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Homotopy Analysis Method to Flow and Heat Transfer in Visco-Elastic Fluid Flow in a Porous Medium over Exponentially Stretching Sheet

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Abstract

This paper presents the analysis of visco-elastic fluid flow and heat transfer in porous medium over exponentially stretching sheet. In this process the governing boundary layer equations are transferred into non-linear ordinary differential equations by using a suitable similarity transformation and then the problem is solved using an analytical technique known as Homotopy analysis method (HAM), which provides in the best possible way to control and adjust the convergence region, using a non-zero auxiliary parameter $\hbar$. The accuracy of the results is shown by making comparison of the results with the results already exist in the literature and are seen in good agreement.

Keywords: visco-elastic fluid, variable porpsity, similarity transformation, exponentially stretching sheet, HAM

INTRODUCTION

In the modern age polymer industry has its significant stand, the Flow of an incompressible viscous fluid over a stretching sheet has important applications in polymer industry. In the line of production of fiber sheet/plate sheet, extrusion of molten polymers through a slit die is an important process in polymer industry. The pioneer work of Sakiadis [1,2] analysed the boundary layer flow over continuous moving sheet.This work is extended to various aspects of momentum and heat transfer characteristics in a visco elastic boundary layer fluid flow over a stretching sheet [3] –[5].However, it is often argued that [6] realistically stretching of plastic sheet may not necessarily be linear. In view of this arument, Kumaran and Ramanaiah [7] considered the viscous boundary layer fluid flow over quadratic stretching sheet.

In contrast to the above experiment, most of the fluid flow applications in polymer processing industries are concerned with Non- Newtonian fluids. Then Ali[8] investigated the thermal boundary layer flow on a power law stretching surface with suction or blowing. Later Elbashbeshy [9] analyzed the problem of heat transfer over an exponentially stretching sheet with suction. Next Sanjayanand and Khan [10, 11] extended the work of Elbashbeshy [9] to viscoelastic fluid flow, heat and mass transfer over an exponentially stretching sheet. And provide that the transport of heat in porous medium has considerable practical applications in geothermal systems, crude oil extraction, and ground water pollution and also in a wide range of bio-mechanical problems. The flow of a steady viscous fluid and heat transfer characterestics in a porous medium by considering different heating processes was studied by Vajravelu [12]. Subhas and Veena [13] found the heat transfer of visco-elastic fluid flow in a porous medium over stretching sheet. Eldabe and Mohamed [14] studied the mass and heat transfer in mhd flow of non – Newtonian fluid with heat source over an accelerated surface through porous medium. K.V.Prasad and M.Subhas[17] Abel considered the momentum and heat transfer in visco-elastic fluid flow in porous medium over an non iso-thermal stretching sheet. Recently, Sajid and Hayet[18] discussed the influence of thermal radiation on the boundary layer flow due to an exponentially stretching sheet.

The present work envisages the study of visco-elastic fluid flow over exponentially stretching sheet in porous medium. This problem was taken from K.V.Prasad et al [17] by added the exponential stretching sheet.

MATHEMATICAL FORMULATION:

We consider the steady two – dimensional incompressible Non- Newtonian visco-elastic fluid in porous medium. The flow is confined to y > 0. Now Two equal and opposite forces are applied along the x-axis, so that the wall is stretched keeping the origin fixed. The basic boundary layer equations for the given flow are :

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-k_0\left\{u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right\}-\frac{\nu}{k^*}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho c_p}\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) \tag{3}$$

Where *u* and *v* are the velocity components of *x* and *y* directions respectively, ν is the kinematic viscosity, k^* is the porous parameter, k_0 is the visco-elastic parameter, ρ is the density, c_p is the specific heat at constant pressure and *k* is the thermal conductivity which is assumed to be variable.

$k=k_\infty\left(1+\varepsilon\theta\right)$.Where ε is the small parameter depends on nature of the fluid.

The basic boundary conditions are:

$$\begin{aligned} &u=U_w(x)=U_0\,e^{\frac{x}{l}},\ v=0, T=T_w=T_\infty+T_0e^{\frac{x}{2l}} && at\ y=0 \\ &u=0\ ,\ u_y=0,\ T=T_\infty && as\ y\to\infty \end{aligned} \tag{4}$$

Here U_0 is the constant, l is the reference length, suffix y denotes the differentiation with respect to y, T_0 is the parameter of the temperature distribution and T_∞ is the temperature far away from the stretching sheet.

The continuity equation is satisfied by the stream function ψ(x, y) can be written as :

$$u=\frac{\partial\psi}{\partial y},\qquad v=-\frac{\partial\psi}{\partial x} \tag{5}$$

Now, Introducing the similarity transformations

$$\eta=y\sqrt{\frac{U_0}{2\nu l}}e^{x/2l} \tag{6}$$

$$\psi(x,y)=\sqrt{2\nu l U_0}\,f(x,\eta)e^{x/2l},\quad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \tag{7}$$

Here f is the dimensionless stream function, $\theta(\eta)$ being the dimensionless temperature, η being the similarity variable and considering $f(x,\eta)=f(\eta)$. Using (5)-(7), the equations (2)-(3) results in a fourth order non- linear ordinary differential equation of the form;

$$2f'^2-ff''-f'''+k_1\left[3f'f'''-\frac{1}{2}ff^{iv}-\frac{3}{2}f''^2\right]+k_2f'=0 \tag{8}$$

$$(1+\varepsilon\theta)\theta''+\varepsilon\theta'^2+\Pr(f\theta'-f'\theta)=0 \tag{9}$$

Where $k_1=\frac{k_0U_w}{\nu l}$ is the dimensionless visco-elastic parameter, $k_2=\frac{2\nu l}{k^*U_w}$ is the porosity parameter $\Pr=\frac{\mu c_p}{k_\infty}$ is the prandtl number.

Transformed boundary conditions in non – dimensional form are as follows:

$$\begin{aligned} &f=0, \quad f'=1, \quad \theta=1 \quad && at\ \eta=0\\ &f'=0, \quad \theta=\infty && as\ \eta\to\infty \end{aligned} \tag{10}$$

ANALYTICAL SOLUTION (HAM)

In this section, we employ HAM to solve the equations (8) – (9) subject to the boundary conditions (10). We choose the initial guesses f_0, θ_0 of f, θ in the following form :

$$f_0(\eta)=1-e^{-\eta} \tag{11}$$

$$\theta_0(\eta)=e^{-\eta} \tag{12}$$

The linear operators are selected as

$$L_1(f)=f'''-f' \tag{13}$$

$$L_2(\theta)=\theta''-\theta' \tag{14}$$

Which have the following properties

$$L_1\left(C_1+C_2e^{-\eta}+C_3e^{\eta}\right)=0 \tag{15}$$

$$L_2\left(C_4e^{-\eta}+C_5\,e^{\eta}\right)=0 \tag{16}$$

Where *C* *i* *(i = 1 to 5)* are arbitrary constants.

If $q\in[0,1]$ is the embedding parameter, $\hbar_1$ *and* $\hbar_2$ are the non – zero auxiliary parameter and $H_1(\eta), H_2(\eta)$ are the auxiliary functions, then we can construct the zeroth – order deformation equations

$$(1-q)L_1\big(f(\eta,q)-f_0(\eta)\big)=q\hbar_1\,H_1(\eta)N_1\big(f(\eta,q)\big) \tag{17}$$

$$(1-q)L_2\big(\theta(\eta,q)-\theta_0(\eta)\big)=q\hbar_2\,H_2(\eta)N_2\big(f(\eta,q),\theta(\eta,q)\big) \tag{18}$$

Subject to the boundary conditions

$$\begin{aligned}&f(0,q)=1, \quad f'(0,q)=1, \quad f'(\infty,q)=0\\&\theta(0,q)=1, \quad \theta'(\infty,q)=0\end{aligned} \tag{19}$$

Where

$$N_1\big(f(\eta,q)\big)=\frac{\partial^3 f}{\partial\eta^3}-2\left(\frac{\partial f}{\partial\eta}\right)^2+\frac{\partial^2 f}{\partial\eta^2}f-k_1\left(3\frac{\partial f}{\partial\eta}\frac{\partial^3 f}{\partial\eta^3}-\frac{1}{2}f\frac{\partial^4 f}{\partial\eta^4}-\frac{3}{2}\left(\frac{\partial^2 f}{\partial\eta^2}\right)^2\right)-k_2\frac{\partial f}{\partial\eta} \tag{20}$$

$$N_2\big(f(\eta,q),\theta(\eta,q)\big)=(1+\varepsilon\theta)\frac{\partial^2\theta}{\partial\eta^2}+\varepsilon\left(\frac{\partial\theta}{\partial\eta}\right)^2+\Pr\left(f\frac{\partial\theta}{\partial\eta}-\frac{\partial f}{\partial\eta}\theta\right) \tag{21}$$

For *q = 0* and *q = 1*, we have

$$\begin{aligned}&f(\eta,0)=f_0(\eta), \quad f(\eta,1)=f(\eta)\\&\theta(\eta,0)=\theta_0(\eta) \quad \theta(\eta,1)=\theta(\eta)\end{aligned} \tag{22}$$

Thus, as q increases from 0 to 1, $f(\eta,q)$ varies from $f(\eta,0)$ to $f(\eta)$ and $\theta(\eta,q)$ varies from
$\theta(\eta,0)$ to $\theta(\eta)$. Then expanding use of Taylor's theorem with respect to *q*, we obtain

$$f(\eta;q) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta) q^m \tag{23}$$

$$\theta(\eta;q) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta) q^m \tag{24}$$

Where

$$f_m(\eta) = \frac{1}{m!} \frac{\partial^m f(\eta;q)}{\partial q^m} \Big|_{q=0} \tag{25}$$

$$\theta_m(\eta) = \frac{1}{m!} \frac{\partial^m \hat{\theta}(\eta;q)}{\partial q^m} \Big|_{q=0} \tag{26}$$

The auxiliary parameters are properly chosen so that series (24) and (25) converges at *q =1*
and thus

$$f(\eta) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta) \tag{27}$$

$$\theta(\eta) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta) \tag{28}$$

The resulting problems at the mth-order deformation are

$$L_1[f_m(\eta) - \chi_m f_{m-1}(\eta)] = h_f R_m^f(\eta), \tag{29}$$

$$L_2[\theta_m(\eta) - \chi_m \theta_{m-1}(\eta)] = h_\theta R_m^\theta(\eta) \tag{30}$$

subject to the boundary conditions

$$f_m(0) = 0,\ f'_m(0) = 0,\ f'_m(\infty) = 0, \theta_m(0) = 0, \theta_m(\infty) = 0 \tag{31}$$

$$R_m^f = f'''_{m-1} + \sum_{k=0}^{m-1} f_k f''_{m-1-k} - 2\sum_{k=0}^{m-1} f_k' f'_{m-1-k} - k_1\left(3\sum_{k=0}^{m-1} f'_{m-1-k}\ f_k''' - \frac{1}{2}\sum_{k=0}^{m-1} f_{m-1-k} f_k'''' - \frac{3}{2}\sum_{k=0}^{m-1} f''_{m-1-k}\ f_k''\right) - k_2 f'_{m-1}. \tag{32}$$

$$R_m^\theta = (1 + \varepsilon\theta_{m-1})\theta''_{m-1} + \Pr(\sum_{k=0}^{m-1} f_k \theta'_{m-1-k} - \theta_k f'_{m-1-k}). + \varepsilon\theta'_{m-1-k}\theta'_k \tag{33}$$

and

$$\chi_m = \begin{cases} 0, m \leq 1, \\ 1, m > 1 \end{cases} \tag{34}$$

We choose the auxiliary function as follows :

$$H_1(\eta)=1 \ , H_2(\eta)=1. \tag{35}$$

If we let $f_m^*(\eta), \theta_m^*(\eta)$ as the special solutions of the mth – order deformation equations, the general solutions are given by

$$f_m(\eta) = f_m^*(\eta) + C_1 + C_2 e^{-\eta} + C_3 e^{\eta} \tag{36}$$

$$\theta_m(\eta) = \theta_m^*(\eta) + C_4 e^{-\eta} + C_5 e^{\eta} \tag{37}$$

Where the integral constants $C_i(i=1 to 7)$ are determined using the boundary conditions (31).

Now it is to solve the linear non-homogenous equations (29) and (30) using MATHEMATICA software one after the other by considering m = 1,2,..............

CONVERGENCE OF HAM SOLUTION

Liao [16] showed that for an analytic solution obtained by HAM, its convergence and rate approximation strongly depend upon the auxiliary parameters $\hbar_1$ and $\hbar_2$.If these parameters are chosen properly, then the solution is effective. Hence, $\hbar$-curves are plotted at 25^{th} order approximation in order to obtain the suitable ranges for $-1.1 \leq \hbar_1 \leq -0.1$, $-1.1 \leq \hbar_2 \leq -0.1$, with convergence of h value is $h = -0.8$.

RESULTS AND DISCUSSION

A systematic study is performed to analyze the impacts of various parameters, visco-elastic parameter *(k_1)* and porosity parameter *(k_2)* on velocity and temperature. Results are compared for Prandtl number, which is applicable to the case of polymer solution. In **fig 1 & 2,** they show that the convergence of h_1 and h_2 curves for the skin friction and temperature.

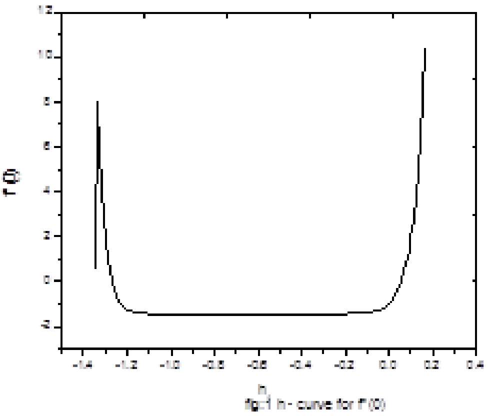


fig:1 h- curve for f''(0)

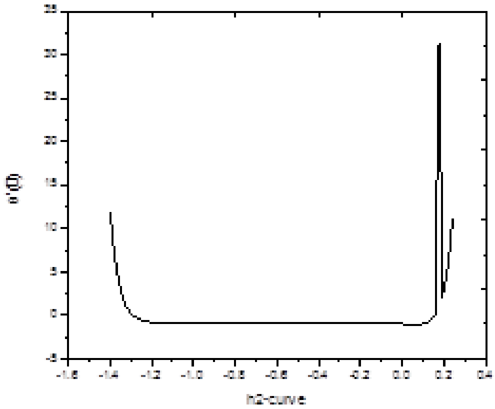


fig:2 h_2 curve for θ

Fig 3 & 4, the effect of visco-elastic parameter on temperature and velocity, it is observed that the dimensionless velocity decreases when the visco- elastic parameter increases. And the dimensionless temperature increase with the increasing in visco-elastic parameter.

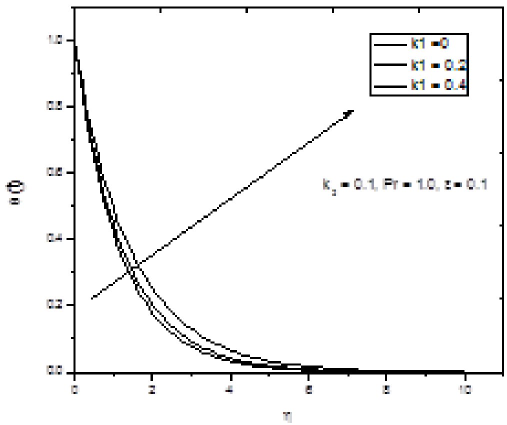


fig 4 : 23th order approximations for θ(η)

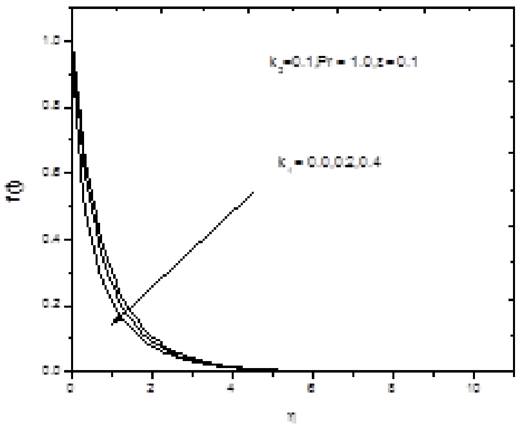
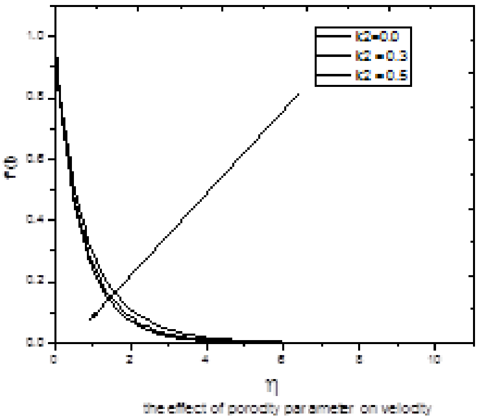
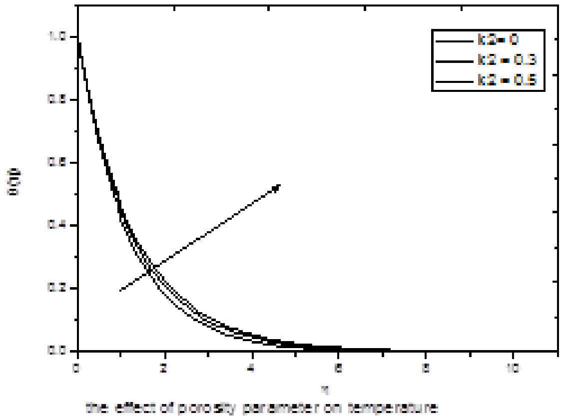


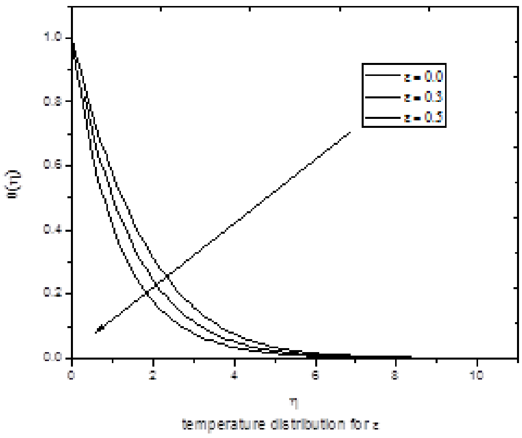
fig 3 : 25th order approximation for f(η)

Fig 5 & 6, the effect of porosity parameter on velocity and temperature, it is observed that the effect of porosity parameter increases with decrease in dimensionless velocity and the effect of porosity parameter increases with increase in dimensionless temperature.





In fig 7 & 8, the effect of prandtl number and small parameter on velocity and temperature, it is observed that dimensionless temperature decreases with increase in small parameter epsilon(the effect flow field) and Prandtl number.



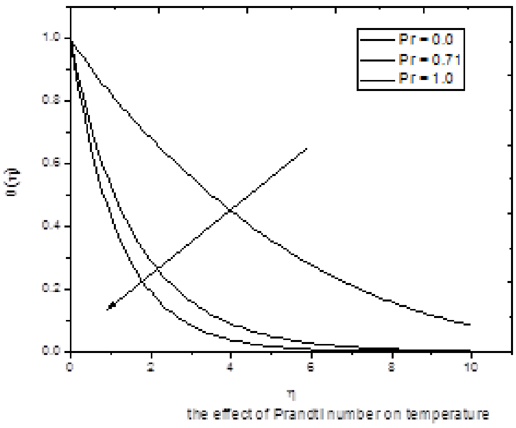


Table 1. Convergence of HAM solution for different orders of approximations when

$k_1 = 0.1, k_2 = 0.1, Pr = 1., \varepsilon = 0.1.$

Order	$-f''(0)$	$-\theta'(0)$
5	1.426370	0.870193
10	1.427150	0.864028
15	1.427157	0.863115
20	1.427156	0.862909
25	1.427156	0.862854
30	1.427156	0.862838
35	1.427156	0.862833
40	1.427156	0.862833
45	1.427156	0.862833
50	1.427156	0.862833

Table 2. Comparison of $-\theta'(0)$ for different values of Pr when $k_1 = k_2 = \varepsilon = 0.0$.

Pr	Bidin and Nazar [19]	HAM
1.0	0.9547	0.954783
2.0	1.4714	1.471460
3.0	1.8691	1.869067

CONCLUSIONS

In this paper, we analyzed two- dimensional steady incompressible laminar flow of Walter's liquid B through a porous medium. Analytical solutions are obtained through HAM. The effects of various parameters on velocity and temperature are analysed and are shown graphically.

The results are summarized as follows

- The effect of increasing visco-elastic parameter is to decrease in dimensionless velocity and increase in temperature.
- The effect of porosity is to increase the wall temperature when it is increases.
- Increase in Prandtl number results in decreases the dimensionless temperature
- Increase in flow field parameter (epsilon) results in decrease in dimensionless temperature.

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