

Total Dominating Color Transversal number of Graphs and Monotonicity

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Abstract

Total Dominating Color Transversal Set of a graph is a Total Dominating Set which is also Transversal of Some χ -Partition of vertices of G. Here χ is the Chromatic number of the graph G. Total Dominating Color Transversal number of a graph is the cardinality of a Total Dominating Color Transversal Set which has minimum cardinality among all such sets that the graph admits. In this paper, we determine a sufficient condition under which this number becomes equal to Total Domination number, if a graph G has k components. We also obtain an upper bound of this number and the sufficient condition under which this number for spanning super graphs increases monotonically.

Key words: Total Dominating Color Transversal number, χ – Partition of a graph, Diameter, Radius.

AMS subject Classification: 05C15, 05C69

1. Introduction

We begin with simple, finite, connected and undirected graph without isolated vertices (unless otherwise stated). In [1], we defined Total Dominating Color Transversal number of a Graph. We know that proper coloring of vertices of graph G partitions the vertex set V of G into equivalence classes (also called the color classes of G). Using minimum number of colors to properly color all the vertices of G yields χ equivalence classes. Transversal of a χ -Partition of G is a collection of vertices of G that meets all the color classes of the χ – Partition. That is, if T is a subset of V (the vertex set of G) and $\{V_1, V_2, \dots, V_\chi\}$ is a χ -Partition of G then T is called a Transversal of this χ -Partition if $T \cap V_i \neq \emptyset, \forall i \in \{1, 2, \dots, \chi\}$. Total Dominating Color

Transversal Set of graph G is a Total Dominating Set with the extra property that it is also Transversal of some such χ -Partition of G .

In [1], We proved that if a graph is bipartite then Total Dominating Color Transversal number and Total Domination number of the graph are equal. In this paper, we extend our result to k components of a graph G and determine a condition under which this number and Total Domination number of G are equal. We also obtain an upper bound of this number.

We know that for a graph G , $\chi(G) \leq \chi(G^2) \leq \chi(G^3) \leq \chi(G^4) \leq \dots \leq \chi(G^d) = \chi(K_n) = n$ and $\gamma_t(G) \geq \gamma_t(G^2) \geq \gamma_t(G^3) \geq \gamma_t(G^4) \geq \dots \geq \gamma_t(G^d) = \gamma_t(K_n) = 2$, where d and n are, respectively, the diameter and the order of the graph G , K_n is a complete graph with n vertices and G^k is a spanning super graph of G with vertices u and v are adjacent if $d(u, v) \leq k$. This implies that Chromatic number and Total Domination number of a graph, respectively, Monotonically increases and Monotonically decreases, for spanning super graphs G^k .

We show that nothing as such is happening in case of Total Dominating Color Transversal number of a graph. In fact, we determine a condition under which after definite number of powers of G this number Monotonically Increases.

2. Definitions

Definition 2.1[4]: (Total Dominating Set) Let $G = (V, E)$ be a graph. Then a subset S of V (the vertex set of G) is said to be a Total Dominating Set of G if for each $v \in V$, v is adjacent to some vertex in S .

Definition 2.2[4]: (Minimum Total Dominating Set/Total Domination number) Let $G = (V, E)$ be a graph. Then a Total Dominating set S is said to be a Minimum Total Dominating set of G if $|S| = \text{minimum } \{|D| : D \text{ is a Total Dominating set of } G\}$. Here S is called γ_t -set and its cardinality, denoted by $\gamma_t(G)$ or just by γ_t , is called the Total Domination number of G .

Definition 2.3[1]: (χ -partition of a graph) Proper coloring of vertices of a graph G , by using minimum number of colors, yields minimum number of independent subsets of vertex set of G called equivalence classes (also called color classes of G). Such a partition of a vertex set of G is called a χ -Partition of the graph G .

Definition 2.4[1]: (Transversal of a χ -Partition of a graph) Let $G = (V, E)$ be a graph with χ -Partition $\{V_1, V_2, \dots, V_\chi\}$. Then a set $S \subset V$ is called a Transversal of this χ -Partition if $S \cap V_i \neq \emptyset, \forall i \in \{1, 2, 3, \dots, \chi\}$.

Definition 2.5[1]: (Total Dominating Color Transversal Set) Let $G = (V, E)$ be a graph. Then a Total Dominating Set $S \subset V$ is called a Total Dominating Color Transversal Set of G if it is Transversal of at least one χ -partition of G .

Definition 2.6[1]: (Minimum Total Dominating Color Transversal Set/Minimum Total Dominating Color Transversal number) Let $G = (V, E)$ be a graph. Then $S \subset V$ is called a Minimum Total Dominating Color Transversal Set of G if $|S| = \text{minimum } \{|D| : D \text{ is a Total Dominating Color Transversal Set of } G\}$. Here S is called γ_{tstd} –Set and its cardinality, denoted by $\gamma_{\text{tstd}}(G)$ or just by γ_{tstd} , is called the Total Dominating Color Transversal number of G .

Definition 2.7: (Eccentricity of a vertex) Let $G = (V, E)$ be a graph. Then eccentricity of a vertex v of G is denoted and defined as $e(v) = \max \{d(u, v) / u \in V\}$, where $d(u, v) =$ the distance between the vertices u and v in G .

Definition 2.8: (Diameter of a graph) Let $G = (V, E)$ be a graph. Then diameter of G is denoted and defined as $\text{diam}(G) = \max\{e(v) / v \in V\}$.

Definition 2.9: (Radius of a graph) Let $G = (V, E)$ be a graph. Then radius of G is denoted and defined as $\text{rad}(G) = \min \{e(v) / v \in V\}$.

3. Main results

Theorem 3.1: Let $G = (V, E)$ be a graph with k – components say G_1, G_2, \dots, G_k and each component is without isolated vertex. If $\chi(G) \leq 2k$ then $\gamma_{\text{tstd}}(G) = \gamma_t(G)$.

Proof: Let $\chi(G) = 2k$. Let $\gamma_t(G_i)$ denote the Total Domination number of G_i ($i = 1, 2, 3, \dots, k$).

Each γ_t – Set has at least two adjacent vertices. So assign two distinct colors to two adjacent vertices in γ_t – Set of component G_1 . Again, assign two arbitrary colors, not used early, to two different vertices in γ_t – Set of component G_2 . Likewise continue assigning two arbitrary distinct colors, not used early, to two distinct vertices of γ_t – Set of components of G till we reach upto to γ_t – Set of component $G_{\frac{k}{2}} (= G_k)$ of G . Hence $\cup_{i=1}^k \gamma_t(G_i) = \gamma_t(G)$ is a transversal of such χ – Partition of G .

Therefore $\gamma_{\text{tstd}}(G) = \gamma_t(G)$.

For $\chi(G) < 2k$, the result is obvious by applying the above method of coloring the γ_t – Set of components of G . ■

Result 3.2: Let $G = (V, E)$ be a graph with k – components say G_1, G_2, \dots, G_k . If $\chi(G) = k$ then there exists some G_i such that $\chi(G_i) = k$. Hence $\chi(G_i) \geq \chi(G_j), \forall j \in \{1, 2, \dots, k\}$.

Theorem 3.3: Let $G = (V, E)$ be a graph with k – components, say G_1, G_2, \dots, G_k , such that $\chi(G_1) \geq \chi(G_i), \forall i \in \{2, \dots, k\}$. Then $\gamma_{\text{tstd}}(G) \leq \gamma_{\text{tstd}}(G_1) + \sum_{i=2}^k \gamma_t(G_i)$.

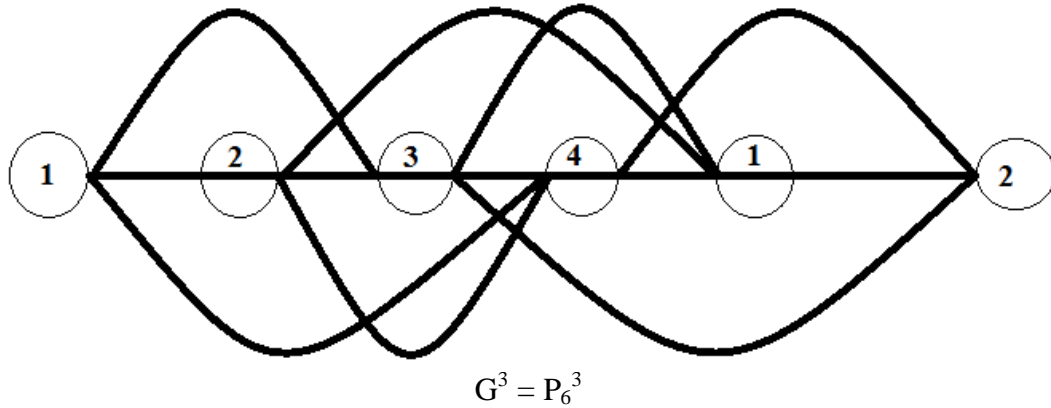


Figure 3

Here $\gamma_t(P_6^3) = 2$. So by theorem 3.5, $\gamma_{tstd}(P_6^3) = \chi(P_6^3) = 4$.
 Hence we have $\gamma_{tstd}(P_6) = 4 \geq 3 = \gamma_{tstd}(P_6^2)$ but $\gamma_{tstd}(P_6^3) = 4 \leq 3 = \gamma_{tstd}(P_6^2)$.
 Now we obtain a condition under which the Total Dominating Color Transversal number Monotonically increases for spanning super graphs after definite number of powers of G.

Theorem 3.7: Let $G = (V, E)$ be a graph with n vertices. If k and d are, respectively, radius and diameter of the graph G then $\gamma_{tstd}(G^k) \leq \gamma_{tstd}(G^{k+1}) \leq \gamma_{tstd}(G^{k+2}) \leq \dots \leq \gamma_{tstd}(G^d) = \gamma_{tstd}(K_n) = n$.

Proof: We are given that k is the radius of the graph G . So there exists a vertex v of G such that eccentricity of v is k . So no vertex of G is farther than distance k from v . That is $d(u, v) \leq k$, for every vertex u in G . Note that in G^k all vertices, whose distance is less than or equal to k , are adjacent. This implies that v dominates all the vertices of G^k . Hence $\gamma(G^k) = 1$ and so $\gamma_t(G^k) = 2$. Therefore by theorem 3.5, $\gamma_{tstd}(G^k) = \chi(G^k)$. Here also note that $\gamma_t(G^k) = \gamma_t(G^{k+1}) = \gamma_t(G^{k+2}) = \dots = \gamma_t(G^d) = \gamma_t(K_n) = 2$. So $\gamma_{tstd}(G^{k+i}) = \chi(G^{k+i})$ for every $i \in \{0, 1, 2, \dots, d - k\}$.
 As $\chi(G) \leq \chi(G^2) \leq \chi(G^3) \leq \chi(G^4) \leq \dots \leq \chi(G^d) = \chi(K_n) = n$, we have $\gamma_{tstd}(G^k) \leq \gamma_{tstd}(G^{k+1}) \leq \gamma_{tstd}(G^{k+2}) \leq \dots \leq \gamma_{tstd}(G^d) = \gamma_{tstd}(K_n) = n$.



Hence the theorem

4. Concluding Remarks

In this paper, we tried to relate Total Dominating Color Transversal number of disconnected graph to Total Dominating number of the graph. In this effort, we obtain an upper bound of Total Dominating Color Transversal number of a graph with k components under some restriction to chromatic number. Property of non Monotonicity of Total Dominating Color Transversal number noticeably distinguish it

from Chromatic number and Total Domination number. In this paper, it was our effort to reflect this property of dissimilarity.

5. References

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