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# Stress Analysis of Functionally Graded Material Plate with Cut-out

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## Abstract

In structures and machine components, operations like cut-outs of different shapes for openings are facilitated. The total area of structures are reduces due to these cut-outs, which becomes the stress risers for particular structure and it works as weakening agent for the structure or machine. Different shapes of cut-out generate different effect of stress concentration, which directly effects on the life of the Structure/machine component. The stress field around such cut-outs must be known under different loading conditions. Here an attempt is made to obtain generalized solution for determining the stress concentration around circular hole, the shape which has minimum effects of stress concentration, in composite infinite plate subjected to arbitrary uniaxial, biaxial and uniform pressure loading at infinity using Muskhelishvili's complex variable method. General solutions are obtained with MATLAB coding and compared with an FEA tool.

**Keywords**: Functionally Graded Materials (FGMs), Circular Cut-out, Complex Variable method, Stress Concentration Factor

## Introduction

Functionally Graded materials (FGM) is the class of composite materials, in which two or more than two materials are composite as per the function or application of the materials where it is going be used. The composition of the materials is in continuously graded form. There is no material in nature which has good strength with brittleness. A material with good thermal resistance (like ceramics) having brittleness which can face the temperature resistance but, it cannot face the high mechanical loading. On the other side, the material with good toughness and strength (like metal) is with good ductility, which can face the mechanical loading conditions but for temperature resistance it is not suitable. In this case we can form a composite material which will participate as a metal at one side and as a ceramic at another side. In this case the FGM composed will be transformed from ceramic to metal from one end to another end in continuously graded form. So FGM is a composite material which is made up of two or more than two material combination with desired physical and chemical properties [1]. The most common functionally graded materials are made of ceramic/metal non homogeneous structure, in which ceramic provides good thermal resistance and metal roles as a superior toughness and hardness. V. G. Ukadgaonker, Vyasraj Kakhandki [2] checked the effect of different shape of cut-out and irregular shapes of hole. General solutions for determining the stress field around circular hole in infinite orthotropic plate subjected to internal pressure were obtained by D S Sharma, Panchal Khushbu, and Patel Nirav [3], using Muskhelisvili's complex variable formulation in 2011. D. V. Kubair, B. Bhanuchandra [4] numerically investigated the effect of the material property in homogeneity on the stress concentration factor (SCF) due to a circular hole in functionally graded panels. Mohammad Abedi and GH. Rahimi [5] introduced analytical solution of stress distribution around circular cutout in orthotropic functionally Graded Composite (FGC) plate.

**Types of FGM**

(1) According to the structure, two types, one is continuously structured FGM and another is discontinues (Layered) FGM.

(2) According to process of manufacturing, one type is Thin FGM and another type is Bulk FGM. Thin FGM are manufactured with different methods like Physical Vapor Deposition (PVD), Chemical Vapor Deposition (CVD), Self propagating High temperature Synthesis (SHS) method etc. While the Bulk FGMs are manufactured by the methods like, Powder Metallurgical Technique, Centrifugal Casting method, Solid Free Foam technique, etc. For current study we concentrate on bulk FGM only, so three main manufacturing techniques are described briefly. In powder Metallurgy method for making bulk fgm, basic steps are weighing and mixing of powder, stacking and ramming of premixed powder, and finally sintering. In Centrifugal method, gravitational force is used through spinning of the mould to form continuous bulk fgm. By the change in mould spinning speed and change in density of powder materials of metal will produces continuously graded structure. A limitation of the method is that, the gradient of materials cannot be tolerated as per requirement, because it depends naturally. Solid Free Foam Fabrication method is a method in which first the modeling of the part is done, then converting the modeled file in. stl (standard triangulation language) file. Now slices of the. stl file model in to two dimensional cross sections. The next step is component building, and finally removal of unnecessary part & finishing. Generally in solid free foam fabrication method, laser based methods are used. Like, laser cladding based method, selective laser sintering (SLS), 3D printing method, Selective laser melting (SLM) etc. This method for manufacturing fgm is most advantageous because it works at high speed production,

less energy intensive, maximum material utilization, flexibility in modeling because we can directly convert the model from cad software, and complex shape formulation is also becomes easy by solid free foam fabrication method. Limitation of this method is that the rough surface finishing. [1]

## COMPLEX VARIABLE FORMULATION [2]

An FGM plate with circular hole is considered under generalized plane stress condition. The plate is assumed under stress in XY plane so here the values of stress for Z axis will becomes zero, so $\sigma_z, \tau_{xz}$ and $\tau_{yz}$. Stress contours for FGM plate with circular cut-out under different loading conditions (e. g. Uniaxial, Biaxial and Uniform pressure around hole) are studied. The area external to a given hole, in Z-plane is mapped conformably to the area outside the unit circle in ζ plane using following mapping function.

$$z_j = \omega_j(\zeta) = \frac{R}{2}\left[ a_j\left(\frac{1}{\zeta} + \sum_{k=1}^{N} m_k \zeta^k\right) + b_j\left(\zeta + \sum_{k=1}^{N} \frac{m_k}{\zeta^k}\right)\right]$$

Where, $a_j = (1 + is_j)$, $b_j = (1 - is_j)$; j=1, 2. Eq. (1)

Depending on the values of $m_k$ different shapes can be mapped outside the unit circle and by applying boundary conditions the stresses is derived.

$$\sigma_x = \frac{p}{2}[(\lambda+1) + (\lambda-1)\cos 2\beta]$$

$$\sigma_y = \frac{p}{2}[(\lambda+1) - (\lambda-1)\cos 2\beta]$$

$$\tau_{xy} = \frac{p}{2}[(\lambda-1)\sin 2\beta] \qquad \text{Eq. (2)}$$

$\lambda = 0$ and $\lambda = 1$ explains uniaxial and biaxial loading conditions, respectively.

## GENERAL SOLUTION FOR UNIFORM PRESSURE AROUND THE HOLE [3]

For the case of uniform pressure *p* around hole, the resultant stress $X_n$, $Y_n$ on the hole boundary are given by

$$X_n = -p\cos(n,x) = -p\frac{dy}{ds},\ Y_n = -p\cos(n,y) = -p\frac{dx}{ds} \qquad \text{Eq. (3)}$$

Where n is the positive direction of the outward normal to the hole boundary. The stress boundary conditions are given by

$$f_1^0 = -\int_0^s Y_n ds + const. = -px,\quad f_2^0 = -\int_0^s X_n ds + const. = -py, \qquad \text{Eq. (4)}$$

The stress functions are obtained using these boundary conditions $(f_1^0, f_2^0)$ into Schwarz formula:

$$\psi(\xi) = \frac{i}{4\Pi(s_1 - s_2)} \int_\gamma \left[ (s_1 f_1^0 - f_2^0) \left\{ \frac{t+\xi}{t-\xi} \right\} \frac{dt}{t} \right]$$

$$\phi(\xi) = \frac{i}{4\Pi(s_1 - s_2)} \int_\gamma \left[ (s_2 f_1^0 - f_2^0) \left\{ \frac{t+\xi}{t-\xi} \right\} \frac{dt}{t} \right] \qquad \text{Eq. (5)}$$

The stress functions and their derivatives obtained are given bellow,

$$\phi(\zeta) = \frac{-ipR}{2(s_1 - s_2)} \left[ \frac{a_2}{\zeta} \right], \ \psi(\zeta) = \frac{-ipR}{2(s_1 - s_2)} \left[ \frac{a_1}{\zeta} \right]$$

$$\phi'(\zeta) = \frac{-ipR}{2(s_1 - s_2)} \left[ \frac{-a_2}{\zeta^2} \right], \ \psi'(\zeta) = \frac{-ipR}{2(s_1 - s_2)} \left[ \frac{-a_1}{\zeta^2} \right] \qquad \text{Eq. (6)}$$

Put these values in stress equations,

$$\sigma_x = 2\,\mathrm{Re}\left[ s_1^2 \phi'(z_1) + s_2^2 \psi'(z_2) \right]$$

$$\sigma_y = 2\,\mathrm{Re}\left[ \phi'(z_1) + \psi'(z_2) \right]$$

$$\tau_{xy} = -2\,\mathrm{Re}\left[ s_1 \phi'(z_1) + s_2 \psi'(z_2) \right] \qquad \text{Eq. (7)}$$

## Results and discussion

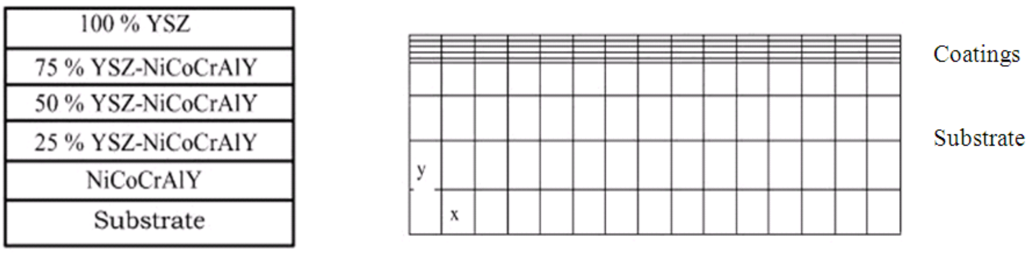
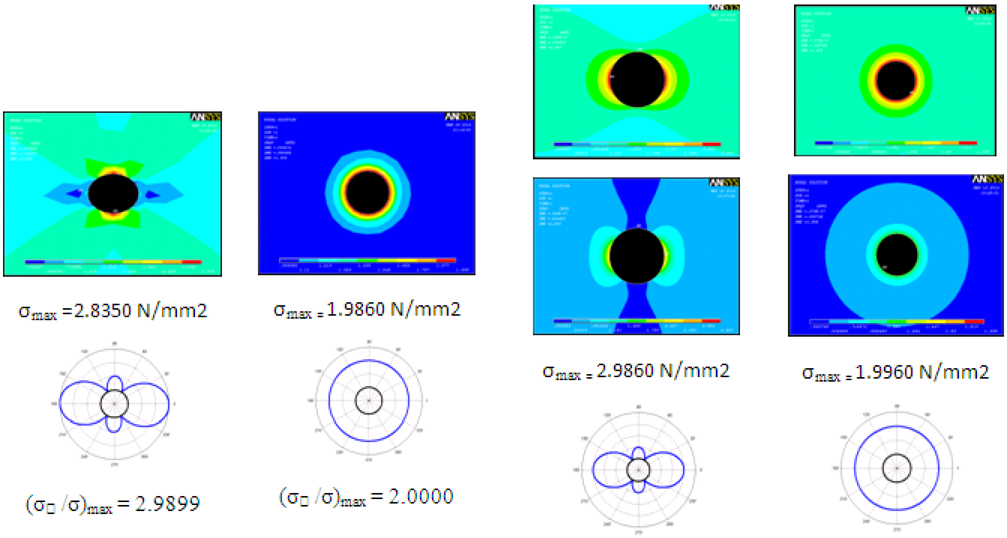



**Figure 1: -** Construction of FGM plate [6]

Measurement of stress as per generalized method, we can check with mathematical coding in MATLAB R2011a, and compare it with FEA software tool (ANSYS 11).

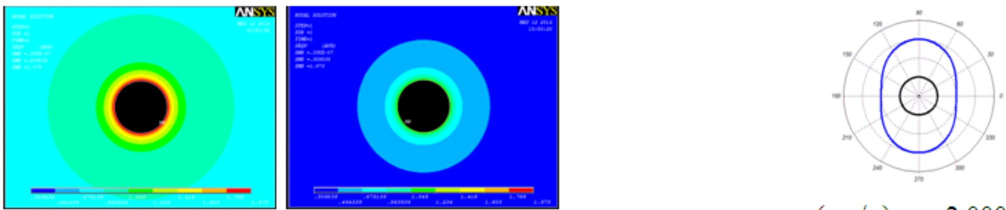


$\sigma_{max}$ =2.8350 N/mm2 | $\sigma_{max}$ = 1.9860 N/mm2 | $\sigma_{max}$ = 2.9860 N/mm2 | $\sigma_{max}$ = 1.9960 N/mm2

$(\sigma_{\square}/\sigma)_{max}$ = 2.9899 | $(\sigma_{\square}/\sigma)_{max}$ = 2.0000 | $(\sigma_{\square}/\sigma)_{max}$ =3.0000 | $(\sigma_{\square}/\sigma)_{max}$ =2.0000

Steel plate with uniaxial loading ($\lambda$=0) | Steel plate with biaxial loading ($\lambda$=1) | FGM plate with uniaxial loding ($\lambda$=0) | FGM plate with biaxial loading ($\lambda$=1)

**Figure 2: -** Stress contours of Steel and FGM plate under Uniaxial and Biaxial Loading conditions



$\sigma_{max}$ = 1.9730 N/mm2

$(\sigma_{\square}/\sigma)_{max}$ =2.0000

**Figure 3: -** Stress contours of FGM plate under uniform pressure on hole boundary

## CONCLUSION

In this paper results for main three loading conditions i. e. Uniaxial, Biaxial and Uniform pressure on hole boundary for Steel and Nickel based FGM material plate with circular cut-out are studied. The results with mathematical formulation in MATLAB R2011a are compared with FEA (Finite Element Analysis) results in ANSYS 11. 0. The comparison concludes that the work Simulation results in ANSYS and mathematical results in MATLAB are nearer and give satisfactory results. The Results obtained in this research work is based on scaled modeling of object and so by taking different scale factor (as per condition) we can check the effect of stress concentration effect.

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