

Study of Vibration Analysis of Laminated Composite Plates Using FEM

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Abstract

The vibrational analysis of laminated composite plates is analysed using finite element method. In this paper Numerical results have been computed for the effect of number of layers, thickness ratio of plate, different boundary conditions, different aspect ratio, and different angle of fibre orientation of laminated composite plate. The analysis for rectangular plate is carried out for thickness ($h/b= 0.001, 0.01, 0.05, 0.1, \text{ and } 0.2$) and different aspect ratios ($a/b=1, 1.5, 2, \text{ and } 2.5$). The problem of free vibration analysis of composite square plates having (3, 5, 7, and 9) lamina layers (angle ply and cross ply) is also considered. The non-dimensional fundamental frequency of vibration is found to increase with increase of angle of fibre orientation and number of layers but non-dimensional fundamental frequency decrease with increase in size ratio and thickness to width ratio. The natural frequencies and mode shapes are compared for different boundary condition. Comparisons are made with the result for thin and thick composite laminated plate.

Keywords Free vibration, Laminate composite plate, Boundary condition,

Introduction

Composite laminated plate have been mostly used in engineering application like mechanical, aerospace, automobile, marine, and civil engineering due to their high strength and flexibility in design. A large number of structural components in engineering structures can be classified as plates. To avoid the resonant behaviour of the structures, free vibration analyses of the laminated composite plates are necessary in different design aspect. The analysis of laminated plates was started by Whitney [1], Whitney and Leissa [2], Reissner [3], Bert and Francis [4], Bert and Mayberry [5], Noor [6], Reddy [7]. Sharma et. al. [8, 11, 15] studied the free vibration analysis of laminated composite plates considering first order shear deformation theory.

Aydogdu et. al. [12] studied the Vibration analysis of crossply laminated square plates with general boundary conditions. Karami et. al. [13] studied DQM free vibration analysis of moderately thick symmetric laminated plates with elastically restrained edges. Wang et. al. [14] studied the free vibration analysis of skew fiberreinforced composite laminates based on first-order shear deformation plate theory. Numerical examples of isotropic and composite rectangular plates having different fiber orientations angles, thickness ratio and aspect ratio have been solved. Long list of references on free vibration analysis of laminated composite rectangular plate given, for example, in Refs. [15-17].

Numerical Result and Discussion

To show the accuracy of our results we compare the results with the previous reference papers as in literature. It is clear from convergence study that there is upto 4% variation in result.

Table 1, Comparison of non-dimensional frequencies with respect to the results given by [11] [12] and [13]for a cross-ply laminate for boundary condition i.e. CCFE.

	Mode							
	1	2	3	4	5	6	7	8
Present	5.4113	10.9971	25.7563	30.8719	34.1487	43.9286	49.1970	63.4234
[11]	5.4691	11.1076	25.9892	31.3080	34.6154	44.4667	49.5819	64.0844
[12]	5.471							
[13]	5.472							

Table 2, Comparison of non-dimensional frequencies with respect to the results given by [11] [12] and [13] for a cross-ply laminate for boundary condition i.e. CSFE.

	Mode							
	1	2	3	4	5	6	7	8
	5.1341	8.9086	21.3748	30.7758	33.2154	41.0802	42.6495	58.0033
[11]	5.1894	9.0006	21.5817	31.2121	33.6750	41.6003	43.0237	58.6416
[12]	5.191							
[13]	5.191							

Table 3, Comparison of non-dimensional frequencies with respect to the results given by [11] [12] and [13] for a angle-ply laminate for boundary condition i.e. CCCC .

M=N	Mode							
	1	2	3	4	5	6	7	8
7	2.2051	3.6155	3.8205	5.0037	5.5057	5.6206	6.5150	6.7627
Ref[11]	2.2483	3.6744	3.9060	5.0836	5.6111	5.7462	6.6012	6.9140
Ref[12]	2.2857	3.7393	3.9813	5.1800	5.7019	5.8455	6.7167	7.0452
Ref[14]	2.2857	3.7392	3.9813	5.1799	5.7019	5.8545	6.7166	7.0449

Table 4, Variation of non dimensional frequencies with the aspect ratio (a/b= 1, 1.5, 2, 2.5) of plate for cross-ply laminate (0⁰/90⁰/0⁰) for CCCC external condition (h/b=0.01)

Mode a/b	1	2	3	4	5	6	7	8	9	10
1	32.51	40.73	59.46	84.34	89.42	89.72	102.29	126.84	132.76	160.89
1.5	16.84	28.98	39.63	47.77	51.34	66.53	75.44	82.16	83.76	97.14
2	12.12	23.99	26.20	34.92	44.23	49.51	53.48	56.73	72.68	73.95
2.5	10.40	17.14	25.28	29.83	30.05	41.24	48.32	48.86	68.04	59.66

Table 5, Variation of non dimensional frequencies with the thickness ratio (h/b= 0.001, 0.01, 0.05, 0.1, 0.2) of plate for cross-ply laminate (0⁰/90⁰/0⁰) for CCCC external condition (a/b=1)

Mode h/b	1	2	3	4	5	6	7	8	9	10
0.001	35.07	51.38	82.78	92.87	118.80	125.12	162.43	174.46	181.95	208.27
0.01	32.51	40.74	59.47	84.35	89.42	89.72	102.29	126.84	132.76	160.89
0.05	23.79	32.04	48.27	48.88	53.95	65.50	70.99	78.80	82.41	84.20
0.1	15.79	23.23	28.87	33.83	35.07	43.05	44.28	47.82	49.54	54.87
0.2	9.42	14.45	15.90	19.44	21.37	23.35	25.06	25.98	29.14	30.44

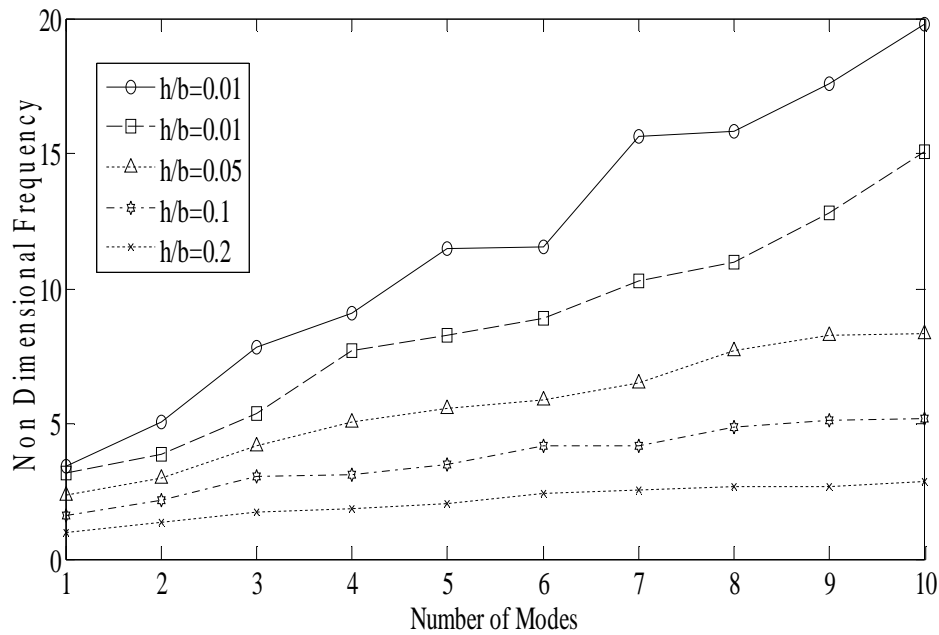


Fig. 1, Variation of non dimensional frequencies with the thickness ratio ($h/b=0.001,0.01,0.05,0.1,0.2$) of plate for cross-ply laminate (15/-15/15) for CCCC external condition ($a/b=1$)

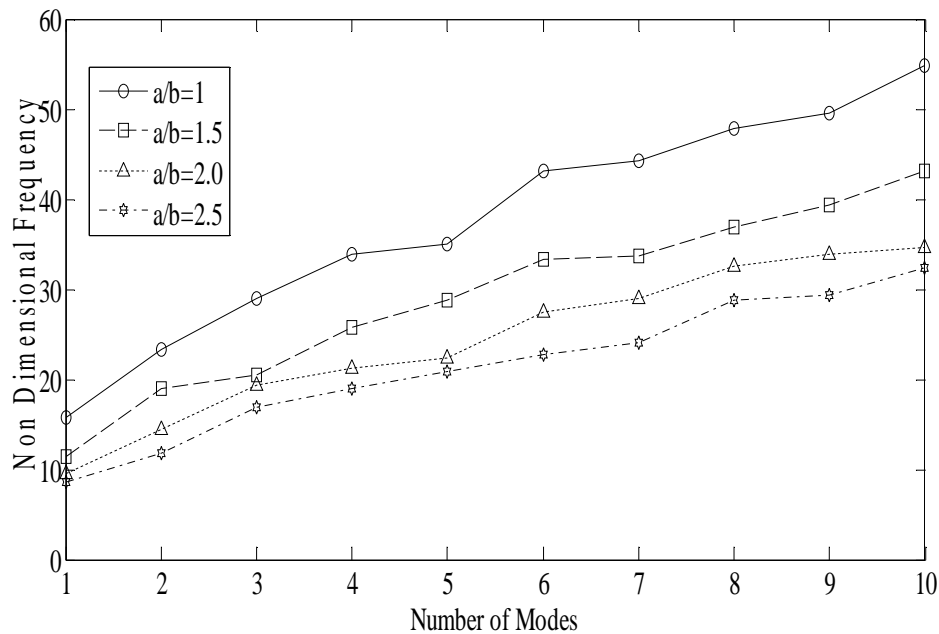


Fig. 2, Variation of non dimensional frequencies with the aspect ratio ($a/b=1, 1.5, 2, 2.5$) of plate for cross-ply laminate (0°/90°/0°) for CCCC external condition ($h/b=0.1$)

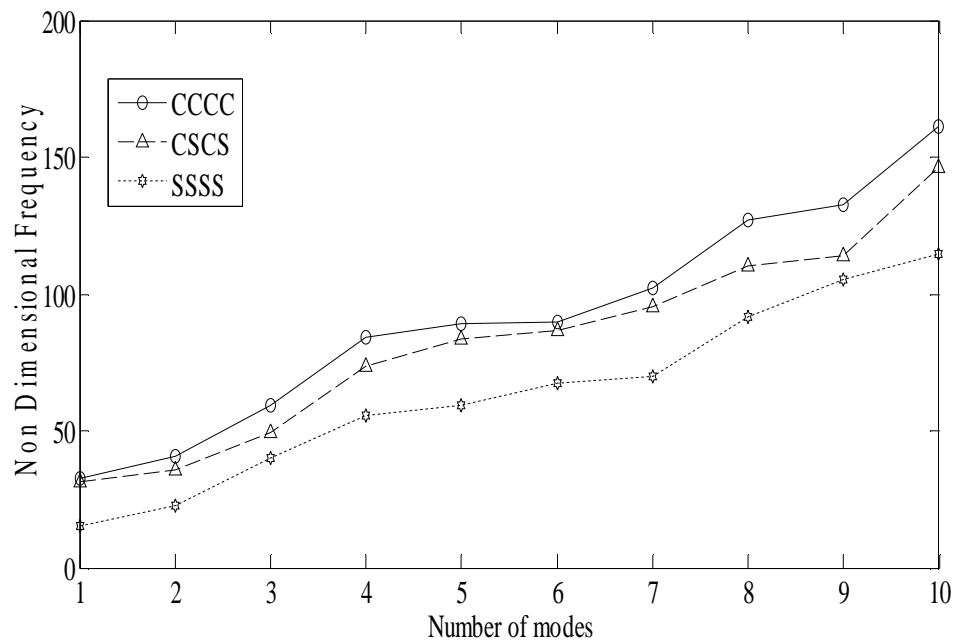


Fig. 3, Variation of non dimensional frequencies with the boundary condition (CCCC, SSSS, CSCS) of plate for cross-ply laminate ($0^0/90^0/0^0$) for CCCC external condition ($a/b=1$, $h/b=0.01$)

Conclusion

The present models lead to the following conclusions. The non-dimensional fundamental frequency of vibration is found to increase with increase of angle of fibre orientation and number of layers but non-dimensional fundamental frequency decrease with increase in aspect ratio and thickness to width ratio. Non-dimensional fundamental frequency of vibration of clamped plate is higher than from simply supported plate. The numerical results show that the FEM can yield convergent and accurate result for thin and thick laminated plate.

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