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Subdivision of Heronian Mean Labeling of Graphs

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Abstract

In this paper, we contribute some new results for Heronian Mean labeling of graphs. We prove that subdivision of Heronian Mean Graphs are Heronian Mean Graphs. We use some standard graphs to derive the results for subdivision of graphs.

Keywords: Graph, Heronian Mean Graph, Comb, Ladder, Triangular Snake.

AMS Subject Classification: 05C78

1. INTRODUCTION

By a graph we mean a finite undirected graph without loops or parallel edges. For all detailed survey of graph labeling, we refer to J.A. Gallian [1]. For all other standard terminology and notations we follow Harary[2].The concept of Mean labeling has been introduced by S. Somasundaram and R. Ponraj [3] in 2004. S.Somasundaram and S.S.Sandhya introduced Harmonic mean labeling [4] in 2012. Motivated by the above works we introduced a new type of labeling called **Heronian Mean Labeling in [5].**

In this paper we investigate the Subdivision of Heronian Mean Labeling of graphs. We will provide brief summary of definitions and other information which are necessary for our present investigation.

A Path $\mathbf{P_n}$ is a walk in which all the vertices are distinct. The graph obtained by joining a single pendant edge to each vertex of a Path is called a **Comb**. The **Ladder** $\mathbf{L_n}$ is the product graph $\mathbf{P_2 \times P_n}$. A **Triangular Snake** $\mathbf{T_n}$ is obtained from a path $\mathbf{u_1, u_2, \ldots\ldots u_n}$ by joining $\mathbf{u_i}$ and $\mathbf{u_{i+1}}$ to a new vertex $\mathbf{v_i}$ for $1 \le i \le n-1$. That is every edge of a path is replaced by a triangle $\mathbf{C_3}$.

Definition 1.1:

A graph **G=(V,E)** with p vertices and q edges is said to be a **Heronian Mean graph** if it is possible to label the vertices $\mathbf{x \in V}$ with distinct labels $\mathbf{f(x)}$ from **1,2,…,q+1** in such a way that when each edge $\mathbf{e = uv}$ is labeled with,

$$\mathbf{f(e=uv)} = \left\lceil \frac{\mathbf{f(u)} + \sqrt{\mathbf{f(u)f(v)}} + \mathbf{f(v)}}{\mathbf{3}} \right\rceil (\mathbf{OR}) \left\lfloor \frac{\mathbf{f(u)} + \sqrt{\mathbf{f(u)f(v)}} + \mathbf{f(v)}}{\mathbf{3}} \right\rfloor$$

then the edge labels are distinct. In this case **f** is called a **Heronian Mean labeling** of G.

Definition 1.2:

If **e=uv** is an edge of G and w is not a vertex of G then e is said to be subdivided when it is replaced by the edges **uw** and **wv**. The graph obtained by subdividing each edge of a graph G is called the subdivision of G and is denoted by **S(G).**

Theorem 1.3: Any Comb $P_n \odot K_1$ is a Heronian mean graph.

Theorem 1.4: Any Ladder L_n is a Heronian mean graph.

Theorem 1.5: Any Triangular Snake T_n is a Heronian mean graph.

2. MAIN RESULTS

Theorem: 2.1

Subdivision of any Comb $P_n \odot K_1$ is a Heronian mean graph.

Proof:

Let $P_n \odot K_1$ be a graph obtained from a path $u_1 u_2 \ldots . u_n$ by joining the vertex u_i to pendant vertices v_i.

Let $G = S(P_n \odot K_1)$ be a graph obtained by subdividing all the edges of $P_n \odot K_1$.

Here we consider the following cases.

Case (i):

Let G be a graph obtained by subdividing each edge $u_i u_{i+1}$ of $P_n \odot K_1$.

Let $w_i, 1 \le i \le n-1$ be the vertices which subdivide u_i and u_{i+1}.

Define a function $f: V(\mathbf{G}) \to \{1,2,3, \ldots\ldots, q+1\}$ by $f(u_i) = 3i-1, 1 \le i \le n.$

$$f(v_i) = 3i-2, 1 \le i \le n.$$

$$f(w_i) = 3i, 1 \le i \le n-1.$$

Edges are labeled with, $f(u_i v_i) = 3i-2, 1 \le i \le n,$

$f(u_i w_i) = 3i - 1, 1 \leq i \leq n$

$f(w_i u_{i+1}) = 3i, 1 \leq i \leq n.$

Clearly f is a Heronian Mean labeling.

The labeling pattern is displayed below.

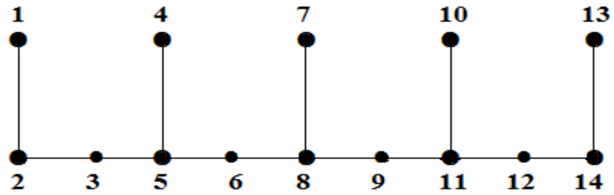

Figure:1

Case (ii):

Let G be a graph obtained by subdividing the edge $u_i v_i$ of $P_n \odot K_1$.

Let t_i, $1 \leq i \leq n$ be the vertices which subdivide u_i and v_i.

Define a function $f: V(\mathbf{G}) \rightarrow \{1,2,3, \ldots\ldots, q+1\}$ by $f(u_i) = 3i - 2, 1 \leq i \leq n.$

$f(v_i) = 3i, 1 \leq i \leq n.$

$f(t_i) = 3i - 1, 1 \leq i \leq n.$

Edges are labeled with, $f(u_i u_{i+1}) = 3i, 1 \leq i \leq n - 1,$

$f(u_i t_i) = 3i - 2, 1 \leq i \leq n$

$f(t_i v_i) = 3i - 1, 1 \leq i \leq n.$

Clearly f is a Heronian Mean labeling.

The labeling pattern is displayed below.

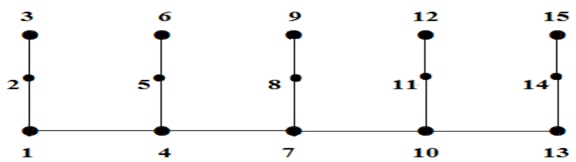

Figure:2

Case (iii):

Let G be a graph obtained by subdividing all the edges of $P_n \odot K_1$.

Let w_i, $1 \leq i \leq n - 1$ be the vertices which subdivide u_i and u_{i+1}.

Let t_i, $1 \leq i \leq n$ be the vertices which subdivide u_i and v_i.

Define a function $f: V(\mathbf{G}) \rightarrow \{1,2,3, \ldots\ldots, q+1\}$ by $f(u_i) = 4i - 3, 1 \leq i \leq n.$

$f(v_i) = 4i - 1, 1 \leq i \leq n.$

$f(w_i) = 4i, 1 \leq i \leq n-1.$

$f(t_i) = 4i-2, 1 \leq i \leq n.$

Edges are labeled with, $f(u_i w_i) = 4i-1, 1 \leq i \leq n-1$

$f(w_i u_{i+1}) = 4i, 1 \leq i \leq n.$

$f(u_i t_i) = 4i-3, 1 \leq i \leq n$

$f(t_i v_i) = 4i-2, 1 \leq i \leq n.$

Clearly f is a Heronian Mean labeling.

The labeling pattern is displayed below.

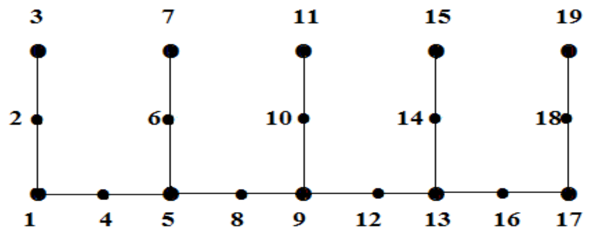

Figure:3

From all the above three cases, we conclude that $G = S(P_n \odot K_1)$ is a Heronian mean graph.

Theorem: 2.2

Subdivision of any Ladder L_n is a Heronian mean graph.

Proof:

Let $\mathbf{L_n}$ be a ladder connecting two paths $u_1 u_2 \ldots u_n$ and $v_1 v_2 \ldots v_n$.

Let $G = S(\mathbf{L_n})$ be a graph obtained by subdividing all the edges of $\mathbf{L_n}$.

Here we consider the following cases.

Case (i):

Let G be a graph obtained by subdividing each edge $u_i u_{i+1}$ and $v_i v_{i+1}$ of $\mathbf{L_n}$.

Let x_i, y_i, $1 \leq i \leq n-1$ be the vertices which subdivide the edges $u_i u_{i+1}$ and $v_i v_{i+1}$.

Define a function $f: V(\mathbf{G}) \to \{1,2,3,\ldots,q+1\}$ by $f(u_i) = 5i-4, 1 \leq i \leq n.$

$f(v_i) = 5i-3, 1 \leq i \leq n.$

$f(x_i) = 5i-2, 1 \leq i \leq n-1.$

$f(y_i) = 5i-1, 1 \leq i \leq n-1.$

Edges are labeled with, $f(u_i v_i) = 5i-4, 1 \leq i \leq n,$

$f(u_i x_i) = 5i-3, 1 \leq i \leq n-1,$

$f(x_i u_{i+1}) = 5i-1, 1 \leq i \leq n-1,$

$f(v_i y_i) = 5i - 2, 1 \leq i \leq n - 1,$

$f(y_i v_{i+1}) = 5i, 1 \leq i \leq n - 1.$

Clearly f is a Heronian Mean labeling.

The labeling pattern is displayed below.

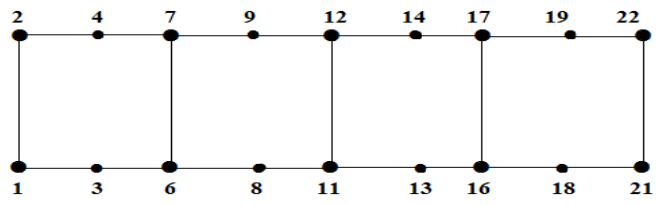

Figure:4

Case (ii):

Let G be a graph obtained by subdividing the edge $u_i v_i$ of $\mathbf{L_n}$.

Let z_i, $1 \leq i \leq n$ be the vertices which subdivide u_i and v_i.

Define a function $f: V(\mathbf{G}) \rightarrow \{1,2,3, \dots\dots, q+1\}$ by $f(u_i) = 4i - 3, 1 \leq i \leq n.$

$f(v_i) = 4i - 1, 1 \leq i \leq n.$

$f(z_i) = 4i - 2, 1 \leq i \leq n.$

Edges are labeled with, $f(u_i u_{i+1}) = 4i - 1, 1 \leq i \leq n - 1,$

$f(v_i v_{i+1}) = 4i, 1 \leq i \leq n - 1,$

$f(u_i z_i) = 4i - 3, 1 \leq i \leq n$

$f(z_i v_i) = 4i - 2, 1 \leq i \leq n.$

Clearly f is a Heronian Mean labeling.

The labeling pattern is displayed below.

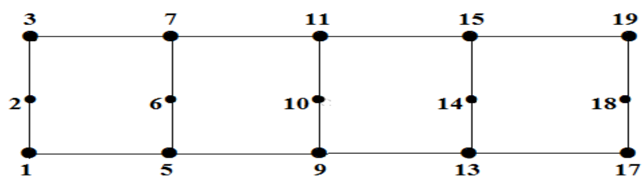

Figure:5

Case (iii):

Let G be a graph obtained by subdividing all the edges of $\mathbf{L_n}$.

Let x_i, y_i, $1 \leq i \leq n - 1$ be the vertices which subdivide the edges $u_i u_{i+1}$ and $v_i v_{i+1}$.

Let z_i, $1 \leq i \leq n$ be the vertices which subdivide u_i and v_i.

Define a function $f: V(\mathbf{G}) \to \{1,2,3, \dots\dots, q+1\}$ by $f(u_i) = 6i - 5, 1 \le i \le n.$

$f(v_i) = 6i - 3, 1 \le i \le n.$

$f(z_i) = 6i - 4, 1 \le i \le n.$

$f(x_i) = 6i - 2, 1 \le i \le n - 1.$

$f(y_i) = 6i - 1, 1 \le i \le n - 1.$

Edges are labeled with, $f(u_i z_i) = 6i - 5, 1 \le i \le n$

$f(z_i v_i) = 6i - 4, 1 \le i \le n.$

$f(u_i x_i) = 6i - 3, 1 \le i \le n - 1,$

$f(x_i u_{i+1}) = 6i - 1, 1 \le i \le n - 1,$

$f(v_i y_i) = 6i - 2, 1 \le i \le n - 1,$

$f(y_i v_{i+1}) = 6i, 1 \le i \le n - 1.$

Clearly f is a Heronian Mean labeling.

The labeling pattern is displayed below.

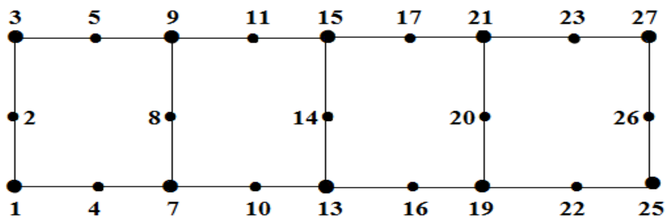

Figure:6

From all the above three cases, we conclude that $G = S(L_n)$ is a Heronian mean graph.

Theorem: 2.3

Subdivision of any Triangular Snake $\boldsymbol{T_n}$ is a Heronian mean graph.

Proof:

Let $\mathbf{T_n}$ be a Triangular Snake obtained from a path $u_1 u_2 \dots . u_n$ by joining u_i and u_{i+1} to a new vertex $v_i, 1 \le i \le n - 1.$

Let $G = S(\mathbf{T_n})$ be a graph obtained by subdividing all the edges of $\mathbf{T_n}$.

Here we consider the following cases.

Case (i):

Let G be a graph obtained by subdividing each edge $u_i u_{i+1}$ of $\mathbf{T_n}$.

Let $w_i, 1 \le i \le n - 1$ be the vertices which subdivide u_i and u_{i+1}.

Define a function $f: V(\mathbf{G}) \to \{1,2,3, \dots\dots, q+1\}$ by $f(u_i) = 4i - 3, 1 \le i \le n.$

$f(v_i) = 4i - 2, 1 \le i \le n - 1.$

$f(w_i) = 4i, 1 \le i \le n - 1.$

Edges are labeled with, $f(u_i v_i) = 4i - 3, 1 \leq i \leq n - 1,$

$$f(u_{i+1} v_i) = 4i - 1, 1 \leq i \leq n - 1,$$
$$f(u_i w_i) = 4i - 2, 1 \leq i \leq n - 1,$$
$$f(w_i u_{i+1}) = 4i, 1 \leq i \leq n - 1.$$

Clearly f is a Heronian Mean labeling.

The labeling pattern is displayed below.

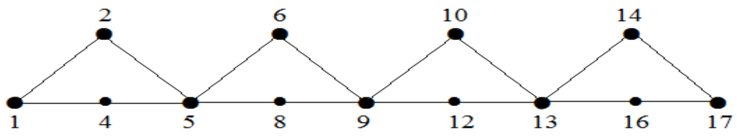

Figure:7

Case (ii):

Let G be a graph obtained by subdividing the edges $u_i v_i$ and $u_{i+1} v_i$ of $\mathbf{L_n}$.

Let x_i and y_i , $1 \leq i \leq n - 1$ be the vertices which subdivide the edges $u_i v_i$ and $u_{i+1} v_i$.

Define a function $f: V(\mathbf{G}) \rightarrow \{1,2,3, \ldots\ldots, q + 1\}$ by $f(u_i) = 5i - 4, 1 \leq i \leq n.$

$$f(v_i) = 5i - 2, 1 \leq i \leq n - 1.$$
$$f(x_i) = 5i - 3, 1 \leq i \leq n - 1.$$
$$f(y_i) = 5i - 1, 1 \leq i \leq n - 1.$$

Edges are labeled with, $f(u_i u_{i+1}) = 5i - 2, 1 \leq i \leq n - 1,$

$$f(u_i x_i) = 5i - 4, 1 \leq i \leq n - 1.$$
$$f(x_i v_i) = 5i - 3, 1 \leq i \leq n - 1.$$
$$f(u_{i+1} y_i) = 5i, 1 \leq i \leq n - 1.$$
$$f(y_i v_i) = 5i - 1, 1 \leq i \leq n - 1.$$

Clearly f is a Heronian Mean labeling.

The labeling pattern is displayed below.

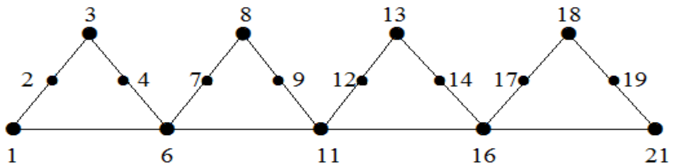

Figure:8

Case (iii):

Let G be a graph obtained by subdividing all the edges of $\mathbf{T_n}$.

Let x_i and y_i , $1 \le i \le n-1$ be the vertices which subdivide the edges u_iv_i and $u_{i+1}v_i$.

Let w_i, $1 \le i \le n-1$ be the vertices which subdivide u_i and u_{i+1}.

Define a function $f: V(\mathbf{G}) \to \{1,2,3, \ldots\ldots, q+1\}$ by $f(u_i) = 6i-5, 1 \le i \le n.$

$$f(v_i) = 6i-3, 1 \le i \le n-1.$$
$$f(w_i) = 6i, 1 \le i \le n-1.$$
$$f(x_i) = 6i-4, 1 \le i \le n-1.$$
$$f(y_i) = 6i-2, 1 \le i \le n-1.$$

Edges are labeled with, $f(u_iw_i) = 6i-3, 1 \le i \le n-1$

$$f(w_iu_{i+1}) = 6i, 1 \le i \le n-1.$$
$$f(u_ix_i) = 6i-5, 1 \le i \le n-1,$$
$$f(x_iv_i) = 6i-4, 1 \le i \le n-1,$$
$$f(u_{i+1}y_i) = 6i-1, 1 \le i \le n-1,$$
$$f(y_iv_i) = 6i-2, 1 \le i \le n-1.$$

Clearly f is a Heronian Mean labeling.

The labeling pattern is displayed below.

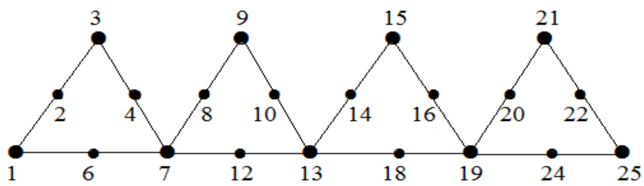

Figure:9

From all the above three cases, we conclude that $G = S(T_n)$ is a Heronian mean graph.

3. CONCLUSION:

The Study of labeled graph is important due to its diversified applications. It is very interesting to investigate subdivision of Heronian mean graphs which admit Heronian Mean Labeling. The derived results are demonstrated by means of sufficient illustrations which provide better understanding. It is possible to investigate similar results for several other graphs.

4. ACKNOWLEDGEMENT:
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