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Optimization of Spur Gears Using Profile Modification

[*1] A. Mandal, [2] S. Panda, [3] P.K. Pradhan, [4] S.N. Panda

[1,2,3,4] Department of Mechanical Engineering, Veer Surendra Sai University of Technology ,Burla ,Odisha , 768018, India
*.*E-mail: abhijeetmandal1994@gmail.com*

Abstract

In this article a profile modification approach is adopted to design a spur gear set. An alternating tooth sum method is used to obtain the desired contact ratio so as to reduce the hertzian contact stress along the path of contact. A case study is performed for a spur gear set with specific center distance and tooth sum of 100 (±4), to estimate the optimal addendum radius. Finally, a CAD model was developed using the gear geometry so as to check the practical feasibility of the design. Then, a FEM analysis was implemented to estimate the stresses induced in the loaded gear set. In order to identified the critical stress region, the FEM analysis was executed and the result obtain are compared with the standard available result.

Keywords- Spur gears, contact ratio, profile shift, contact stress, operating pressure angle.

I. Introduction

Design of a gear that can withstand almost all failure modes in its running condition is a complex task. In the industry, there is a need of comparatively compact gear drive with smother transmission. Gears tooth profiles are either cycloidal or involutes. But involutes profile finds frequent application due its simple design and tooling requirements for manufacturing. Another important feature of involutes profile is that they are insensitive to change in center distance. So the tooth profile may be shifted along the path of contact expediently.

The process of shifting the profile is also known as profile correction. A profile shift has influence on the tooth thickness and profile. The purpose of profile shift is to avoid under cutting, improve rolling of gears, increase bending fatigue and surface fatigue, reduce backlash and change center distance.

The contact stress is a significant parameter to maintain durability against surface fatigue in meshing gears (Nabih, et al.(1)). Hence, stress analysis in the contact area and determination of the crtically stressed section is gaining interest among the research community for optimum design og transmission gears (Ruben, et al. (2)). This process of identification of critical section involves complex analysis. In this background, various attempts were made by different researchers to estimate the hertizian contact stresses on non-conformal contact like gears.(Stachowiak and Batchelor (3)).. By using analytical equations based on the elasticity theory developed by Hertz.Miryam, et al. (4) has developed an analytical model for load distribution the line of contact to calculate contact stress in spur and helical gears. The authors have reported that the load distribution was not uniform (due to the change in rigidity of the pair of teeth) along the path of contact and has a critical influence on the location and magnitude of the contact stress .Fatih and Stephen (5) revealed that the wear at the beginning and end of the tooth mesh due to instantaneous contact loads and Hertz pressures is more severe and affects the performance of gear drive.

II. Nomenclature

a = Center distance (mm)	r_{a2} = Addendum circle radius (gear) (mm)
b = Face width (mm)	r_{b1} = Base circle radius (pinion) (mm)
E_1 = Young's modulus (pinion) (MPa)	r_{b2} = Base circle radius (gear) (mm)
E_2 = Young's modulus (gear) (MPa)	X = Profile shift coefficient (mm)
F_n = Normal force (N)	X_1 = Profile shift coefficient (pinion) (mm)
Ft= Tangential force (N)	X_2 = Profile shift coefficient (gear) (mm)
m = Module (mm)	ε = Contact ratio
P_{bn} = Base pitch (mm)	σ_{max} = Contact stress (MPa)
Q = Load (N)	υ_1 = Poisson's ratio (pinion)
R_1 = Radius of curvature (pinion) (mm)	υ_2 = Poisson's ratio (gear)
R_2 =Radius of curvature (gear) (mm)	ø = Standard pressure angle (°)
r = Radius (mm)	$ø_w$ = Working pressure angle (°)
r_{11} = Pitch circle radius (pinion) (mm)	Z_e = the altered tooth-sum
r_{12} =Pitch circle radius (gear) (mm)	Z = standard tooth-sum
r_{a1} = Addendum circle radius (pinion) (mm)	

Further, in few ref. (Ali (6); Sfakiotakis, et al. (7)), it is reported that the gear teeth have an elliptical stress distribution pattern with the maximum stresses lying in the middle plane. The contact stresses are evaluated using Eq. (1):

$$\sigma_{max} = \sqrt{\frac{Q\left(\frac{1}{R1}+\frac{1}{R2}\right)}{b\pi\left[\left(\frac{1-\vartheta 1}{E1}\right)+\left(\frac{1-\vartheta 2}{E2}\right)\right]}} \qquad [1]$$

An improved method was provided by (Johnson et al.(8)). Further, the authors have pointed out that the contact stress is guided by the adhesiveness of the surfaces in contact, the correlation between the contact area, the elastic material properties, and interfacial interaction strength.

In most of the gears pitting, scoring, scuffing, and abrasive wear are influenced by the different operating condition, fatigue life etc .These wear phenomena were guided by the maximum Hertz surface pressure (Anders and Soren (9)) and fluctuating load. During operation of gear under full load, very high contact pressure occurs at the mesh interference. This high contact pressure under high load and low speed condition leads to partial breakdown of the lubricant and causes mild wear, scuffing, scoring, sapling, and pitting (Amarnath and Sujatha (10)).

It has been found from the literature review that, analysis of the contact stress of altered tooth-sum gearing is limited. The contact stress is influenced by profile shift, so some attempts are made in this study to predict accurately degree of change of the contact stress along the path of contact due to change in profile shift. In this article a case study has been performed with the gears used in private vehicle and having an alter tooth-sum of 100 (± 4), to find its contact ratio and its contact stress along its path of contact.

III. Methodology

Parameters consideration for modeling

The geometrical parameters of the gear considered for estimation of hertizian contact stresses are module (m) of 2 mm, face width (b) of 20 mm, and pressure angles (φ) of 20° and 25°. The gear material for the gear considered in this study is steel (C40) with Young’s modulus of the as 200 GPa. The applied tangential tooth load per unit millimeter of face width was 10 N.

Estimation of contact stress

The following design procedure has been adopted to estimate the contact stress for altered tooth-sum for different values of profile shift. The maximum contact stress induced was calculated using Eq. [2] (Fernandes and Mcduling; Moldovan, et al. (11)):

$$\sigma_{max} = \sqrt{\frac{0.35\ Q\left(\frac{1}{R1}+\frac{1}{R2}\right)}{b\left[\left(\frac{1-\vartheta 1}{E1}\right)+\left(\frac{1-\vartheta 2}{E2}\right)\right]}} \qquad [2]$$

The profile shift (X) for every altered tooth sum has been computed using Eq. [3] (Gitin(12)):

$$x = z_e \frac{\left[inv\varnothing_w - inv\varnothing\right]}{2\tan\varnothing} \qquad [3]$$

The points A, B, C, D, and E as shown in Fig. 1 (a) are various points along the path of contact of the meshing gear pair. The load distribution along the path of contact with contact ratio, ranging from 1.2-2 and even above 2 are shown in Fig. 1(b) & (c).

The point C is the pitch point and it maintains a constant position on the center line of gear. Whereas the points A,E and B,D, the single pair meshing segment is used to change their position along the segment T_1T_2 (i.e. path of contact). The position of points A, E and B, D is influenced by the profile shift distribution between the gear pairs and gear ratio. The points A, E and B, D occupy the position along the path of contact in such a way that the normal addendum and clearance between teeth is maintained. In this study the impact of profile shift on these points AE and the point BD have been analyzed. The following sets of Esq. (4-14) are used to estimate the radius of curvature and maximum contact stress at various points along the path of contact. These following steps are adopted for the evaluation of radius of curvature (Gitin (12)).

$$T_1T_2 = a \sin \emptyset_w \quad [4]$$

$$T_1A = T_1T_2 - \sqrt{{r_{a2}}^2 - {r_{b2}}^2} \quad [5]$$

$$T_2A = T_1T_2 - T_1A \quad [6]$$

$$T_1B = \sqrt{{r_{a1}}^2 - {r_{b1}}^2} - P_{bn} \quad [7]$$

Where $P_{bn} = \pi m \cos \emptyset$ [8]

$$T_1C = \frac{a \sin \emptyset_w}{2} \quad [9]$$

$$T_2C = T_1T_2 - T_1C \quad [10]$$

$$T_1D = T_1T_2 - \sqrt{{r_{a2}}^2 - {r_{b2}}^2} + P_{bn} \quad [11]$$

$$T_2D = T_1T_2 - T_1D \quad [12]$$

$$T_1E = \sqrt{{r_{a1}}^2 - {r_{b1}}^2} \quad [13]$$

$$T_2E = T_1T_2 - T_1E \quad [14]$$

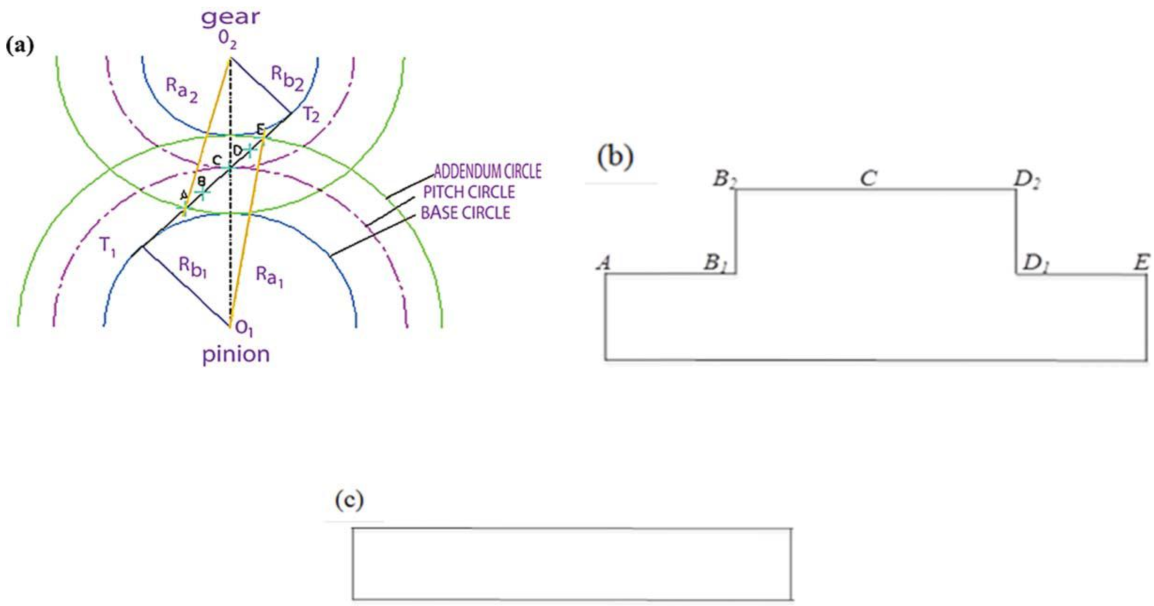

Fig. 1— (a) Path of contact (b) & (c) load distribution for different contact ratio

The contact ratio (∈) is evaluated as follows (Gitin (12)):

$$\in = \frac{T_1E - T_1A}{P_{bn}} \quad [15]$$

The addendum radii for pinion and gear are given as follows: (Gitin(20)):

$$R_{a1} = (a + m - x_2 m) - r_{12} \quad [16]$$

$$R_{a2} = (a + m - x_1 m) - r_{11} \quad [17]$$

The radius of curvature at different points along the path of contact is given by the following equation: (Fernandes and Mcduling(11)):

$$(RTR)_x = \frac{T_1X(T_1T_2 - T_1X)}{T_1T_2} \quad [18]$$

The normal force at different location along the path of contact is give by Eq. (19):

$$FNX = \frac{F_t}{\cos\left(tan^{-1}\left(\frac{T_1X}{r_{b1}}\right)\right)} \quad [19]$$

Where F_t is the tangential force (N).
The load at various points along path of contact is estimated using Eq. (20):

$$QX = \frac{FNX}{b} \quad [20]$$

The maximum contact stress at various points along path of contact is given by the Eq. (21):

$$(\sigma_{max})_X = con\sqrt{\frac{QX}{(RTR)_x}} \quad [21]$$

Where, $con = \sqrt{\frac{0.35\, E}{2}}$ [22]

Here the pinion and gear material is same.

IV. Result and discussion

Contact stress

In this study the length of path of contact T_1T_2 of for altered tooth sum of 100 (altered by ± 4) for a gear system with pressure angles 20°and 25°. The values of length of path of contact for various altered tooth sum are reported in table 1.

The point of maximum stress i.e.σ_{max} shifts along the path of contact depending on the profile shift and the position of pitch point C. The Fig.1a shows that$T_1C > T_1B$; hence, the ratio $(T_1C/T_1B)>1$ and the point of maximum stress (σ_{max}) for the pinion appears at point B. Else, if$T_1C \leq T_1B$, the ratio $T_1C/T_1B \leq 1$ and the point of maximum stress (σ_{max}) for the pinion appears at point C. Similarly, ifT_2C>T_2D, and if $T_2C \leq T_2$D, then point of maximum stress (σ_{max}) for the gear appears at points D and C, respectively (Moldovan, et al. (13)).

The corresponding contact ratio of alter tooth sum of 100($\pm$4) can be calculated by using the Eq.[15]. In this study the contact ratio is found to be in the range of 1.24 to 2.08, and it is reported in table 2.

Table 1—Length of path of contact (mm) for tooth-sum of 100 altered by $\pm$4 teeth

Altered tooth sum	96	97	98	99	100	101	102	103	104
Length of T_1T_2(Ø=20)	43.14	41.11	38.97	36.68	34.2	31.49	28.5	25.12	21.18
Length of T_1T_2(Ø=25)	49.28	47.65	45.93	44.14	42.26	40.25	38.12	35.85	33.4

The results reported in table 2 indicate that the profile shift (x_1) for 20° pressure angle lies in the range 0.26-1.13 and -0.24 to -0.89for negative and positive alteration tooth-sum, respectively. In similar way the profile shift for25°pressure angle varies in the range 0.25 to 1.08 and -0.24 to -0.89 for negative alteration and positive alteration tooth-sum, respectively. For a pressure angle of 20° with profile shift as 1.13 and altered tool sum of 96, the ratio of $\frac{T_1C}{T_1B}$ =1.11 points towards the fact that the maximum contact stress σ_{max} shift to point B. Similarly $\frac{T_2C}{T_2D}$ =1.11, indicates that the point of maximum contact stress σ_{max} appeared at point D.

Table 4 represents the values of $\sigma_{max}$for different altered tooth sum. It can be seen that a profile shift of 1.13 the σ_{max} is in single pair mesh (i.e. point B). It was found that the σ_{max} is 40.8MPa, for 20°pressure angle with x_1=1.13 and altered tooth sum of 96. In this case the length of a segment along path of contact T_1C, T_1C>T_1B(i.e. 21.57 mm >19.41 mm) and T_1C<T_1E (21.57mm<25.32 mm), the point of maximum stress lies at point C, which is between points B&D in single pair mesh.

Further it can be seen that for a profile shift of 1.13, with pressure angle of 25°, the maximum value of contact stress at point B&C are lower as compared to the maximum value of contact stress at point B&C for a pressure angle of 20°with a same profile shift. The length of path of contact for a 25° of pressure angle gear system is more as compared to the length of path of contact for a 20° of pressure angle gear system. This point towards the fact that the radius of curvature is high for 25° of pressure angle gear system, but the contact stress is lee foe this gear system.

Table 2: Comparisons of Results of Altered Tooth Sum Gearing.

No. of teeth Altered	Altered tooth sum, Ze	No of teeth on pinion, Z1	No of teeth on the Gear, Z2	Operating pressure angle Degree, $\emptyset_w$		Profile shift X		Profile shift on gears $x_1 = x_2$		Contact ratio ε	
				Ø=20	Ø=25	Ø=20	Ø=25	Ø=20	Ø=20	Ø=20	Ø=25
-4	96	48	48	25.56	29.53	2.27	2.17	1.13	1.08	1.27	1.24
-3	97	48	49	24.28	28.46	1.65	1.57	0.82	0.78	1.65	1.57
-2	98	49	49	22.94	27.35	1.06	1.04	0.53	0.52	1.4	1.34
-1	99	49	50	21.52	26.2	0.52	0.51	0.26	0.25	1.65	1.47
0	100	50	50	20	25	0	0	0	0	1.76	1.54
1	101	50	51	18.36	23.74	-0.48	-0.48	-0.24	-0.24	1.85	1.58
2	102	51	51	16.56	22.41	-0.92	-0.95	-0.46	-0.47	1.93	1.61
3	103	51	52	14.56	21.01	-1.31	-1.39	-0.65	-0.7	2	1.67
4	104	52	52	12.23	19.51	-1.65	-1.79	-0.83	-0.89	2.08	1.67

Table 3: Values of Ratio $\frac{T_1C}{T_1B}$ and $\frac{T_2C}{T_2D}$ At σ_{max} Point for $\emptyset$ =20° and $\emptyset$ = 25°.

Altered tooth sum	For $\emptyset$ = 20°			For $\emptyset$ = 25°		
	x_1	T_1C/T_1B	T_2C/T_2D	x_1	T_1C/T_1B	T_2C/T_2D
96	1.13	1.11	1.11	1.08	1.09	1.09
97	0.82	1.10	1.08	0.78	1.09	1.07
98	0.53	1.07	1.07	0.52	1.07	1.07
99	0.26	1.07	1.04	0.25	1.08	1.06
100	0	1.04	1.04	0	1.06	1.06
101	-0.24	1.04	1.01	-0.24	1.07	1.05
102	-0.46	1.01	1.01	-0.47	1.06	1.06
103	-0.65	1.01	0.98	-0.7	1.06	1.04
104	-0.83	0.97	0.97	-0.89	1.06	1.06

It is found that for Ø=20°, x_1= 0, and tooth-sum of 100, the value of maximum contact stress at pitch point C is 46.67MPa. Also the length of T_1C>T_1B (17.10 mm >16.39 mm) andT_1C<T_1D (17.10 mm <17.8 mm). This implies that the pitch point C lies between B&D and hence it lies in single pair mesh. In this case, the contact ratio is 1.76. In a similarly way it can be seen that for a gear set with Ø=25°and x_1= 0, the maximum contact stress at pitch point is 42.75MPa and the pitch point lies between B&D. However,$\sigma_{max}$for a pressure angel of Ø=25°, is less as compared to a gear system with pressure angle ofØ=20°.

Further, from table 4, it is clear that the values of contact stress at point B_1 and B_2 increases as the altered tooth sum increases for a gear system with pressure angle of $\emptyset$=20°as well as $\emptyset$=25°. The point of maximum stress is B_2 for all altered tooth sum except for altered tooth sum of 104 for which the point of maximum contact stress is B_1 in place of B_2.

We can plot the maximum contact stress along the path of contact then we found that as the tooth sum increases the maximum contact stress shifted to B_1 point from B_2 as follow in fig(2).

Table 4: The Values and Point of σ_{max} for Altered Tooth-Sum Gearing.

Altered tooth-sum	For =20°				For =25°			
	x_1	σ_{max} at $B_1(Mpa)$	σ_{max} at $B_2(Mpa)$	σ_{max} at $C\ (Mpa)$	x_1	σ_{max} at $B_1(Mpa)$	σ_{max} at $B_2(Mpa)$	σ_{max} at $C(Mpa)$
96	1.13	41.05	43.05	40.85	1.08	40.15	41.00	40.41
97	0.82	43.07	43.78	43.27	0.78	40.66	41.56	40.92
98	0.53	44.04	44.49	44.13	0.52	41.20	41.85	41.41
99	0.26	45.22	45.62	45.31	0.25	41.89	42.55	42.07
100	0.00	46.63	46.83	46.67	0	42.60	43.08	42.75
101	-0.24	48.35	48.55	48.41	-0.24	43.49	43.99	43.62
102	-0.46	50.6	50.65	50.63	-0.47	44.48	44.83	44.55
103	-0.65	53.62	53.67	53.65	-0.7	45.66	46.05	45.75
104	-0.83	58.2	58.14	58.17	-0.89	47.08	47.37	47.15

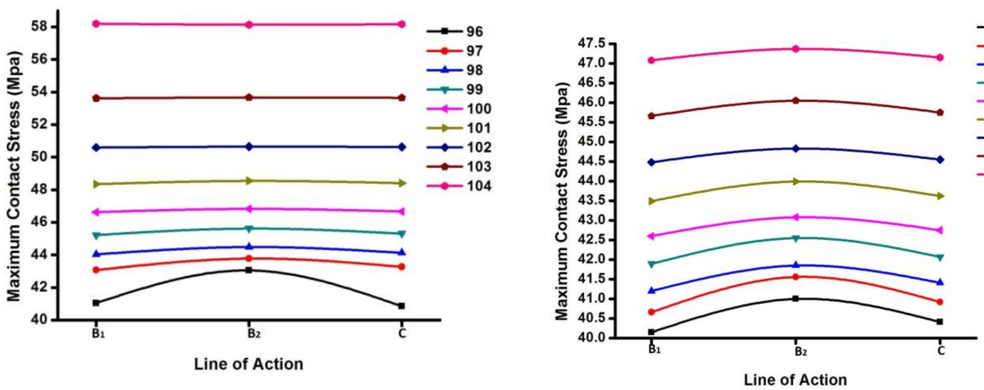

Fig 2: Contact stress on path of contact for pressure angle 20°(a) and 25°(b)

V. Conclusions

In this study the maximum Hertzian contact stress for altered tooth sum of 100 ($\pm$4) is evaluated for point B&D. The following conclusions are drawn from this analysis.

- The performance of negative altered tooth sum gear system is smoother as compared to the standard gear system. As the contact stresses induced in the negative altered tooth sum gear system less as compared to the standard gear system. Further it is observed that the contact stress in positive altered tooth sum is more as compared to a standard gear system for both 25° and 20°pressure angles.
- The contact stress point can be altered by optimizing the addendum circle diameter. Also in this profile shift design approach the designer has the flexibility in selecting the range of contact ratios.
- In future a stress analysis can be performed using FEM so as to find out the critical regions of stress and further the design can be improved to achieve higher fatigue and creep life for the gear set
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