

## **Error Compensation and Accuracy Improvement in Five-Axis CNC Machine Tool**

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### **Abstract**

The complex structures of the five-axis machine tools produce an inaccuracy at the tool tip which is caused by kinematics parameter deviation resulting mainly from manufacturing error and assembly error. Here, all linear axes are theoretically perpendicular (dot product,  $\cos 90^\circ = 0$ ) to each other and directed along the X, Y, Z coordinate, but in working machines, the axes are nearly perpendicular ( $\cos 89.9^\circ \neq 0$ ) because of the reasons mentioned above. This kind of error can be taken into consideration only by the precise description of the actual kinematics of the machine tool. This paper attempts to develop a generalized error model for the effects of positioning errors of the components of the kinematic chain of a machine in the work space and the results obtained by this model have been verified experimentally. The effect of positioning error has been studied further for the machining of rear hub and an improvement in quality of hub has been observed.

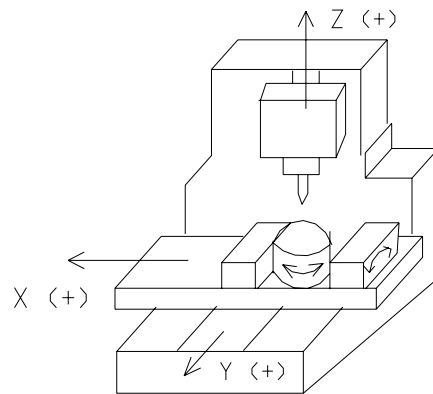
**Keywords:** Kinematics, Positioning error, Error model, Rear hub, Quality improvement.

### **1. Introduction**

Now a day, five-axis CNC machine tools have become popular because of their ability of efficiently machining geometrically complex work-pieces, with higher dimensional accuracies. This is due to the machine's ability to take full advantage of conventional tool geometry by controlling the orientation of the cutting edge. Five-axis machine tools have three translational (T) and two rotational (R) axes. Commonly used

configurations involve a double turntable which rotates the work-piece. An example is a  $\theta_x$  –rotation of the part, followed by a  $\theta_z$  –rotation, X-translation, Y-translation and a final Z-translation (RRTTT). Other common configurations are RTTTR and TTTRR.

The five-axis vertical machining centre (Fig.1) considered in this paper has three translational axes with two rotational axes on the table. The motion in all five axes is controlled simultaneously. This configuration allows for the control of position, like a traditional three-axis machine tool, with the two extra rotational axes providing the added benefit of controlling the orientation of the tool with respect to the work-piece.



**Fig. 1:** Five-axis vertical machining center (RRTTT)

According to Hocken [1], error is the difference between the actual and the anticipated response of the machine to a command issued, according to the machine's accepted protocol. Furthermore, accuracy is defined as the maximum translational or rotational error between any two points in the machine's work volume [2]. Several factors, which may produce the errors on the work piece, are: geometric-kinematics errors of the machine tool; thermally induced error on the machine tool; static and dynamic loading error; error due to clamping; and spindle errors of the machine tool. Considerable work on the general area of machine tool accuracy is reported in literature [3-10]. Most of the authors (3-10) have studied the working accuracy of a CNC machine tool taking into consideration the influence of geometrical errors and / or thermal deformations. Berman and Sen [11] have discussed the techniques to measure the parametric errors of CNC CMM in the form of linear, angular, straightness, squareness, flatness of the machine bed and the diagonal errors by a laser calibration system. Hsu and Wang [12] proposed the method of compensation based on the model that considers the tool orientation error related to the motion of machine rotation axes, and it further calculates the error compensation for rotation axes and linear axes separately. Weidong et. al. [13] have established the B-spline mathematical model to represent the component error friction, and the least-squares fitting method to measure data point. Finally, based on the component error extraction method, numerical error compensation experiments were conducted. Jha and Anjani [14] have developed the generalized error model for three-axis CNC machine tool.

The present paper discusses the development of a generalized positioning error model associated with five axes CNC machine tool with its experimental verification. An improvement in quality of Rear Hub has been observed by compensation of positioning error, using the present model, in five axes CNC machine tool.

## 2. Error Modeling for Five-Axis CNC Machine Tool

A kinematic model on the basis of the characterization of errors, takes into consideration the deviations in motions and alignment of the structural members of a machine. This coupled with schemes, for tracking changes in model parameter and introducing compensations, forms a comprehensive system for compensating effects of the errors on a machine's accuracy. The error models developed are discussed in following steps.

### 2.1 Transformation Model of a Single Joint-link Combination

Links are considered to be rigid body connected either by revolute or prismatic joints. A coordinate frame may be assigned to each link and the relationship between succeeding links may be established using transformations for each of the four variables, namely,  $d$ ,  $x$ ,  $a$  and  $t$ . Using the transformations, nominal relative relationship between adjacent links can be expressed as

$$T_{i-1,i} = T_{z,d} \times T_{z,x} \times T_{x,a} \times T_{x,t} \dots(1)$$

$$= \begin{bmatrix} \cos x & -\cos t \sin x & \sin t \sin x & a \cos x \\ \sin x & \cos t \cos x & -\sin t \cos x & a \sin x \\ 0 & \sin t & \cos t & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

### 2.2 Kinematics of Five-Axis Machining Centre

Kinematics of five axes machining centre are shown in Fig. 2, following the D-H representation rules; and its corresponding D-H parameters are listed in Table 1.

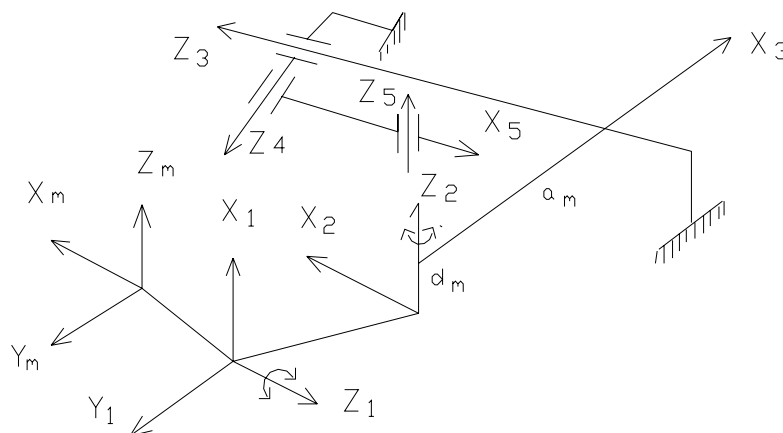


Fig. 2: Kinematics diagram of five-axis machine tool.

**Table 1:** D-H parameters for five-axis machining centre.

Link /pair.	M	1	2	3	4	5	T
di	dm	d1	d2	x*	y*	z*	
ai	am	a1	a2	a3	a4	a5	
xi	-90o	θ1*	θ2*	-90o	90o	-90o	L
ti	-90o	90o	-90o	-90o	-90o	-90o	

\* indicate the variables.

### 2.3 Transformation of Coordinates

Suppose  $P_p$  is the position of the cutter in programming coordinate system;  $P_m$  is the position of the cutter in the machine coordinate system;  $P_t$  is the position of the cutter in the tool coordinate system;  $T_{p,m}$  is the transformation matrix between programming coordinate system and the machine coordinate system; and  $T_{m,t}$  is the transformation matrix between the machine coordinate system and the tool coordinate system. It follows that:

$$P_p = T_{p,m} \times P_m$$

$$P_m = T_{m,t} \times P_t$$

$$P_p = T_{p,m} \times T_{m,t} \times P_t$$

$$P_p = T_{p,t} \times P_t \quad \dots(2)$$

Assuming that the transformation between the programming and machine coordinate system is purely translational, then:

$$T_{p,m} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \dots \quad (3)$$

Substituting the D-H parameters, listed in Table 1, into equation (1), the transformation matrices along each link/joint can be established as follows (note: use of "cos (x)" = "Cx" and " sin (x)" = "Sx") :

$$T_{p,5} = T_{p,m} \times T_{m,1} \times T_{1,2} \times T_{2,3} \times T_{3,4} \times T_{4,5}$$

$$T_{p,t} = T_{p,5} \times T_{5,t}$$

$$T_{p,t} =$$

$$\begin{bmatrix} S\theta_2 & 0 & -C\theta_2 & \left\{ \begin{array}{l} (a_2 + a_5 + Y)S\theta_2 \\ +(X - a_4)C\theta_2 + d_1 \end{array} \right\} + t_x \\ -C\theta_1 C\theta_2 & -S\theta_1 & -C\theta_1 S\theta_2 & \left\{ \begin{array}{l} (X - Y - a_2)C\theta_1 C\theta_2 - (d_2 + a_3)S\theta_1 \\ -(a_1 + a_4 S\theta_2)C\theta_1 - a_m \end{array} \right\} + t_y \\ -S\theta_1 C\theta_2 & C\theta_1 & -S\theta_1 S\theta_2 & \left\{ \begin{array}{l} (-a_2 - Y - a_5)S\theta_1 C\theta_2 + (d_2 + a_3 - Z)C\theta_1 \\ + (-a_1 + X S\theta_2 - a_4 S\theta_2)S\theta_1 + d_m \end{array} \right\} + t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

$$\begin{bmatrix} x_p \\ y_p \\ z_p \end{bmatrix} = [T_{p,t}] \begin{bmatrix} x_t \\ y_t \\ z_t \\ 1 \end{bmatrix} \quad \dots (5)$$

When, the tool length is 1 unit

$$\begin{bmatrix} x_p \\ y_p \\ z_p \end{bmatrix} = \begin{bmatrix} i \\ j \\ k \end{bmatrix} = [T_{p,t}] \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad (6)$$

Where,  $[i,j,k]^T$  is the specified direction cosine. By simplifying the above equation, we get

$$\begin{bmatrix} i \\ j \\ k \end{bmatrix} = \begin{bmatrix} -C\theta_2 \\ -C\theta_1 S\theta_2 \\ -S\theta_1 S\theta_2 \end{bmatrix} \quad \dots (7)$$

$$\theta_1 = \tan^{-1}(1); \quad \theta_2 = \cos^{-1}(-i)$$

When the tool length is L, and considering the programmed position as being located at the tip of the tool

$$\begin{bmatrix} x_p \\ y_p \\ z_p \end{bmatrix} = [T_{p,t}] \begin{bmatrix} 0 \\ 0 \\ L \\ 1 \end{bmatrix} = \begin{bmatrix} -LC\theta_2 + (a_2 + a_5 + Y)S\theta_2 \\ + (X - a_4)C\theta_2 + d_1 + t_x \\ -LC\theta_1 S\theta_2 + (X - Y - a_2)C\theta_1 C\theta_2 - (d_2 + a_3)S\theta_1 \\ -(a_1 + a_4 S\theta_2)C\theta_1 - a_m + t_y \\ -LS\theta_1 S\theta_2 + (-a_2 - Y - a_5)S\theta_1 C\theta_2 + (d_2 + a_3 - Z)C\theta_1 \\ + (-a_1 + X S\theta_2 - a_4 S\theta_2)S\theta_1 + d_m + t_z \end{bmatrix} \quad (8)$$

The values of X, Y and Z can be written in matrix form as:

$$\begin{bmatrix} x_p - [-LC\theta_2 + (a_2 + a_5 + Y)S\theta_2 \\ + (-a_4)C\theta_2 + d_1 + t_x] \\ y_p - [-LC\theta_1 S\theta_2 + (-a_2)C\theta_1 C\theta_2 - (d_2 + a_3)S\theta_1 \\ -(a_1 + a_4 S\theta_2)C\theta_1 - a_m + t_y] \\ z_p - [-LS\theta_1 S\theta_2 + (-a_2 - a_5)S\theta_1 C\theta_2 + (d_2 + a_3)C\theta_1 \\ + (-a_1 - a_4 S\theta_2)S\theta_1 + d_m + t_z] \end{bmatrix} = \begin{bmatrix} C\theta_2 & S\theta_2 & 0 \\ C\theta_1 C\theta_2 & -C\theta_1 C\theta_2 & 0 \\ S\theta_1 S\theta_2 & -S\theta_1 C\theta_2 & -C\theta_1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \quad (9)$$

### 2.4 Error Model for Five-Axis Machine Tools

The joint axes orientations vary with the degree of misalignment. If we find the value of i and k in the reference coordinate system, then j is given by

$$j = \sqrt{1 - i^2 - k^2}$$

The location of the actual joint varies with the location of the plane.

In the zero-position-modeling, the machine zero position and the coordinate system are defined before defining the unit vector of each axis. The unit vector from Fig.2 has been tabulated in Table 2, may be defined as follows:  $[i,j,k]$  are the direction cosines of individual axis in the reference coordinate system. The rows 1-5 are for the five axes of the machine. The above model (Table 2) is for the ideal case. According to Table 2, the X-axis should pass through the point  $P_1 [P_{1x}, P_{1y}, P_{1z}]$  with a direction cosine  $[1, 0, 0]$ . If  $P_{1r}$  is the real point  $[P_{1r} \neq P_{1r}]$  through which the X-axis passes, then

$$\Delta P_1 = 1 i + (P_{y1r} - P_{y1}) j + (P_{z1r} - P_{z1}) k$$

where,  $\Delta P_{1y} = (P_{y1r} - P_{y1})$ , and  $\Delta P_{1z} = (P_{z1r} - P_{z1})$  are small deviations in the position of the origin in the base Y- and Z- directions, respectively. The actual direction cosine of joint one is given by

$$\left[ \sqrt{1 - j_1^2 - k_1^2}, j_1, k_1 \right]$$

The results have been obtained, by extending the same idea to all the joints, and tabulated in Table 3. The unknown direction and position values can be found by the calibration process.

**Table 2:** Position of frame in zero position (ideal case)

Coord/Link	i	j	K	$P_x$	$P_y$	$P_z$
1	1	0	0	$p_{1x}$	$p_{1y}$	$p_{1z}$
2	0	0	1	$p_{2x}$	$p_{2y}$	$p_{2z}$
3	1	0	0	$p_{3x}$	$p_{3y}$	$p_{3z}$
4	0	1	0	$p_{4x}$	$p_{4y}$	$p_{4z}$
5	0	0	1	$p_{5x}$	$p_{5y}$	$p_{5z}$

Table 3 describes the real machine geometry with either arbitrary rotation or a displacement axis.  $[P_x, P_y, P_z]^T$  is the transformation at the zero position.  $[P_{xr}, P_{yr}, P_{zr}]^T$  is the actual location (offsets) of the axis. If we know the direction cosine of the actual axis and the offsets we can find the tool tip and the orientation of the tool axis vector for a given displacement of joints.

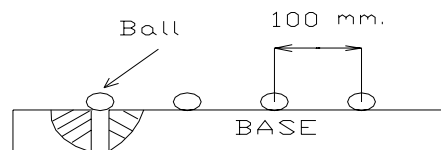
Table 3: Exact parameters for a five-axis machine tool for zero reference model.

Coord./Link	i	j	k	$p_x$	$p_y$	$p_z$
1	$\sqrt{1 - j_1^2 - k_1^2}$	$j_1$	$k_1$	$p_{1x}$	$\Delta p_{1y}$	$\Delta p_{1z}$
2	$i_2$	$j_2$	$\sqrt{1 - i_2^2 - j_2^2}$	$\Delta p_{2x}$	$\Delta p_{2y}$	$p_{2z}$
3	$\sqrt{1 - j_3^2 - k_3^2}$	$j_3$	$k_3$	$p_{3x}$	$\Delta p_{3y}$	$\Delta p_{3z}$
4	$i_4$	$\sqrt{1 - i_4^2 - k_4^2}$	$k_4$	$\Delta p_{4x}$	$p_{4y}$	$\Delta p_{4z}$
5	$i_5$	$j_5$	$\sqrt{1 - i_5^2 - j_5^2}$	$\Delta p_{5x}$	$\Delta p_{5y}$	$p_{5z}$

### 3. Experimental Error Measurement

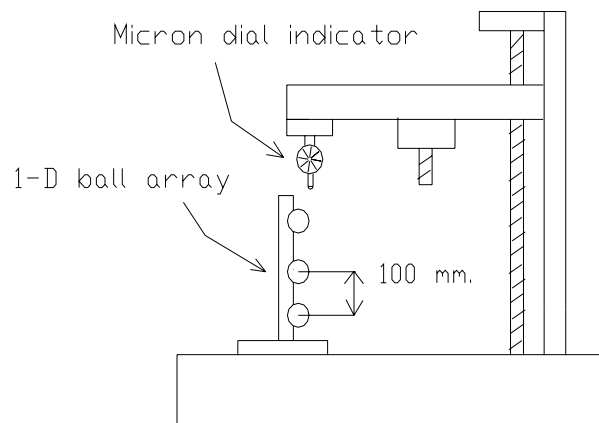
#### 3.1 Calibration of Machine Tool

The calibration of the machine tool geometry has been done, using 1-D ball array [15], with micron dial indicator. The construction design of the 1-D ball array has been shown schematically in Fig. 3. A series of balls with the same diameter and small sphericity errors are fixed on a rigid bar. The manufacturing process of this type of 1-D ball array is very simple. A series of cone holes are bored by a jig boring machine. Then the balls are glued in these holes.



**Fig. 3:** Structure of 1-D ball array.

The distance between the balls of 1-D ball array are calibrated by a specially designed device consisting of a laser interferometer, similar to that described by Rademacher [16]. The positioning errors of the machine tool along its motion axis has been measured by direct comparison of the readings from CRT display of the machine tool with the calibrated data of the ball array, when the later is aligned along the motion axis. The measuring principle is shown in Fig. 4. The micron dial with magnetic stand has been mounted on the spindle box and a 1-D ball array has been fixed on the working table.



**Fig. 4:** Measurement set-up.

#### 3.2 Machine Specification

For the verification of errors and its calculation, Five-axis Vertical Machining Centre (Trade Name: VMC-50), is used, detail of which is given as below:

Specification of the machine

Machine type	Control type	Axis Movement	Feed Movement
VMC-50	FANUC 0M	X = 800 mm Y = 600 mm Z = 800 mm $\theta 1 = +/- 3600$ $\theta 2 = +/- 3600$ $\theta 2 = +/- 3600$	1-4000 mm/min

3.3 Error Calculation

Errors for the movement along each linear axes and its effect on the other axes have been calculated. Then the average error has been calculated and it has been pre-compensated through NC part program. The errors of the machine tool before compensation, and after compensation are graphically presented in Fig. 5.

The effect of positioning errors associated with five-axis machine tool in machining of rear hub for an automobile company, shown in Fig.6, has been studied .The quality of rear hub depends upon the machine tool errors. A practical verification has been done, by machining a rear hub under two conditions (1) without compensating the error of the machine tool, and (2) after pre-compensating the average errors in the work reference point of the machine tool. Now, both the components have been inspected on column type Coordinate Measuring Machine (CMM). The improvement in the quality of rear hub has been noticed and graphically presented in Fig.7.

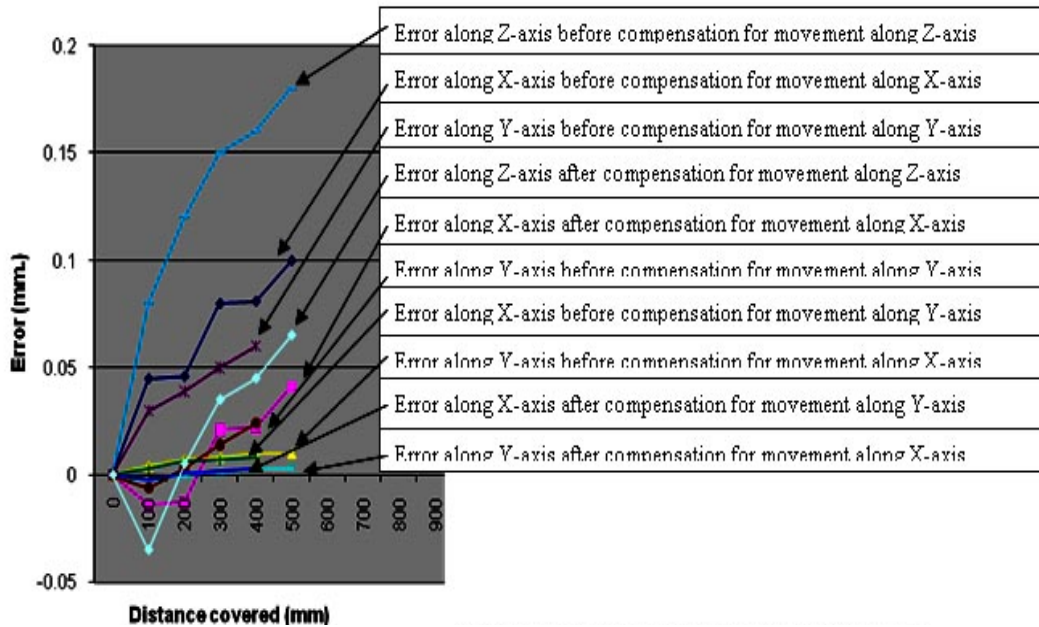


Fig. 5: Error before compensations and after compensation.

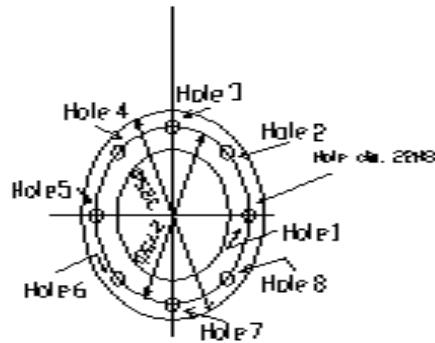


Fig. 6: Rear Hub.

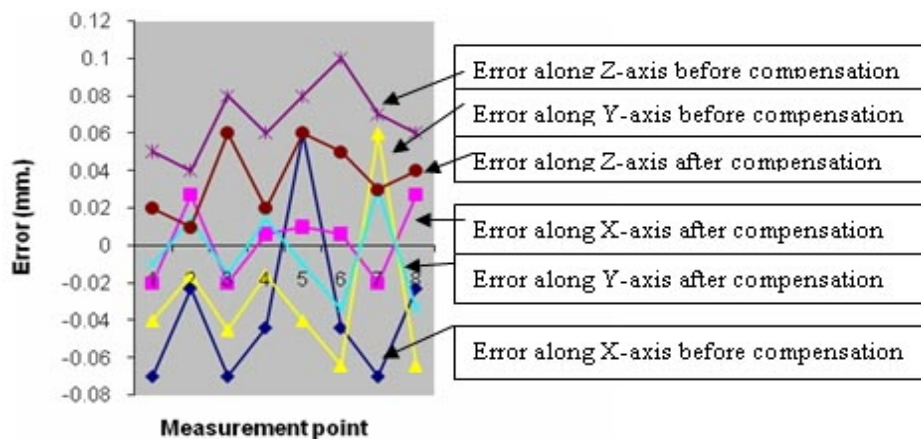


Fig. 7: Errors at various points of rear hub.

#### 4. Conclusions

This paper describes a generalized error model at the cutter throughout the workspace due to positioning errors of individual components on a five-axis CNC machine tool. The theoretical errors associated has been practically verified and tested on the CNC machine using a 1-D ball array and micron dial indicator. By correcting the CNC part machining program, the positioning errors can be pre-compensated more easily before machining, and without any additional measuring device and hardware modification. The present method is more suitable and it can be applied more easily as compared to other methods available in literature.

The positioning error of the machine tool has been compensated, by the present error model, in machining a rear hub of an automobile industry. The quality of hub, after machining, has been checked up by CMM in both the cases, i.e. without compensating the errors of CNC machine and after compensating the errors of CNC machine. An improvement in the quality of hub has been observed by application of the present error model and error compensation. In addition, the application of the present error model makes the production processes more flexible, automatic and controllable.

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