\left[1+\log\left(\frac{\bar{x}+\rho}{\bar{X}+\rho}\right)\right]$	1	ρ
$\psi_{r9}=w_2\bar{y}\left[1+\log\left(\frac{\bar{X}+\beta_2(x)}{\bar{x}+\beta_2(x)}\right)\right]$	$\psi_{p9}=w_2\bar{y}\left[1+\log\left(\frac{\bar{x}+\beta_2(x)}{\bar{X}+\beta_2(x)}\right)\right]$	1	$\beta_2(x)$
$\psi_{r10}=w_2\bar{y}\left[1+\log\left(\frac{C_x.\bar{X}+\rho}{C_x.\bar{x}+\rho}\right)\right]$	$\psi_{p10}=w_2\bar{y}\left[1+\log\left(\frac{C_x\bar{x}+\rho}{C_x\bar{X}+\rho}\right)\right]$	C_x	ρ
$\psi_{r11}=w_2\bar{y}\left[1+\log\left(\frac{\rho\bar{X}+C_x}{\rho\bar{x}+C_x}\right)\right]$	$\psi_{p11}=w_2\bar{y}\left[1+\log\left(\frac{\rho\bar{x}+C_x}{\rho\bar{X}+C_x}\right)\right]$	ρ	C_x
$\psi_{r12}=w_2\bar{y}\left[1+\log\left(\frac{\beta_2(x).\bar{X}+\rho}{\beta_2(x).\bar{x}+\rho}\right)\right]$	$\psi_{p12}=w_2\bar{y}\left[1+\log\left(\frac{\beta_2(x).\bar{x}+\rho}{\beta_2(x).\bar{X}+\rho}\right)\right]$	$\beta_2(x)$	ρ
$\psi_{r13}=w_2\bar{y}\left[1+\log\left(\frac{\rho\bar{X}+\beta_2(x)}{\rho\bar{x}+\beta_2(x)}\right)\right]$	$\psi_{p13}=w_2\bar{y}\left[1+\log\left(\frac{\rho\bar{x}+\beta_2(x)}{\rho\bar{X}+\beta_2(x)}\right)\right]$	ρ	$\beta_2(x)$

5. EMPIRICAL STUDY

To examine the merits of the suggested class of estimators $'T'$ we considered a natural population data set. The description of the population is given below.

Population 1: (Singh, D and Chaudhary, F.S). The data concerns the no. of bearing lime trees and the area reported under lime, in each of the 22 villages growing lime in one of the tehsils of Nellore district. Pg.no.141

y : area reported under lime

x : no. of bearing lime trees

$N = 22$; $n = 15$; $\overline{Y} = 22.62091$; $\overline{X} = 1467.545$

$S_y = 33.04697$; $S_x = 2562.145$; $\rho = 0.902147$

Population2: (Koyuncu and Kalidar, 2009a). The data concerns primary and secondary school for 923 districts of Turkey in 2007. The variables are defined as follows:

y : number of teachers in both primary and secondary school

x : number of students in both primary and secondary school.

$N = 923$; $n = 180$; $\overline{Y} = 436.4345$; $\overline{X} = 11440.4984$

$S_y = 749.9395$; $S_x = 2131.1315$; $\rho = 0.9543$

	Population 1		**Population 2**	
ESTIMATORS	**MSE**	**PRE**	**MSE**	**PRE**
$\overline{y}$	23.16581	100	2515.168	100
$\overline{y}_R$	6.299483	367.74	267.6515	939.7177
$\overline{y}_P$	106.2017	21.813	10685.4	23.53837
T_K	4.311873	537.26	224.6335	1119.677
η_K	4.275842	541.78	224.3689	1120.997
ξ	4.311873	537.26	224.6335	1119.677
ψ	4.303488	538.3	221.5432	1135.295
T^*	4.311873	537.26	224.6335	1119.677
T	**4.311873**	**537.26**	**224.6335**	**1119.677**

Note: The above calculations are made on the basis of the optimum values of the characterising parameters (α, β) and $(g = 1)$. Others calculations can be made on the similar lines.

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