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Low velocity impact performance of CNTs-reinforced composite plate

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Abstract:

In the present analysis, finite element method is employed to investigate the transient response of four types of carbon nanotubes (CNTs) reinforced composite plates subjected to low velocity impact. The plates are reinforced by single walled carbon nanotubes (SWCNTs) and the material properties are assumed to be graded across the thickness direction. The panel is modeled using 8 noded isoparametric plate element comprising five degrees of freedom per node. The deformation of the plate is based on First-order shear deformation theory (FSDT) and the effective mechanical properties of the carbon nanotubes-reinforced composite plate is obtained using extended rule of mixture. The Hertzian contact law, which accounts for permanent indentation is used for the computation of contact force at the impact point. The dynamic equilibrium equations of both the plate and the striker are derived from Lagrange's equation of motion and the solutions of these equations are obtained by Newmark's time integration algorithm at a suitable time step. The effectiveness and accuracy of the mathematical model is validated by comparing the converged results with the corresponding solutions available in the open literature. The optimum design of a structure can only be obtained by understanding the impact behavior and the roles of various parameters affecting the response. Hence parametric studies have been carried out to investigate the transient response of the panel under simply supported boundary conditions.

Keywords: CNTs, Impact, Hertz contact, Finite element

1. INTRODUCTION

The applications of carbon nanotubes reinforced composite materials are extensively found in many industries [1] owing to the high specific mechanical, electrical and thermal properties over the conventional fiber reinforced composite materials. Many investigators have reported the free vibration of the CNTs-reinforced plates and shells [2-6] while a few researchers have investigated the low velocity impact behavior of such panels. A summary of such works related to CNTs reinforced composites is described in the next.

Jam and Kiani [7] reported the low velocity impact response of functionally graded carbon nanotube reinforced composite beam under thermal loading based on Timoshenko beam theory wherein the parametric study shows the variation of contact force and beam displacement. Malekzadeh and Dehbozorgi [8] presented the variation of contact force, plate displacement and impactor displacement of the functionally graded carbon nanotubes reinforced composite skew plates under low velocity impact. Bayat et al.[9] carried out the nonlinear impact analysis of nanotubes reinforced functionally graded composite cylindrical shell in thermal environment based on higher order shear deformation theory with consideration of von Karman-type nonlinearity. Zarei et al. [10] reported the transient response of carbon nanotubes reinforced plates subjected to low velocity impact of multiple masses in thermal environment while Azizi et al. [11] studied the response of a thick sandwich truncated conical shell subjected to low-velocity impact with nanocomposite face sheets which are reinforced with single walled carbon nanotubes with agglomeration effects. Wang et al. [12] carried out the nonlinear dynamic response of the nanotube reinforced composite plates which includes both single layer plate and sandwich plates in thermal environment.

The literature review reveals that few investigations of the low velocity impact response of CNTs-reinforced composite plates/shells have been reported by the researchers but the impact induced stress results have not been reported by any of the authors. Hence, the present investigation is aimed at investigation the dynamic response of CNTs-reinforced composite plate subjected to low velocity impact under simply supported boundary conditions employing finite element method. An eight noded isoparametric plate element with five degrees of freedom per node have been considered to model the panel while Hertzian contact law has been used for the computation of contact force. The governing equation of motion is derived from Lagrange's equation and the solution is obtained by Newmark's time integration algorithm. The parametric study of the panel is reported with respect to simply supported boundary conditions.

2. THEORETICAL FORMULATION

The displacements at any point in the plate are defined in the FSDT mid-plane kinematics with five degrees of freedom as

$$u\ (x,y,z) = u_0\ (x,y) + z\,\theta_x\ (x,y) \tag{1}$$

$$v\,(x,y,z) = v_0\ (x,y) + z\,\theta_y\ (x,y) \tag{2}$$

$$w(x,y,z) = w_0\ (x,y) \tag{3}$$

where, u_0,v_0 and w_0 are the mid-plane displacements along *x*-,*y*- and *z*-directions, respectively while θ_x and θ_y are the rotations of transverse normal about *y*- and *x*-axis, respectively.

The generalized strains composed of mid-plane strains and curvature of the plate are defined as

$$\{\varepsilon_x \ \varepsilon_y \ \gamma_{xy} \ \gamma_{xz} \ \gamma_{yz}\}^T = \{\varepsilon_x^0 \ \varepsilon_y^0 \ \gamma_{xy}^0 \ \gamma_{xz}^0 \ \gamma_{yz}^0\}^T + z\,\{\kappa_x \ \kappa_y \ \kappa_{xy} \ 0 \ \ 0\}^T \tag{4}$$

where $\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0, \gamma_{xz}^0$ and γ_{yz}^0 are mid-surface strains, $\kappa_x, \kappa_y, \kappa_{xy}$ are the curvatures of the laminated plate.

The constitutive relation under plane stress of the plate can be written as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & 0 & 0 & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & 0 & 0 & 0 \\ 0 & 0 & \bar{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & 0 \\ 0 & 0 & 0 & 0 & \bar{Q}_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \tag{5}$$

where, $\bar{Q}_{11} = \dfrac{E_{11}}{1-\nu_{12}\nu_{21}}, \bar{Q}_{12} = \dfrac{\nu_{12}E_{22}}{1-\nu_{12}\nu_{21}}, \bar{Q}_{22} = \dfrac{E_{22}}{1-\nu_{12}\nu_{21}}, \bar{Q}_{66} = G_{12}, \bar{Q}_{44} = G_{13}$ and $\bar{Q}_{55} = G_{23}$.

The effective longitudinal and transverse Young's modulus, and also, shear modulus of the plate is estimated numerically using the extended rule of mixture as [10]

$$E_{11} = \eta_1 V_{CNT}(z) E_{11}^{CNT} + V_m(z) E^m \tag{6}$$

$$\frac{\eta_2}{E_{22}} = \frac{V_{CNT}(z)}{E_{22}^{CNT}} + \frac{V_m(z)}{E^m} \tag{7}$$

$$\frac{\eta_3}{G_{12}} = \frac{V_{CNT}(z)}{G_{12}^{CNT}} + \frac{V_m(z)}{G^m} \tag{8}$$

where, V_{CNT} and V_m are the volume fraction of CNTs and matrix, which are graded along the thickness direction in such way that the total volume of composite in each layer is the sum of individual contribution of the CNTs and matrix and expressed as

$$V_{CNT}(z) + V_m(z) = 1 \tag{9}$$

$E_{11}^{CNT}, E_{22}^{CNT}, G_{12}^{CNT}$ and $\eta_i (i = 1, 2, 3)$ are the longitudinal modulus, transverse modulus, shear modulus of elasticity and the efficiency parameters of the CNTs, respectively and are computed by using molecular dynamics simulation. Similarly E^m and G^m are the Young's modulus and shear modulus of the polymer matrix, respectively.

The effective mass density and Poisson's ratio of the CNT-reinforced plate are expressed as

$$\rho = V_{CNT}(z) \rho^{CNT} + V_m(z) \rho^m \tag{10}$$

$$\nu_{12} = V_{CNT}(z)\, \nu_{12}^{CNT} + V_m(z)\, \nu^m \tag{11}$$

where, ρ^{CNT} and ρ^{m} are the mass density of CNTs and polymer matrix, respectively while ν_{12}^{CNT} and ν^{m} are the corresponding Poisson's ratio of CNTs and polymer matrix.

In the present study, four types of linear CNT grading are considered along the thickness direction and illustrated in Figure 1. For UD type, CNTs are uniformly distributed while for FG-X type, CNTs are symmetrically distributed above and below the mid-plane wherein both bottom and top surfaces are CNT rich. In FG-O type, the mid-plane is CNT rich, and volume fraction of CNT reduces towards top and bottom, while in FG-V type, the top surface of the plate is CNT rich, and gradually decreases towards bottom.

The effective volume fraction of CNT in the four types of grading are expressed as:

$$V_{CNT}(z) = V_{CNT}^{*} \qquad \text{UD} \tag{12}$$

$$V_{CNT}(z) = 2\left(\frac{2|z|}{h}\right)V_{CNT}^{*} \qquad \text{FG-X} \tag{13}$$

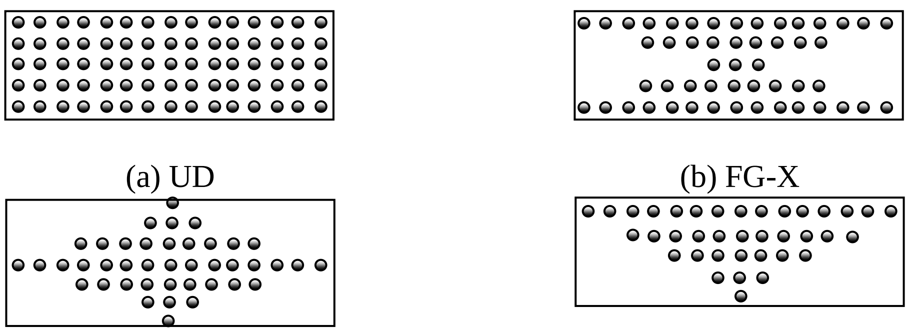
$$V_{CNT}(z) = 2\left(1-\frac{2|z|}{h}\right)V_{CNT}^{*} \qquad \text{FG-O} \tag{14}$$

$$V_{CNT}(z) = \left(1+\frac{2z}{h}\right)V_{CNT}^{*} \qquad \text{FG-V} \tag{15}$$

where, V_{CNT}^{*} is the total volume fraction of CNT and is expressed as

$$V_{CNT}^{*} = \frac{w_{CNT}}{w_{CNT} + \left(\frac{\rho^{CNT}}{\rho^{m}}\right) - \left(\frac{\rho^{CNT}}{\rho^{m}}\right)w_{CNT}} \tag{16}$$

where, w_{CNT} is the mass fraction of CNT, ρ^{CNT} is the density of CNT and ρ^{m} is the density of the matrix.



(a) UD (b) FG-X (c) FG-O (d) FG-V

Fig. 1. Different types of CNT grading in plates.

An eight noded isoparametric quadratic plate element with five degrees of freedom (u, v, w, θ_x and θ_y) per node is employed for modeling the plate. The stiffness and mass matrices of the plate element are determined using standard procedure of finite element method [13] and given by

$$[K_e]=\int_{-1}^{1}\int_{-1}^{1}[B]^T[D][B]|J|d\xi d\eta \tag{17}$$

$$[M_e]=\int_{-1}^{1}\int_{-1}^{1}[N]^T[m][N]|J|d\xi d\eta \tag{18}$$

where, [*B*], [*D*], [*N*] and [*m*] are the strain displacement matrix, elasticity matrix, shape function matrix and general inertia matrix per unit area, respectively.

In case of low velocity impact, it is assumed that the vibration of the hard impactor can be neglected. The contact forces based on Hertzian contact law during loading and unloading cycle are determined as [12]

$$F_C = k\alpha^{1.5} \qquad 0<\alpha\le\alpha_m \qquad \text{loading,} \tag{19}$$

$$F_C = F_m\left[\frac{\alpha-\alpha_0}{\alpha_m-\alpha_0}\right]^{2.5} \qquad \text{unloading} \tag{20}$$

where k is the contact stiffness, α is the local indentation, F_m is the maximum contact force, α_m is the maximum indentation and α_0 is the permanent indentation in a loading/unloading cycle. The contact stiffness k of the plate is defined by

$$k=\frac{4}{3}\frac{\sqrt{r_i}}{\left[\dfrac{(1-\nu_i^2)}{E_i}+\dfrac{1}{E_2}\right]} \tag{22}$$

where r_i, ν_i and E_i are the radius, Poisson's ratio and modulus of elasticity of the spherical impactor. E_2 is the modulus of elasticity of the plate transverse to fiber direction.

The local indentation at the contact point is determined as [12]

$$\alpha(t)=w_i-w_s \tag{23}$$

where w_s and w_i are the displacements of the mid-plane of the plate and the spherical impactor at the impact point in the direction of impact.

The contact force vector obtained during impact between the plate and impactor is given by

$$\{F\}=\{0 \quad 0 \quad 0 \quad \cdots \quad F_{Ci} \cdots \quad 0 \quad 0 \quad 0\}^T \tag{24}$$

The governing dynamic equilibrium equation derived using Lagrange's equation of motion in global form of the plate at time ($t+\Delta t$), neglecting effects of damping is expressed as

$$[M]\{\ddot{\delta}\}^{t+\Delta t}+[K]\{\delta\}^{t+\Delta t}=\{F\}^{t+\Delta t} \tag{25}$$

The equation of motion of the rigid impactor is given by

$$m_i\ddot{w}_i+F_C=0 \tag{26}$$

The solution of the Eqs. (25) and (26) are obtained by Newmark's time integration algorithm[14] by selecting a suitable time step.

3. RESULTS AND DISCUSSION

The accuracy of the present formulation is validated by comparing its converged results with those available in the literature. For this purpose the example considered by Wang et al. [12] is solved and illustrated in Figure 2. The results are obtained with a conversed mesh size of 8x8 and time step of 1 μs. The results of contact force history of the three types of CNTs-reinforced composite plate's shows an excellent agreement with the published results.

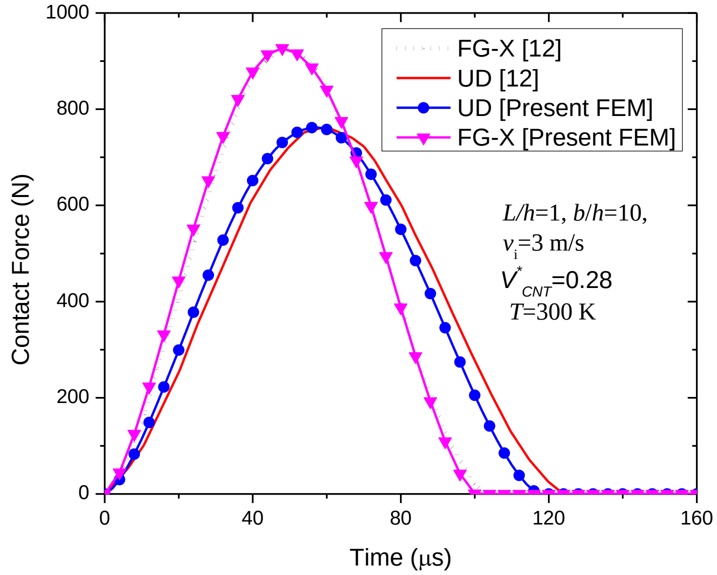


Fig.2. Comparisons of impact response of two types of CNTs-reinforced square plate with Wang et al [12].

The geometrical properties of the plate considered for the entire analysis are L/b=1, and b/h=10. The material properties of the steel impactor are E_i=207 GPa, ν_i=0.3 and ρ_i=7960 kg/m^3. The spherical impactor of radius r_i= 6.35 mm and having initial velocity of 5 m/s is assumed to be impacted at the center of the plate. Poly (methyl methacrylate), referred as (PMMA) is considered as matrix and its material properties are E^m =2.5 GPa, ρ^m =1150 kg/m^3, ν^m =0.34. The SWCNTs of armchair (10, 10) pattern are used as reinforcement phase and its material properties at room temperature are furnished in Table 1 while the volume fraction of CNT (V^*_{CNT}) is taken as 0.12. For V^*_{CNT}=0.12, the efficiency parameters are η_1=0.137, η_2=1.022 and η_3=0.715, respectively. Simply supported boundary conditions are adopted for the analysis.

Table 1: Material properties of (10,10) SWCNT at room temperature. (L = 9.26 nm, R = 0.68 nm, h = 0.067 nm, ν_{12}^{CNT}=0.175, ρ^{CNT} = 1400 kg/m^3)

Temp (K)	E_{11}^{CNT} (TPa)	E_{22}^{CNT} (TPa)	G_{12}^{CNT} (TPa)
300	5.6466	7.0800	1.9445

The transient response of the four types of CNTs-reinforced composite plates is illustrated in Figure 3. It is evident that contact force is obtained maximum in FG-X type graded plate while it is noticed minimum in both FG-O and FG-V type. It may be noticed from the contact force history that the period of loading and unloading also found minimum for FG-X type graded plate. From the history of central deflection it reveals that minimum deflection is observed in FG-X type graded plate and maximum value of central deflection is obtained in FG-O type plate. Both the in plane stresses at top surface of the target point is found to be compressive in nature in the four types of plates. The maximum compressive stress (σ_x) is observed in FG-V type plate while minimum value is found in FG-O type plate. Similar variation of σ_y is also observed for the plates. But comparing both the stresses it may be noticed that the value of σ_x is found much more higher than that of σ_y.

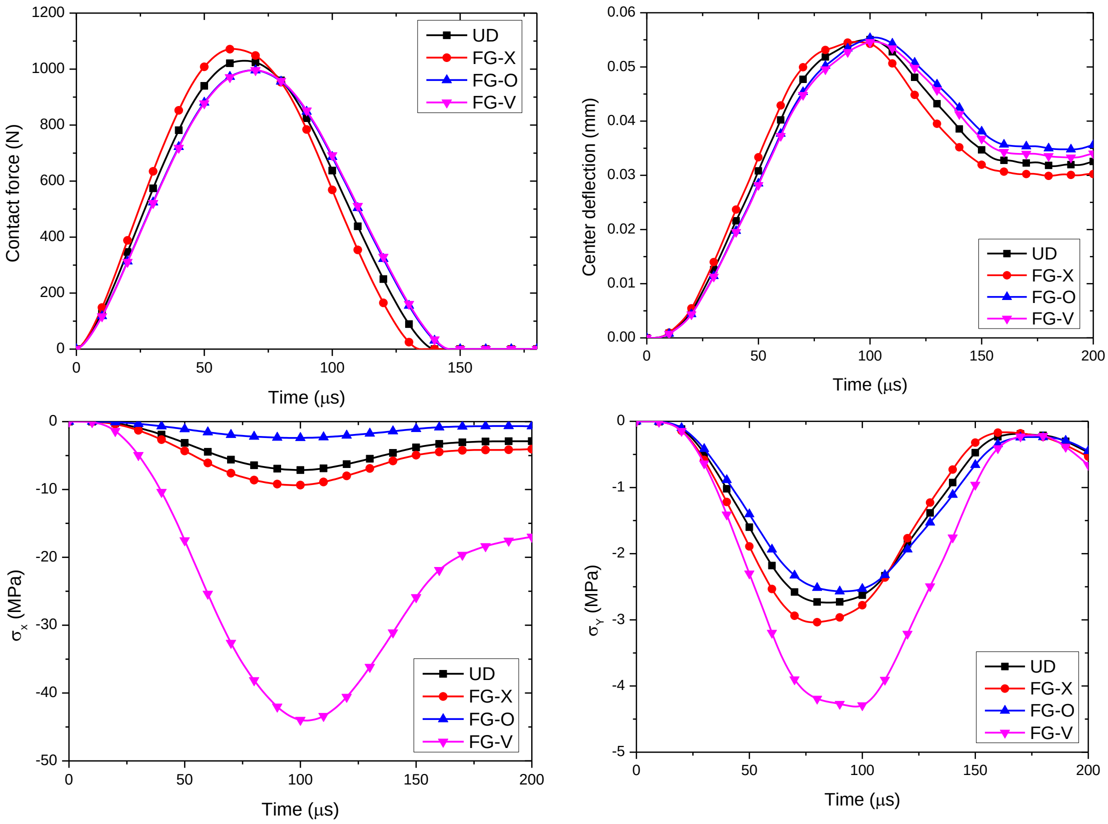


Fig.3. Histories of contact force, Center displacement, in-plane normal stresses at the target point.

4. CONCLUSION

The finite element formulation and validation of the four types of CNTs-reinforced composite plates is presented. From the parametric study it is concluded from the contact force and plate displacement histories that FG-X type graded plate have maximum flexural stiffness than others. Impact induces in-plane stresses are found maximum in FG-V type plates while the stress values are observed minimum in FG-O type plates.

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