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Exponential Variation of Pressure for the Rotation of Porous Journal Bearing

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Abstract

The second order rotatory theory of hydrodynamic lubrication was supported on the expression obtained by retentive the terms containing up to second powers of rotation number within the extended generalized Reynolds equation. Within this paper, there are some new glorious basic solutions for the porous bearings within the second order rotatory theory of hydrodynamic lubrication. The expressions for exponential variation of the pressure with respect to rotation number M and viscosity μ are obtained by taking the constant values of the ratio L/D and e. The analysis of equation for pressure, tables and graphs reveal that pressure is not independent of viscosity and will increase slightly with viscosity. The pressure increases with increasing values of rotation number. Within the absence of rotation, the equation of pressure reduces to the classical solutions of the classical theory of hydrodynamic lubrication.

Keywords: Permeability, Porous bearing, Pressure, Reynolds equation, Rotatory theory.

1. INTRODUCTION

Within the theory of hydrodynamic lubrication, two-dimensional classical theories were 1st given by O. Reynolds [1]. In 1886, with reference to a classical experiment by B. Tower [2], he developed a very important equation, which was referred to as Reynolds Equation [2]. The formation and basic mechanism of the fluid film was analyzed by that experiment on taking some vital assumptions [3] i.e., the fluid film thickness is incredibly little as compare to the axial and longitudinal dimensions of fluid film and if the stuff layer is to transmit pressure between the shaft and therefore the bearing, the layer should have varied thickness [4]. Later O. Reynolds himself derived an improved version of Reynolds Equation legendary as: "Generalized Reynolds Equation" that depends on viscosity, film thickness, density and surface or transverse velocities [5], [6], [7]. The rotation of the fluid film regarding an axis that lies within the film offers some new ends up in lubrication issues of hydrodynamics. The origin of rotation is derived by bound general theorems associated with vorticity within the rotating fluid dynamics [7]. The rotation induces an element of vorticity within the direction of rotation of fluid film and therefore the effects arising from it are predominant, for giant Taylor's number, it ends up in the streamlines changing into confined to plane transverse to the direction of rotation of the film [8], [9].

The new extended version of "Generalized Reynolds Equation" is alleged to be "Extended Generalized Reynolds Equation" that takes into consideration of the consequences of the uniform rotation regarding an axis that lies within the fluid film and depends on the rotation number M, [10], [11] i.e. the root of the standard Taylor's number. The generalization of the classical theory of hydrodynamic lubrication is understood because the "Rotatory Theory of hydrodynamic Lubrication" [12], [13]. The "Second Order Rotatory Theory of hydrodynamic Lubrication" was given by retentive the terms containing up to second powers of M by neglecting higher powers of M [12], [14].

In the case of porous bearing, the bearing is infinitely short, so the pressure gradient in x-direction is too smaller than in y-direction. In y-direction the gradient $\frac{\partial P}{\partial y}$ is of order of $\left(\frac{P}{L}\right)$ and in the x-direction, is of order of $\left(\frac{P}{B}\right)$ i.e., $L << B$ then $\frac{P}{L} >> \frac{P}{B}$, so $\frac{\partial P}{\partial x} << \frac{\partial P}{\partial y}$. Hence, the terms of $\frac{\partial P}{\partial x}$ can be neglected with respect to $\frac{\partial P}{\partial y}$ in the expanded version of Generalized Reynolds Equation**.**

2. BOUNDARY CONDITIONS AND SOLUTION OF DIFFERENTIAL EQUATION

The Extended Generalized Reynolds Equation in view of second order rotatory theory of hydrodynamic lubrication, in ascending powers of rotation number M and by retaining the terms containing up to second powers of M and neglecting higher powers of M, can be written as:

$$\frac{\partial}{\partial x}\left[F_1\rho\frac{\partial P}{\partial x}\right]+\frac{\partial}{\partial y}\left[F_1\rho\frac{\partial P}{\partial y}\right]+\frac{\partial}{\partial x}\left[F_2\frac{\partial P}{\partial y}\right]-\frac{\partial}{\partial y}\left[F_2\frac{\partial P}{\partial x}\right]=-\frac{\partial}{\partial x}\left[\frac{\rho U}{2}\{h-M\,F_2\}\right]-\frac{\partial}{\partial y}\left[\frac{M\rho^2 U}{2}F_1\right]-\rho W^* \tag{1}$$

Where, $F_1=\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\right], F_2=-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)$

In the expressions; x, y and z are coordinates, P is the pressure, ρ is the fluid density, U is the sliding velocity, μ is the viscosity and W^* is fluid velocity in z-direction. Let we choose $h=h(x)$, $U=-U$, $P=P(y)$ *and* $W^* = -\frac{\partial P}{\partial z}\Big|_{z=0}\frac{\phi}{\mu}$. Where $\frac{\partial P}{\partial z}$ represents the pressure gradient at the bearing surface and ϕ is the property called permeability that varies with porosity and size of pores. As the requirements of continuity, we have for the porous matrix $\frac{\phi}{\mu}\nabla W^* = \nabla^2 P=0$ *i.e.,* $\nabla^2 P=0$. Now the problem is to solve the governing equation (1) for the pressures in oil film simultaneously with that of Laplace for the porous matrix with a common pressure gradient $\frac{\partial P}{\partial z}$ at boundary, we have the equation of continuity i.e., $\frac{\partial^2 P}{\partial x^2}+\frac{\partial^2 P}{\partial y^2}+\frac{\partial^2 P}{\partial z^2}=0.$ We have use the assumptions that bearing is infinitely short and $\frac{\partial P}{\partial z}$ is linear across the matrix and is zero at the outer surface of the porous bearing shell to solve the equation (2), we have $\frac{\partial^2 P}{\partial x^2}=0$, $\frac{\partial^2 P}{\partial z^2}=K$ (constant), $\frac{\partial^2 P}{\partial y^2}=-K$ *i.e.,* $\frac{\partial P}{\partial z}\Big|_{Z=0}=KH=\frac{\partial^2 P}{\partial y^2}\Big|_{Z=0}H$. The H is the wall thickness of porous bearing. Now the equation (2) becomes

$$[F_1\rho]\frac{d^2P}{dy^2}-\left(\frac{d}{dx}F_2\right)\frac{dP}{dy}=\frac{d}{dx}\left[\frac{\rho U}{2}\{h-M\,F_2\}\right]-\rho\left(-\frac{dP}{dz}\Big|_{z=0}\frac{\phi}{\mu}\right) \tag{2}$$

The film thickness 'h' and 'y' can be taken as; $h=C(1+e\cos\theta)$, $y=R\,\theta$, where e is the eccentricity, θ is the angular coordinates measured from x-direction and R is the radius of bearing. For the determination of pressure the boundary conditions are taken as: $P=0$, $y=\pm\frac{L}{2}$.

The solution of the differential equation (1) under the boundary conditions gives the pressure for porous bearing as follows:

$$P = \frac{(3\mu CUesin\,\theta + 12KH\phi R)(L^2 - 4y^2)}{4(1+ecos\theta)^3 R} + \frac{\rho Cesin\theta(U\mu + 4KH\phi)(L^2 y - 4y^3)}{8\mu R(1+ecos\theta)^2} M + \frac{\begin{pmatrix} 53U\mu\rho^2 Cesin\theta(1+ecos\theta) - \\ 68RKH\phi\rho^2(1+ecos\theta \end{pmatrix}(L^2 - 4y^2)}{2240\mu^2 R} M^2 \quad (3)$$

3. NUMERICAL SIMULATIONS AND GRAPHICAL ANALYSIS

By taking the values of different mathematical terms in C.G.S. system as follows: θ=30^o, μ=0.0002, *C*=0.0067, ρ=0.9, *U*=10^2, *h*=0.00786, *y*=1, *H*=0.05, ϕ=0.0025, *R*=3.35; the calculated values of pressure and load capacity with respect to *M,* by taking µ= 0.0002, are given by table 1.

Table 1.

e↓	*L/D*↓	*M*→	0.2	0.4	0.6	0.8	1.0
0.2	0.5	*P*	844.7764864	897.5040214	950.1928974	10002.844914	1055.458273
0.2	1.0	*P*	4782.680583	5081.196179	5379.496305	5677.580961	5975.450148
0.9	0.5	*P*	1185.885315	1423.933488	1662.326572	1901.064569	2140.147477
0.9	1.0	*P*	6713.859539	8061.563214	9411.219599	10762.82869	12116.3905

Also by taking the values of different mathematical terms in C.G.S. system as follows:
θ=30^o, e=0.9, *C*=0.0067, ρ=0.9, *U*=10^2, *h*=0.00786, *H*=0.05, ϕ=0.0025, *R*=3.35, *L/D*=1; the calculated values of pressure and load capacity with respect to μ by taking *M*=constant=1.0, are given by Table 2.

Table 2.

µ	**0.0002**	**0.0003**	**0.0004**	**0.0005**	**0.0006**
P	12091.54659	12653.37738	13218.21638	13784.39107	14351.27039

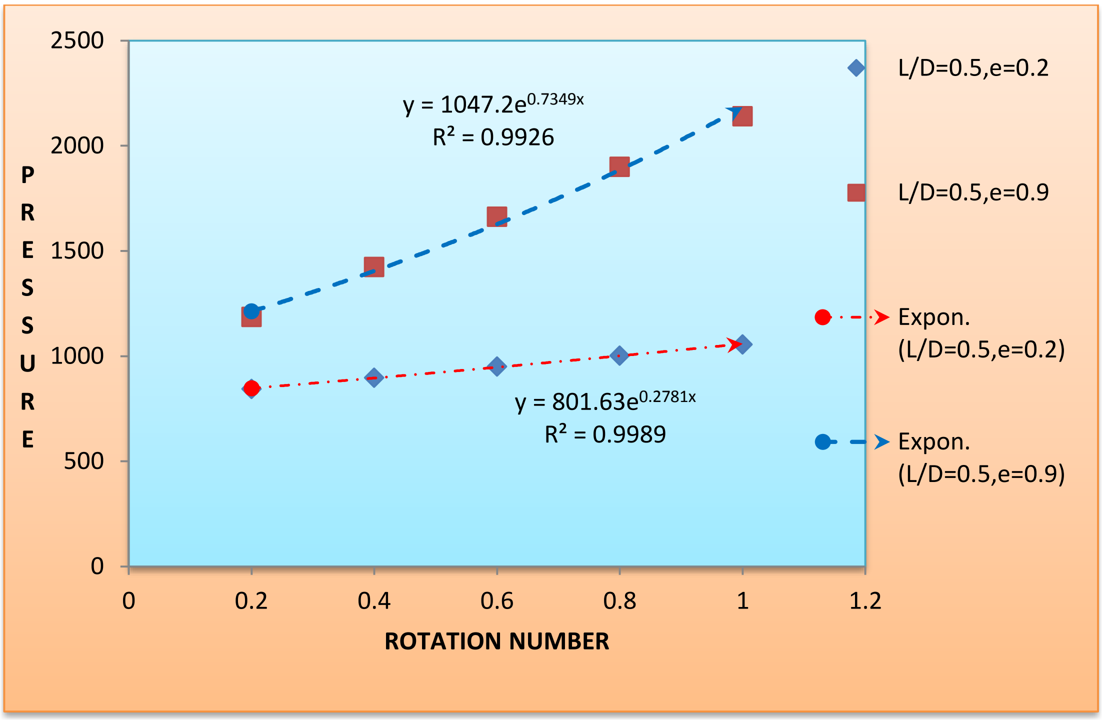

FIG.1. Variation of pressure with respect to M for e=0.2, 0.9; L/D=0.5.

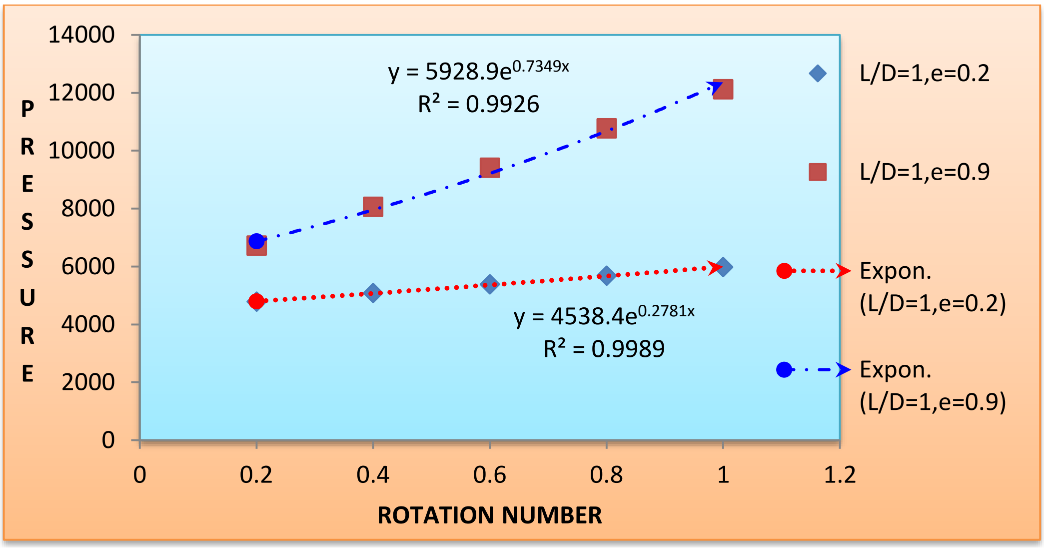

FIG.2. Variation of pressure with respect to M for e=0.2, 0.9; L/D=1.0.

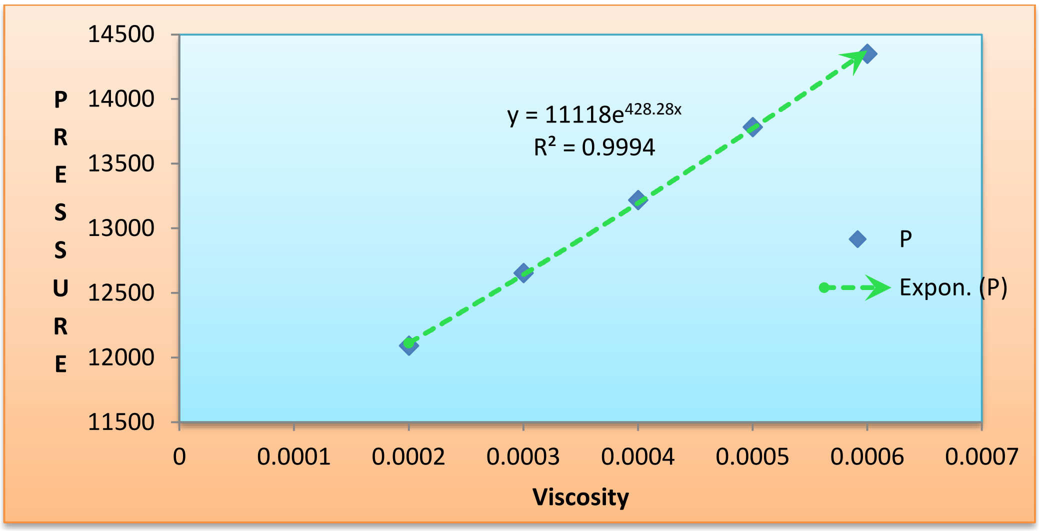

FIG.3. Variation of pressure with respect to μ for e=0.9; L/D=1.0.

4. CONCLUSIONS

The exponential variation of the pressure with respect to rotation number M and viscosity μ are obtained by taking the constant values of the ratio L/D and e, i.e., are as follows:

$P=1047\ e^{0.734M}$ $(L/D=0.5,\ e=0.9)$; $P=801.6\ e^{0.278M}$ $(L/D=0.5,\ e=0.2)$; $P=5928\ e^{0.734M}$ $(L/D=1.0,\ e=0.9)$;

$P=4538\ e^{0.278M}$ $(L/D=1.0,\ e=0.2)$; $P=11118\ e^{428.2\mu}$ $(L/D=1.0,\ e=0.9)$

Hence, $P \propto \alpha e^{\beta M}$, where α and β are arbitrary constants.

The expression of pressure shows that it increases linearly with μ, whereas the expression, tables and graphs show that in second order rotatory theory, the pressure does not changes linearly with μ, it slightly increases with μ due to presence of the permeability factor in the numerator and denominator. The pressure increases with rotation number M, when viscosity is taken as constant. The classical pressure equation for porous bearing was

$$P = \frac{3U\mu e\ sin\theta}{RC^2[\ (1+e\ cos\theta)^3+12\psi]}\left(\frac{L^2}{4} - y^2\right)\ ,\text{ where } \psi = \frac{H\phi}{C^3}.$$

This equation does not give infinite pressure at $\theta=\pi$ if e=1 due to the presence of the term (12ψ), whereas the pressure equation shows that it gives infinite pressure at $\theta=\pi$ if e=1. On taking (M=0) i.e., in the absence of rotation, we get the classical solutions.

REFERENCES

[1]. Reynolds, O., 1886, Phil. Trans. Roy. Soc. London, Part I, 177.

[2]. Reynolds, O., 1886, "On the Theory of Lubrication and its Application to Mr. Beauchamp Tower's Experiment," Phil. Trans. Roy. Soc. London, 177 (I), 157.

[3]. Shaw, M. C., and Macks, E. F., 1949, "Analysis and Lubrication of Bearings," Mc. Graw Hill Book Company, Inc., New York.

[4]. Cameron, A., 1958, "The Viscous Wedge Trans.," ASME, 1, 248.

[5]. Pinkus, O., and Sternlicht, B. (1961). Theory of Hydrodynamic Lubrication, Mc. Graw Hill Book Company, Inc. New York, 5-64.

[6]. Dowson, D., 1962, "A Generalized Reynolds Equations for Fluid Film Lubrication," Int. J. Mech. Sci., 4, 159.

[7]. Chandrasekhar, S., 1970, "Hydrodynamic and Hydro magnetic Stability," Oxford University Press, London, 83.

[8]. Saibel, E. A. and Macken, N.A., 1973, "Annual Review of Fluid Mechanics," Inc. Palo. Alto, Cal. Vol.5.

[9]. Halling, J., 1975, "Principles of Tribology," The Macmillan Press Ltd., London, 369.

[10]. Banerjee, M. B., Gupta, R. S. and Dwivedi, A. P., 1981, "The Effects of Rotation in Lubrication Problems," WEAR, 69, 205.

[11]. Banerjee, M. B., Chandra, P. and Dube, G. S., 1981, "Effects of Small Rotation in Short Journal Bearings," Nat. Acad. Sci. Letters, Vol. 4, No.9.

[12]. Banerjee, M. B., Dube, G. S., Banerjee, K., 1982, "The Effects of Rotation in Lubrication Problems: A New Fundamental Solutions," WEAR, 79, pp. 311-323.

[13]. Cameron, A., 1981, "Basic Lubrication Theory," Ellis Harwood Limited, Coll. House, Watergate, Chicester, p. 45-162.

[14]. Dube, G. S. and Chatterjee, A., 1988, "Proc. Nat. Acad. Sci. India," 58(58), I: 79.