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On Fuzzy Soft Perfectly Disconnected Spaces

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Abstract

The aim of this paper is to introduce and study the concept of fuzzy soft perfectly disconnected spaces on fuzzy soft topological spaces are introduced and several characterizations are discussed.

Keywords: fuzzy soft nowhere dense set ,fuzzy soft regular open set ,fuzzy soft perfectly disconnected space , fuzzy soft weakly Baire spaces , fuzzy soft extremely disconnected space .

Introduction:

In 1965, L.A. Zadeh [14] introduced the concept of fuzzy sets as a new approach for modeling uncertainities In 1968,C.L.Chang [2] introduced the concept of fuzzy topological spaces. The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts Perfectly disconnected space in classical topology was defined and studied by Eric K. Van Douwen [4].The concept of fuzzy extremally disconnected spaces was defined and studied by G.Balasubramaniam [3]. Molodtsov [6] initiated the theory of soft sets as a new mathematical tool for dealing with uncertainities which traditional mathematical tools cannot handle. He has shown several applications of this theory in solving any practical problems in economics, engineering, social sciences, medical science etc. Later other authors like Maji et al. [5] have further studied the theory of soft sets and used this theory to solve some decision making problems. The concept of fuzzy soft nowhere dense sets and Fuzzy soft weakly Baire space was introduced and studied by E.Poongothai and S.Divyapriya[7 ,8]. The concept of fuzzy perfectly disconnected spaces in topological spaces was introduced by G.Thangaraj and S.Murugannathan [9]. In this paper the notion of fuzzy soft perfectly disconnected space is introduced and several characterizations of fuzzy soft perfectly disconnected spaces are established.

2. PRELIMINARIES

In section 2 we have given some basic definitions and notion are self-contained.

Definition 2.1[5]:

The fuzzy soft set $F_\phi \in FS(U, E)$ is said to be null fuzzy soft set and it is denoted by ϕ, if for all $e \in E$, $F(e)$ is the null fuzzy soft set $\overline{0}$ of U, where $\overline{0}(x) = 0$ for all $x \in U$.

Definition 2.2 [5]:

Let $FE \in FS(U, E)$ and $FE(e) = \overline{1}$ all $e \in E$, where $\overline{1}(x) = 1$ for all $x \in U$.Then F_E is called absolute fuzzy soft set. It is denoted by $\bar{E}$.

Definition 2.3 [5]:

A fuzzy soft set F_A is said to be a fuzzy soft subset of a fuzzy soft set G_B over a common universe U if $A \subseteq B$ and $F_A(e) \subseteq G_B(e)$ for all $e \in A$, *i.e.*, if $\mu^e F_A(x) \leq \mu^e G_B(x)$ for all $x \in U$ and for all $e \in E$ and denoted by $F_A \tilde{\subseteq} G_B$.

Definition 2.4[5]:

Two fuzzy soft sets F_A and G_B over a common universe U are said to be fuzzy soft equal if F_A is a fuzzy soft subset of G_B of G_B is a fuzzy soft subset of F_A.

Definition 2.5[5]:

The union of two fuzzy soft sets F_A and G_B over the common universe U is the fuzzy soft set HC, defined by $H_C(e) = \mu^e_{H_C} = \mu^e_{F_A} \cup \mu^e_{G_B}$ for all $e \in E$, where $C = A \cup B$. Here we write $H_C = F_A \tilde{\vee} G_B$

Definition 2.6[6]:

Let F_A and G_B be two fuzzy soft set, then the intersection of F_A and G_B is a fuzzy soft set H_C, defined by $(e) = \mu^e_{F_C} = \mu^e_{F_A} \cap \mu^e_{G_B}$ for all $e \in E$, where $C = A \cap B$. Here we write $H_C = F_A \tilde{\wedge} G_B$.

Lemma 2.1[1]:

For a family A = $\{\lambda_\alpha\}$ of fuzzy sets of a fuzzy space X, $\vee(\text{cl}(\lambda_\alpha)) \leq \text{cl}(\vee(\{\lambda_\alpha\})$. In case A is a finite set, $\vee(\text{cl}(\lambda_\alpha)) = \text{cl}(\vee((\lambda_\alpha))$. Also $\vee(\text{int}(\lambda_\alpha)) \leq \text{int}(\vee(\lambda_\alpha))$.

Definition 2.7[10]:

Let $F_A \in FS(U, E)$ be a fuzzy soft set. Then the complement of F_A, denoted by F_A^C,defined by

$$F_A^C(e) = \begin{Bmatrix} \overline{1} - \mu^e_F A, & if \; e \in A \\ \overline{1}, & if \; e \notin A \end{Bmatrix}$$

Definition 2.8[10]:

Let ψ be the collection of fuzzy soft sets over U. Then ψ is called a fuzzy soft topology on U if ψ satisfies the following axioms:

(i) ϕ,$\bar{E}$belong to ψ.

(ii) The union of any number of fuzzy soft sets in ψ belongs to ψ.

(iii) The intersection of any two fuzzy soft sets ψ belongs to ψ. The triplet (U, E, ψ) is called a fuzzy soft topological space over U. The members of ψ are called fuzzy soft open sets in U and complements of them are called fuzzy soft closed sets in U.

Definition 2.9[10]:

The union of all fuzzy soft open subsets of F_A over (U, E,) is called the interior of F_A and is denoted by$int^{fs}(F_A)$.

Proposition 2.1[10]:

$int^{fs}(F_A \barwedge G_B) = int^{fS}(F_A) \barwedge int^{fs}(G_B)$.

Definition 2.10 [1]:

LetF_A∈FS(U,E) be a fuzzy soft set. Then the intersection of all closed sets, each containing F_A, is called the closure of F_A and is denoted by $cl^{fs}(F_A)$.

Remarks 2.11 [11]:

(1) For any fuzzy soft set F_A in a fuzzy soft topological space (U, E, ψ), it is easy to see that $((F_A))^c = int^{fs}(F_A^C)$ and $(int^{fs}(F_A))^c = cl^{fs}(F_A^C)$.

(2) For any fuzzy soft F_A subset of a fuzzy soft topological space (U, E, ψ) we define the fuzzy soft subspace topology on F_A by $K_D \epsilon \psi_{F_A}$if$K_D = F_A \barwedge G_B$ for some $G_B \epsilon \psi$.

(3) For any fuzzy soft H_C in F_A fuzzy soft subspace of a fuzzy soft topological space, we denote to the interior and closure of H_C in F_A by $int^{fs}_{F_A}(H_C)$ and $cl^{fs}_{F_A}(H_C)$, respectively.

Definition2.12[8]:

A fuzzy soft set F_A in a FSTS (U, E,ψ)is called a Fuzzy Soft Nowhere Dense set if there exist no non-zero fuzzy soft open set G_B in (U,E, ψ) such that G_B<$cl^{fs}(F_A)$. ie), $int^{fs}cl^{fs}(F_A)=0$.

Definition 2.13[8]

A fuzzy soft set F_A in a FSTS (U, E,ψ)is called fuzzy soft dense if there exist no fuzzy soft closed set G_B in (U, E,ψ) such that F_A<G_B<1. ie) $cl^{fs}(F_A)=1$.

Definition 2.14[8]:

Let (U, E, ψ) be a fuzzy soft topology . A fuzzy soft set F_A in (U, E,) is called fuzzy soft first category . If $F_A = \vee_{i=1}^{\infty}(F_{A_i})$ where (F_{A_i})'s are fuzzy soft nowhere dense sets in (U, E,ψ). Any other fuzzy soft set in (U, E,) is said to be of fuzzy soft second category.

Definition 2.15[9]:

A fuzzy soft set F_A in a FSTS (U,E,ψ) is called a fuzzy soft σ - boundary set, if $F_A = \vee_{i=1}^{\infty}(G_{B_i})$, where $G_{B_i} = cl^{fs}(F_{A_i}) \wedge (1- F_{A_i})$ and (F_{Ai})' s are fuzzy soft regular open sets in (U,E,ψ) (short form fuzzy soft regular open sets , fsros).

Definition 2.16[9]:

A fuzzy soft set F_A in a Fsts (U,E,ψ) is called a fuzzy soft pre F_σ– set, if $F_A = \vee_{i=1}^{\infty}(G_{B_i})$, where(G_{B_i})' s are fuzzy soft pre closed sets in (U,E,ψ).

3. FUZZY SOFT PERFECTLY DISCONNECTED SPACES.

Definition 3.1:

If for any two non-zero Fuzzy Soft Sets F_A and G_B are defined on U with $F_A \leq 1-G_B$, $cl^{fs}(F_A) \leq 1-cl^{fs}(G_B)$, in Fuzzy Soft Topological Spaces (U,E,ψ) is called a fuzzy soft perfectly disconnected space.

Example 3.1.1:

The fuzzy soft sets F_E ,G_E ,H_E, L_D, are defined on U as follows.

F_A:X→[0, 1]defined as F_A (a) = 0.4; F_A (b) = 0.7; F_A (c) = 0.5;

G_B:X→[0, 1]defined as G_B (a) = 0.5; G_B (b) = 0.6, G_B (c) = 0.6;

H_c:X→[0, 1]defined as H_c (a) = 0.6; H_c (b) = 0.6; H_c (c) = 0.5;

J_E:X→[0, 1]defined as J_E (a) = 0.3; J_E (b) = 0.5; J_E (c) = 0.4.

L_D:X→[0, 1]defined as L_D (a) = 0.4; L_D (b) = 0.4; L_D (c) = 0.2.

Now T={0, F_A, G_B, H_c,($F_A \vee G_B$),($F_A \vee H_c$),($G_B \vee H_c$),($F_A \wedge G_B$), ($F_A \wedge H_c$),($G_B \wedge H_c$), $F_A \vee$($G_B \wedge H_c$), $G_B \wedge$($F_A \vee H_c$), $H_c \vee$($F_A \wedge G_B$), $F_A \vee G_B \vee H_c$,1} . Then $H_c \leq 1- L_D$,$cl^{fs}(H_c) \leq 1-cl^{fs}(L_D)$, in (U,E,$\psi$).

Hence (U,E,ψ)is a fuzzy soft perfectly disconnected space.

Proposition 3.1

If (U,E,ψ) is a Fuzzy soft pre dense set and $F_A \leq 1-G_B$ for any two non-zero Fuzzy soft sets F_A and G_B defined on U then $cl^{fs}(F_A) \neq 1$ and $cl^{fs}(G_B) \neq 1$ in FSTS (U,E,ψ).

Proof:

Suppose that $F_A \leq 1-G_B$ any two non-zero Fuzzy Soft Sets F_A and G_B defined on U then (U,E,ψ) is a Fspds, we have $(F_A) \leq 1-cl^{fs}(G_B)$, in FSTS (U,E,$\psi$). If $cl^{fs}(F_A)=1$, then $1 \leq 1- cl^{fs}(G_B)$ it implies that $cl^{fs}(G_B) =0$ .That is $cl^{fs}(G_B) = 0$ in (U,E,ψ), if $G_B = 0$, a contradiction to $G_B \neq 0$. If $cl^{fs}(G_B) = 1$ then $cl^{fs}(F_A) \leq 1 - 1 = 0$. That is $cl^{fs}(F_A)= 0$. If $F_A = 0$, which is contradiction to

$F_A \neq 0$,thus ,if $F_A \leq 1\text{-}G_B$, then $cl^{fs}(F_A) \neq 1$ and $cl^{fs}(G_B) \neq 1$ in Fuzzy Soft Topological Spaces (U,E,ψ).

Proposition 3.2:

If F_A and $F_A \leq 1\text{-}G_B$ is a fuzzy soft closed set in fuzzy soft pre dense sets (U,E,ψ), then there exists a Fuzzy soft open sets H_c in (U,E,ψ). Such that $F_A \leq H_C \leq 1 - G_B$.

Proof:

Let $F_A \leq 1\text{-}G_B$ and F_A is a Fuzzy soft closed sets in Fuzzy soft pre dense sets (U,E,ψ). Then $cl^{fs}(F_A) \leq 1 - cl^{fs}(G_B)$ in FSTS (U,E,ψ) .F_A is a Fuzzy soft closed sets in (U,E,ψ) , we have $cl^{fs}(F_A)= 1$, and $F_A \leq 1\text{-}\ cl^{fs}(G_B)$, then $1 - cl^{fs}(G_B) \leq 1 - G_B$, in (U,E,ψ) .Therefore , $F_A \leq 1\text{-}\ cl^{fs}(G_B) \leq 1\text{-}G_B$, $H_c = 1\text{-}\ cl^{fs}(G_B)$ .Then H_C is a Fuzzy soft open set in (U,E,ψ), Hence $F_A \leq H_C \leq 1 - G_B$ in (U,E,ψ) ,where $H_C \epsilon T$.

Proposition 3.3 :

IF G_B and $F_A \leq 1\text{-}G_B$ is a fuzzy soft closed set in fuzzy soft pre dense sets (U,E,ψ), then there exists a Fuzzy soft closed sets L_D in (U,E,ψ) such that $F_A \leq L_D \leq 1 - G_B$.

Proof :

Suppose that $F_A \leq 1\text{-}G_B$ and G_B is a Fscs in (U,E,ψ) is a Fuzzy soft pre dense sets ,we have $cl^{fs}(F_A) \leq 1 - cl^{fs}(G_B)$,then $F_A \leq cl^{fs}(G_B) \leq 1\text{-}G_B$. Let $L_D = cl^{fs}(F_A)$. Then L_D is a Fuzzy soft closed sets in (U,E,ψ).

Proposition 3.4:

If $F_A \leq 1\text{-}G_B$ for any two FSS F_A and G_B defined on U , in a Fuzzy soft pre dense sets (U,E,ψ), then there exist a Fuzzy soft open sets H_c in (U,E,ψ). Such that $int^{fs}\, cl^{fs}\,(F_A) \leq H_C \leq 1\text{-}\ cl^{fs}\,[int^{fs}(G_B)]$ and $int^{fs}(G_B)$ is not a fuzzy soft dense set in (U,E,ψ).

Proof:

Let $F_A \leq 1\text{-}G_B$ in FSTS (U,E,ψ).Then $cl^{fs}\,(F_A) \leq cl^{fs}(1\text{-}G_B\,)$ and hence $cl^{fs}\,(F_A) \leq 1 - int^{fs}(G_B)$. Since $cl^{fs}\,(F_A)$ is a fuzzy soft closet set in FSTS (U,E,ψ), By prop 3.2, there exist a fuzzy soft open set H_C in (U,E,ψ). Such that $cl^{fs}(F_A) \leq H_C \leq [1\text{-}int^{fs}(G_B)]$. Then , $int^{fs}[cl^{fs}(F_A)] \leq int^{fs}(H_c) \leq int^{fs}[1\text{-}\ int^{fs}(G_B)]$ and hence $int^{fs}cl^{fs}\,(F_A) \leq H_C \leq 1\text{-}\ cl^{fs}\,[int^{fs}(G_B)]$ in (U,E,ψ). Then $int^{fs}\{1\text{-}\ cl^{fs}[int^{fs}(G_B)]\} \neq 0$. $\Rightarrow$ $1\text{-}\ cl^{fs}\, cl^{fs}[int^{fs}(G_B)] \neq 0$ and then $cl^{fs}int^{fs}\,(G_B) \neq 1$. Thus $int^{fs}(G_B)$ is not a fuzzy soft dense set in (U,E,ψ).

Proposition 3.5:

If $F_A \leq 1\text{-}G_B$,where G_B is a Fuzzy soft closed sets in Fuzzy soft pre dense sets (U,E,ψ), then there exist a Fuzzy soft regular open set H_c in (U,E,ψ) such that $int\ ^{fs}(F_A) \leq H_c \leq 1\text{-}G_B$.

Proof:

Suppose that $F_A \leq 1\text{-}G_B$, then we say G_B is a Fuzzy soft closed sets in Fuzzy Soft Topological Spaces (U,E,ψ) . By prop 3.3, there exist a Fuzzy soft closed sets L_D in FSTS . such that $F_A \leq L_D \leq 1 - G_B$. $\Longrightarrow$ $int^{fs}(F_A) \leq int^{fs}(L_D) \leq int^{fs}(1\text{-}G_B)$, in FSTS (U,E,ψ) . Then , $int^{fs}(F_A) \leq int^{fs}(L_D) \leq \mathbf{1 -} cl^{fs}\,(G_B) = 1 - G_B$, in (U,E,ψ) . Let $H_C = int^{fs}(L_D)$, the interior of a fuzzy soft regular open set in a FSTS [Thm2.1] H_C is a fsrops in (U,E,ψ) . Hence there exist a Fuzzy soft regular open sets H_c in (U,E,ψ) such that $int\ ^{fs}(F_A) \leq H_c \leq 1\text{-}G_B$.

Proposition 3.6:

If $F_A \leq 1\text{-}G_B$, where F_A is a Fuzzy soft closed sets in Fuzzy soft pre dense sets (U,E,ψ), then there exist a Fuzzy soft regular open sets L_D in (U,E,ψ) such that $F_A \leq L_D \leq 1 - int^{fs}(G_B)$.

Proof:

Let $F_A \leq 1\text{-}G_B$, F_A is a Fscs in (U,E,ψ) then by prop 3.2, there exist a Fsops H_C in (U,E,ψ) such that $F_A \leq H_C \leq 1 - G_B$ **.** $\Rightarrow$ $cl^{fs}(F_A) \leq cl^{fs}(G_B) \leq cl^{fs}(1\text{-}G_B)$ in (U,E,ψ). Then $F_A \leq cl^{fs}(H_c) \leq 1\text{-}\ int^{fs}(G_B)$, in (U,E,ψ) . Let $L_D = cl^{fs}(H_C)$ the closure of a Fuzzy soft open sets is a Fuzzy soft regular closed sets in (U,E,ψ). Then there exist a regular closed set L_D in (U,E,ψ) such that $F_A \leq L_D \leq 1 - int^{fs}(G_B)$.

Proposition 3.7:

If $F_A \leq 1\text{-}\ G_B$,for any two FSS F_A and G_B defined on (U,E,ψ), then

(a) If F_A is a fuzzy soft dense set in (U,E,ψ), then G_B is not a fuzzy soft open set in (U,E,ψ).

(b) If G_B is a fuzzy soft dense set in (U,E,ψ), then F_A is not a Fuzzy soft open sets in (U,E,ψ).

(c) If F_A is a fuzzy soft open sets in (U,E,ψ), then G_B is not a fuzzy soft dense set in (U,E,ψ).

(d) If G_B is a Fuzzy soft open sets in (U,E,ψ), then F_A is not a fuzzy soft dense set in (U,E,ψ).

(e) If $cl^{fs}int^{fs}(G_B) =1$, then F_A is a fuzzy soft nowhere dense set in (U,E,ψ).

Proof:

(i) Let $F_A \leq 1\text{-}\ G_B$, in (U,E,ψ), then $cl^{fs}(F_A) \leq 1\text{-}\ int^{fs}(G_B)$ in FSTS .Where F_A is a fuzzy soft dense set in (U,E,ψ), $cl^{fs}(F_A) = 1$ and $1 \leq 1\text{-}\ int^{fs}(G_B)$. $\Rightarrow$ $int^{fs}(G_B) = 0$ and Hence G_B is not a fuzzy soft open set in (U,E,ψ).

(ii) Now $F_A \leq 1\text{-}\ G_B$ in (U,E,ψ) i.e) $cl^{fs}int^{fs}(F_A) \leq cl^{fs}int^{fs}(1\text{-}G_B)$. Then $cl^{fs}int^{fs}(F_A) \leq 1\text{-}\ int^{fs}[cl^{fs}(G_B)]$ in (U,E,ψ). we say G_B is a fuzzy soft dense set in (U,E,ψ), $cl^{fs}(G_B) = 1$,Hence $int^{fs}cl^{fs}(G_B) = int^{fs}(1)=1$ in (U,E,ψ). Then $cl^{fs}int^{fs}(F_A) \leq 1 \mathbf{-1} =0$ i.e) $cl^{fs}int^{fs}\,(F_A) = 0$.$\Rightarrow$ $int^{fs}(F_A) = 0$, in (U,E,ψ). Hence F_A is not a fsops in (U,E,ψ).

(iii) Let $F_A \leq 1\text{-}\ G_B$, in (U,E,ψ), it shows that $int^{fs}(F_A) \leq int^{fs}(1\text{-}G_B)$ and $cl^{fs}(F_A) \leq 1\text{-}\ cl^{fs}(G_B)$ then F_A is a fuzzy soft open sets ,$int^{fs}(F_A) = F_A$in FSTS (U,E,ψ).Then $F_A \leq$ 1- $cl^{fs}(F_A)$.$\Rightarrow cl^{fs}(G_B) \leq 1-F_A$ and hence $cl^{fs}(G_B) \neq 1$ in (U,E,ψ),therefore G_B is not a fuzzy soft dense set in (U,E,ψ).

(iv) since $F_A \leq$ 1- G_B, where $cl^{fs}(F_A) \leq cl^{fs}(1-G_B)$ and hence $cl^{fs}(F_A) \leq 1- int^{fs}(G_B)$ then G_B is a fuzzy soft openset in (U,E,ψ), $int^{fs}(G_B) = G_B$ and thus $cl^{fs}(F_A) \leq 1-G_B$. Hence $cl^{fs}(F_A) \neq 1$, in (U,E,ψ).

$\therefore$ F_A is not a fuzzy soft dense set in (U,E,ψ).

(v) Let $F_A \leq$ 1- $G_B \Rightarrow int^{fs}\, cl^{fs}(F_A) \leq int^{fs}\, cl^{fs}(1-G_B)$ and then $int^{fs}\, cl^{fs}(F_A) \leq$ 1- $cl^{fs}\, int^{fs}(G_B)$ in (U,E,ψ). since $cl^{fs}int^{fs}(G_B)=1$, $int^{fs}\, cl^{fs}(F_A)$1-1=0 (I.e) $int^{fs}\, cl^{fs}(F_A)=0$ in (U,E,ψ).

$\therefore$ F_A is a fuzzy soft nowhere dense set in (U,E,ψ).

Proposition 3.8:

If F_A is a FSS in Fuzzy soft pre dense sets (U,E,ψ) then $int^{fs}(F_A)$ is a Fuzzy soft closed sets in (U,E,ψ).

Proof:

Let F_A be a Fss defined on U the fuzzy soft set 1-F_A , 1-$F_A \leq cl^{fs}(1-F_A)$.then 1-$F_A \leq cl^{fs}(1-F_A)$. Then $(1-F_A) \leq 1- int^{fs}(F_A)$. since $int^{fs}(F_A)$ is a Fuzzy soft open sets in (U,E,ψ) by prop 3.7, 1-F_A is not a fuzzy soft dense set in (U,E,ψ). ie) $cl^{fs}(1-F_A) \neq 1$ in (U,E,ψ) and $int^{fs}(F_A) \neq 0$ in (U,E,ψ). since (U,E,ψ) is a Fuzzy soft pre dense sets $(1-F_A) \leq 1- int^{fs}(F_A)$ in (U,E,ψ), $cl^{fs}(1-F_A) \leq 1- cl^{fs}[int^{fs}(F_A)]$ and then 1- $int^{fs}(F_A) \leq 1-cl^{fs}[int^{fs}(F_A)]$ and hence $cl^{fs}[int^{fs}(F_A)] \leq int^{fs}(F_A)$. But $int^{fs}(F_A) \leq cl^{fs}[int^{fs}(F_A)]$ in (U,E,ψ). This implies that $cl^{fs}[int^{fs}(F_A)]= int^{fs}(F_A)$ therefore $int^{fs}(F_A)$ is a Fscs in (U,E,ψ).

Proposition 3.9:

IF F_A is a Fuzzy Soft Sets in Fuzzy Soft pre dense sets (U,E,ψ) ,then F_A is a fuzzy soft pre –closed set in (U,E,ψ).

Proof:

Let F_A be a Fuzzy Soft sets defined on U in a Fuzzy Soft Pre dense Sets (U,E,ψ). For prop 3.8, $int^{fs}(F_A)$ is a Fuzzy soft closed sets in (U,E,ψ).Let $G_B = int^{fs}(F_A)$ .since G_B is a Fuzzy soft closed sets , $cl^{fs}(G_B) = G_B$, in (U,E,ψ). Then $cl^{fs}\, int^{fs}(F_A) = int^{fs}(F_A)$ and then $cl^{fs}\, int^{fs}(F_A)= int^{fs}(F_A) \leq F_A$ in (U,E,ψ). since $cl^{fs}\, int^{fs}(F_A) \leq F_A$,this implies that F_A is a fuzzy soft pre –closed set in (U,E,ψ).

Remark 3.1:

In view of the above proposition following results "IF F_A is a Fuzzy Soft Sets in a Fuzzy soft pre dense sets (U,E,ψ) ,then 1-F_A is a fuzzy soft pre –open set in (U,E,ψ).

Remark 3.2

In a FSTS (U,E,φ) the non-zero Fuzzy soft open sets (F_{A_i})'s are not Fuzzy soft nowhere dense sets ,$F_A = int^{fs}(F_A) \leq int^{fs}cl^{fs}(F_A)$ and thus $int^{fs}cl^{fs}(F_A) \neq 0$ in (U,E,ψ) .

Proposition3.10

If F_A is a Fuzzy soft open sets in Fuzzy soft pre dense sets (U,E,ψ), then F_A is not a fuzzy soft nowhere dense set in (U,E,ψ).

Proof:

The proof follows from remark 3.2

Proposition3.11:

If F_A is a fuzzy soft regular open set in Fuzzy soft pre dense sets (U,E,ψ), then F_A is not a fuzzy soft nowhere dense set in (U,E,ψ).

Proof:

The proof follows from remark 3.2.

Proposition 3.12:

If F_A is not a fuzzy soft nowhere dense set in a Fuzzy Soft Pre dense (U,E,ψ), then $int^{fs}[cl^{fs}(F_A)]$ is a Fuzzy Soft Closed Sets in (U,E,ψ).

Proof :

Let F_A is not a Fuzzy soft nowhere dense sets in (U,E,ψ), then $int^{fs}cl^{fs}(F_A) \neq 0$ in (U,E,ψ) . Now $int^{fs}cl^{fs}(F_A) \leq cl^{fs}(F_A)$ in (U,E,ψ) and then $int^{fs}cl^{fs}(F_A) \leq 1-[1-cl^{fs}(F_A)]$ in (U,E,ψ), its fuzzy soft perfectly disconnectedness of (U,E,ψ). We have $cl^{fs}[int^{fs}cl^{fs}(F_A)] \leq 1-cl^{fs}[1-cl^{fs}(F_A)]$ and then $cl^{fs}[int^{fs}cl^{fs}(F_A)] \leq 1-[1-int^{fs}cl^{fs}(F_A)]$ in (U,E,ψ) . This implies $cl^{fs}[int^{fs}cl^{fs}(F_A)] \leq int^{fs}[cl^{fs}(F_A)]$. But $int^{fs}[cl^{fs}(F_A)] \leq cl^{fs}[int^{fs}cl^{fs}(F_A)]$ in (U,E,ψ) thus $cl^{fs}[int^{fs}cl^{fs}(F_A)]= int^{fs}cl^{fs}(F_A)$. Then , $int^{fs}cl^{fs}(F_A)$ .Then $int^{fs}cl^{fs}(F_A)$ is a Fuzzy soft closed sets in (U,E,ψ).

Proposition 3.13:

If F_A is a Fuzzy soft regular open sets in a Fuzzy soft pre dense sets (U,E,ψ) ,then $int^{fs}cl^{fs}(F_A)$ is a Fuzzy soft closed sets in (U,E,ψ).

Proof:

Let F_A be a non-zero Fuzzy soft open sets in (U,E,ψ) is a Fuzzy soft pre dense sets ,by propo 3.10 ,F_A is not a fuzzy soft nowhere dense set in (U,E,ψ). Then ,by prop 3.12, $int^{fs}cl^{fs}(F_A)$ is a Fuzzy soft closed sets in (U,E,ψ).

Proposition 3.14

If F_A is a Fuzzy soft regular open sets in a Fuzzy soft pre dense sets (U,E,ψ),then F_A is a Fuzzy soft closed sets in (U,E,ψ).

Proof:

Let F_A be a Fuzzy soft regular open sets in (U,E,ψ) is a Fuzzy soft pre dense sets ,by prop 3.11, F_A is not a Fuzzy soft nowhere dense sets in (U,E,ψ) and hence by prop 3.13, $int^{fs}cl^{fs}(F_A)$ is a Fuzzy soft closed sets in (U,E,ψ) ,F_A is a fuzzy soft regular open sets in (U,E,ψ), $int^{fs}cl^{fs}(F_A) = F_A$ and F_A is a Fuzzy soft closed sets in (U,E,ψ).

Proposition 3.15:

If F_A is a Fuzzy soft regular closed sets in a Fuzzy soft pre dense sets (U,E,ψ), then F_A is a Fuzzy soft open sets in (U,E,ψ).

Proof:

Let F_A be a Fuzzy soft regular closed sets in (U,E,ψ), 1- F_A is a Fuzzy soft regular open sets in (U,E,ψ) is a Fuzzy soft pre dense sets, by prop 3.14, 1-F_A is a Fuzzy soft closed sets in (U,E,ψ). F_A is a Fuzzy soft open sets in (U,E,ψ).

Proposition 3.16:

If $F_A = [\vee_{i=1}^{\infty}(F_{A_i})]$,where (F_{A_i}) are Fuzzy soft sets defined on a Fspds (U,E,ψ), then F_A is a fuzzy soft pre F_σ – set in (U,E,ψ).

Proof:

Let $F_A = [\vee_{i=1}^{\infty}(F_{A_i})]$. where (F_{A_i})'s are fuzzy soft sets defined on the Fspds (U,E,ψ). Then prop 3.9, (F_{A_i})'s are fuzzy soft pre -closed sets in (U,E,ψ) and hence F_A is a fuzzy soft pre F_σ – set in (U,E,ψ).

Proposition 3.17:

If (U,E,ψ) is a Fuzzy soft pre dense sets ,then

(i) $Cl^{fs}int^{fs}(F_A) + cl^{fs}(1-F_A) \leq 1$, for a Fss F_A defined on U .

(ii) For any two Fss F_A and G_B ,with $F_A + G_B \leq 1$,$cl^{fs}(F_A) + cl^{fs}(G_B) \leq 1$.

(iii) For any two Fss F_A and G_B with $F_A + G_B \leq 1$, $cl^{fs}int^{fs}(F_A) + cl^{fs}int^{fs}(G_B) \leq 1$.

Proof:

Let F_A be a Fuzzy soft sets defined on U ,$int^{fs}(F_A) \leq F_A$ and thus $int^{fs}(F_A) \leq 1-(1-F_A)$ in (U,E,ψ) is a Fspds ,$cl^{fs}int^{fs}(F_A) \leq 1-cl^{fs}(1-F_A)$, in (U,E,ψ) and hence $Cl^{fs}int^{fs}(F_A) + cl^{fs}(1-F_A) \leq 1$.

(ii) Suppose that $F_A + G_B \leq 1$, for any two Fss F_A and G_B defined on U. Then $F_A \leq 1-G_B$, in (U,E,ψ) is a Fspds ,$cl^{fs}(F_A) \leq cl^{fs}(1-G_B)$, in (U,E,ψ) .Then $int^{fs}(F_A) \leq 1-int^{fs}(G_B)$ in (U,E,ψ), Hence $cl^{fs}(F_A) + cl^{fs}(G_B) \leq 1$.

(iii)suppose that $F_A + G_B \leq 1$, F_A and G_B are two subsets its defined on U .Then $F_A \leq 1-G_B$, in (U,E,ψ),then $int^{fs}(F_A) \leq 1-int^{fs}(G_B)$ in (U,E,ψ) is a Fuzzy soft pre dense sets , $cl^{fs}int^{fs}(F_A) \leq 1-cl^{fs}int^{fs}(G_B)$ in (U,E,ψ) and hence $cl^{fs}int^{fs}(F_A) + cl^{fs}int^{fs}(G_B) \leq 1$.In Fuzzy soft pre dense sets ,Fuzzy soft open sets are not Fuzzy soft dense sets .

Proposition 3.18:

If ($\lambda \neq 1$) is a Fuzzy soft open sets in a Fuzzy soft pre dense sets (U,E,ψ), then F_A is not a Fuzzy soft pre dense sets in (U,E,ψ).

Proof:

Let F_A be a Fsops (U,E,ψ) .Now by prop 3.17, $cl^{fs}int^{fs}(F_A) + cl^{fs}(1-F_A) \leq 1$ in (U,E,ψ). Then we say

$cl^{fs}int^{fs}(F_A) + 1 - int^{fs}(F_A) \leq 1$ *.This implies that* $cl^{fs}int^{fs}(F_A) \leq int^{fs}(F_A)$. But $int^{fs}(F_A) \leq cl^{fs}int^{fs}(F_A)$ Hence $cl^{fs}[int^{fs}(F_A)] = int^{fs}(F_A)$ .F_A is a Fuzzy soft open sets ,$int^{fs}(F_A) = F_A$ and then $cl^{fs}(F_A) = F_A \neq 1$,F_A is not a Fuzzy soft dense sets in (U,E,ψ).

4. FUZZY SOFT DISCONNECTED SPACES ON FUZZY SOFT TOPOLOGICAL SPACES

Proposition 4.1:

If (U,E,ψ) is a Fuzzy soft pre dense sets ,then (U,E,ψ) is a fuzzy soft extremally disconnected space.

Proof:

Suppose that F_A be a Fss defined on U . (U,E,ψ) is a Fspds and $(1-F_A) \leq 1- [int^{fs}(F_A)]$ in (U,E,ψ) $\Rightarrow cl^{fs}(1-F_A) \leq 1 - cl^{fs}[int^{fs}(F_A)]$ and then $1- int^{fs}(F_A) \leq 1- cl^{fs}int^{fs}(F_A)$ and hence $cl^{fs}int^{fs}(F_A) \leq int^{fs}(F_A)$. But $int^{fs}(F_A) \leq cl^{fs}[int^{fs}(F_A)]$ in (U,E,ψ).$cl^{fs}int^{fs}(F_A) = int^{fs}(F_A)$, in (U,E,ψ) . Let $L_D = int^{fs}(F_A)$, then G_B is a Fuzzy soft open sets in (U,E,ψ). Now $cl^{fs}(G_B) = L_D$ and $G_B \in$ T this implies that $cl^{fs}(G_B) \in T$ $if\ G_B \in T, then\ cl^{fs}(G_B) \in T$. (U,E,ψ) is a Fuzzy soft extremally disconnected spaces.

Proposition 4.2:

If (U,E,ψ) is a Fuzzy soft pre dense sets, then (U,E,ψ) is not a fuzzy soft weakly Baire space.

Proof:

Let (U,E,ψ) be a Fspds and (F_{A_i})'s are (i = 1 to∞) be non-zero regular open sets in (U,E,ψ). Let $G_{B_i} = cl^{fs}(F_{A_i}) \wedge (1 - F_{A_i})$. $Then\ int^{fs}\ (G_{B_i}) = int^{fs}[cl^{fs}(F_{A_i}) \wedge (1 - F_{A_i})] = int^{fs}[cl^{fs}(F_{A_i})] \wedge int^{fs}[(1 - F_{A_i})]$. $since\ the\ FSS(F_{A_i})$'s are fuzzy soft regular open sets in (U,E,ψ).$int^{fs}[cl^{fs}(F_{A_i})] = F_{A_i}$, $and\ thus\ int^{fs}\ (G_{B_i}) = (F_{A_i}) \wedge int^{fs}\ [(1 - F_{A_i})]$. $By\ prop$ 3.14, $(F_{A_i})'s$ $are\ FSCS\ in\ (U,E,\psi)$. $By\ prop$ 3.8, $[int^{fs}(1 - F_{A_i})]'s$ $are\ Fscs\ in\ (U,E,\psi)$. $Then$ { $F_{A_i} \wedge int^{fs}((1 - F_{A_i})$}$'s\ are$ FSCS in (U,E,ψ) and thus $[int^{fs}(G_{B_i})]$'s are non-zero Fuzzy Soft Closed Sets in (U,E,ψ), $\Rightarrow \vee_{i=1}^{\infty} int(G_{B_i}) \neq 0$. $By\ lemma$ 2.2, $\vee_{\infty}^{i=1} int(G_{B_i}) \leq int^{fs}[\vee_{i=1}^{\infty}(G_{B_i})] \neq 0$, $where\ G_{B_i} = (F_{A_i}) \wedge (1 - F_{A_i})$ $and\ (F_{A_i})'s\ are\ Fsrops$ in (U,E,ψ), is not a fuzzy soft weakly baire space.

Proposition 4.3:

If Pint^{fs}[$\vee_{i=1}^{\infty}(F_{A_i})] = 0, where\ \ pint(\,F_{A_i}) = 0$, for the Fuzzy Soft Sets ($F_{A_i}\,)'s\ are\ Fspds$ in (U,E,ψ), then (U,E,ψ) is a fuzzy soft pre -Baire space.

Proof: Let $(F_{A_i}\,)'s$ $(i = 1\ to\ \infty)$ be FSS defined on a Fspds (U,E,ψ). Then prop 3.9, $and\ (F_{A_i}\,)'s\ are\ Fuzzy\ soft\ pre\ -\ closed\ sets$ in (U,E,ψ). Now pint^{fs}[pcl^{fs}(F_{A_i})] = pint^{fs}(F_{A_i}) = 0 (by hypothesis), hence $(F_{A_i})'s$ are fuzzy soft pre-nowhere dense sets in (U,E,ψ), Hence Pint^{fs}[$\vee_{i=1}^{\infty}(F_{A_i})] = 0, (\,F_{A_i})'s\ \ are\ fuzzy\ soft\ pre\ nowhere\ dense\ sets\ in$,(U, E,ψ) this implies that (U,E,ψ) is a fuzzy soft pre-Baire space.

Proposition 4.4:

If (U,E,ψ) is a Fuzzy Soft pre dense sets ,then (U,E,ψ) is not a fuzzy soft hyper connected space.

Proof:

Let F_A be a fuzzy soft open sets in (U,E,ψ) by prop 3.18, F_A is not a fuzzy soft dense set in (U,E,ψ). Hence (U,E,ψ) is not a fuzzy soft hyper connected space .

REFERENCE

[1] K. K. Azad, On Fuzzy Semi-Continuity, Fuzzy Almost Continuity And Fuzzy Weakly Continuity, J. Math. Anal. Appl. 82(1981), 14-32.

[2] G. Balasubramanian, Fuzzy Disconnectedness and its Fuzzy Stronger Forms, Indian J. Pure Appl. Math., 24(1) (1993).

[3] C. L. Chang, Fuzzy Topological Spaces, J. Math. Anal. Appl. 24 (1968), 182-190.

[4] E. K. Van Douwen, Applications Of Maximal Topologies, Topo. And Appl., 51 (1993), 125-139.

[5] Maji, P.K., Roy, A.R., Biswas, R, 2001. Fuzzy Soft Sets. Journal Of Fuzzy Mathematics 9 (3), 589-602.

[6] Molodtsov, D, 1999. Soft Set Theory First Results. Comput. Math. Appl. 37, 19-31.

[7] On Fuzzy Soft Weak Baire Spaces E. Poongothai, s. Divyapriya .Advances in mathematics: scientific journal 9 (2020), no.8, 5945–5952 issn: 1857-8365 (printed); 1857-8438 (electronic) https://doi.org/10.37418/amsj.9.8.62.

[8] On Fuzzy Soft Nowhere Dense Sets E Poongothai, S Divyapriya International journal of scientific research https://www.worldwidejournals.com/international-journal-of-scientific-research-(IJSR)/article/on-fuzzy-soft-nowhere-dense-sets/MjUwMTQ=/. Volume-9 | Issue-4 | April-2020 | PRINT ISSN No. 2277 - 8179 | DOI: 10.36106/ijsr.

[9] G.Thangaraj And S.Muruganantham On Fuzzy Perfectly Disconnected spaces International Journal Of Advances In Mathematics Volume 2017, Number 5, Pages 12-21, 2017.

[10] Roy, S., Samanta, T.K, 2013. An introduction to open and closed sets on fuzzy soft topological spaces. Ann. Fuzzy Math. Inform., 6 (2), 425-431.

[11] Tanay, B., Kandemir, M.B. 2011.Topological structure of fuzzy sets comput. Math.Appl., 61(10), 2952-2957.

[12] L. A. Zadeh, Fuzzy Sets, Inform. And Control,8(1965), 338-353.