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Partition Dimension of Binary Tree Based Architectures

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Abstract

For a vertex v of a connected graph G and a subset S of $V(G)$, the distance between v and S is $d(v, S) = min\{d(v, x) \mid x \in S\}$. Let $\Pi = \{S_1, S_2 \ldots S_k\}$ be an ordered k-partition of $V(G)$. The representation of v with respect to Π is the k-vector $r(v|\Pi) = (d(v, S_1), d(v, S_2) \ldots d(v, S_k))$. A resolving partition is a k-partition Π with distinct k-vectors $r(v|\Pi)$ for all $v \in V(G)$. The minimum k for which there is a resolving k-partition of $V(G)$ is called the partition dimension $pd(G)$ of G. In this paper, partition dimension of X-tree and slim tree are determined.

Keywords: Resolving partition, partition dimension, X-tree, slim tree.
AMS Subject Classification: 05C12

1. INTRODUCTION

Resolving partition and partition dimension for a graph was introduced [2] based on the distance concept. This concept has wide application in the field of chemistry, problems of pattern recognition, image processing and navigation of robots [4, 11, 13].
Let $M = \{v_1, v_2 \ \ldots \ v_n\}$ be an ordered set of vertices in a graph G. Then $(d(u, v_1), d(u, v_2) \ldots d(u, v_n))$ is called the M-coordinates of a vertex u of G. If the M-coordinates are distinct for all vertices in G, then the set M is called a $metric$ $basis$ of G. A minimum metric basis is a set M with minimum cardinality. The cardinality of a minimum metric basis of G is called $minimum$ $metric$ $dimension$ and is denoted by $\beta(G)$; the members of a metric basis are called $landmarks$ [4,10,11].

For $v \in V(G)$ and $S \subset V(G)$, the distance $d(v,S)$ between v and S is defined as $d(v,S) = min\{d(v,x) \mid x \in S\}$. The representation of v with respect to Π is defined as the k-vector $r(v|\Pi) = (d(v,S_1), d(v,S_2) \ldots d(v,S_k))$, where Π is an ordered k-partition $\{S_1, S_2 \ldots S_k\}$ and $v \in V(G)$. The partition Π is called a $resolving\ partition$ for G if the distinct vertices of G have distinct representation with respect to Π. The minimum k for which there is a resolving k-partition of $V(G)$ is a $partition\ dimension\ pd(G)$ of G. A resolving partition of $V(G)$ containing $pd(G)$ elements is called a $minimum\ resolving\ partition$ [2,3].

Let $\Pi = \{S_1, S_2, \ldots S_k\}$ be a resolving partition of $V(G)$. If $u \in S_i$, $v \in S_j$, where $1 \leq i,j \leq k$ and $i \neq j$, then $r(u|\Pi) \neq r(v|\Pi)$ since $d(u,S_i) = 0$ but $d(v,S_i) \neq 0$. Thus, while determining whether a given partition Π of $V(G)$ is a resolving partition for G, we only need to verify if the vertices of G belonging to the same set of Π have distinct representation with respect to Π.

If G is a non-trivial connected graph, then $pd(G) \leq \beta(G) + 1$ [3]. For a connected graph G of order $n \geq 2$, $pd(G) = 2$ if and only if $G = P_n$ and $pd(G) = n$ if and only if $G = K_n$ [3].

Chartrand et al. [3] proved that for a graph G which is neither a path nor a complete graph with order $n \geq 4$, then $3 \leq pd(G) \leq n - 1$, further, they have exhibited graphs with partition dimension $n - 1$. A connected bipartite graph G with partite sets V_1 and V_2 of cardinalities r and s respectively has $pd(G) \leq r + 1$, if $r = s$ and $pd(G) \leq max\{r,s\}$, if $r \neq s$ [3].

The partition dimension problem has been studied for circulant networks [9], hexagonal and honeycomb networks [13], pyramid networks [5], honeycomb derived networks [6], tree [14], cartesian and strong product graphs [15,16].

A *binary tree* is a rooted tree in which each vertex has at most two children and a *complete binary tree* is a binary tree with exactly two children except at the last level. We consider architectures namely, X-tree and slim tree obtained from complete binary tree.

2. X-TREES

An X-tree is obtained from a complete binary tree by connecting consecutive vertices on the same level of the tree. Edges are added to the tree so that the vertices on each level are connected, from left to right, in a path. These edges are called as *horizontal edges*. Horizontal edges can be classified as *sibling edges* and *cousin edges*. A sibling edge denotes a horizontal edge that connects two vertices with the same parent and a cousin edge denotes any of the remaining horizontal edges. Two sibling edges are said to be *adjacent* if there is exactly one cousin edge between them. The term *vertical* *edge* designates a tree edge. Vertices with the same parent are called siblings [1,12].

The vertices at level n are called as *leaf vertices*. An n-level X-tree has $2^{n+1}-1$ vertices and $2^{n+2}-n-4$ edges. The root of $X(n)$ is considered to be at level 0. The vertices of $X(n)$ other than the root and the leaf vertices are called *internal vertices*. Let V_L and V_I denote the set of leaf and internal vertices of $X(n)$ respectively. An X-tree with level 4 is shown in Figure 1.

Lemma 1. A partition $\Pi = \{S_1, S_2 \dots S_m\}$ is not a resolving partition for $X(n)$ if $V_L \subseteq S_i$ for some $i \leq m$.

Proof. Let v_j, v_{j+1} be the siblings in V_L. It is clear that $d(v_j, x) = d(v_{j+1}, x)$, for all $x \in V_I$. Thus, the representation of v_j and v_{j+1} are identical with respect to Π. W
The vertices of Level i in $X(n)$ are labeled from left to right as v_j^i, $1 \leq j \leq 2^i$. The labeling of the graph $X(4)$ is shown in Figure 1.

The minimum resolving partition for an X-tree $X(n)$, $1 \leq n \leq 3$ is 3. The vertices of $X(4)$ are partitioned as $S_1 = \{x;\ x \in V_I\}$, $S_2 = \{v_1^4, v_2^4, v_3^4, v_4^4\}$, $S_3 = \{v_5^4, v_6^4, v_7^4, v_8^4, v_9^4, v_{10}^4, v_{11}^4, v_{12}^4\}$, $S_4 = \{v_{13}^4, v_{14}^4, v_{15}^4, v_{16}^4\}$ and $\Pi = \{S_1, S_2, S_3, S_4\}$. Consider any two vertices v_j^i, $v_k^r \in S_1$ and it is observed that for $m = 2,4$, either $d(v_j^i, S_m) \neq d(v_k^r, S_m)$ or $d(v_j^i, S_3) \neq d(v_k^r, S_3)$. For $v_1^4 \in S_2$, $d(v_1^4, S_3) = 1 + d(v_2^4, S_3) = 2 + d(v_3^4, S_3) = 3 + d(v_4^4, S_3)$. It is clear that $d(v_8^4, S_2) = 1 + d(v_7^4, S_2) = 2 + d(v_6^4, S_2) = 3 + d(v_5^4, S_2)$, $d(v_9^4, S_2) = d(v_{10}^4, S_2)$ but $d(v_9^4, S_4) = 1 + d(v_{10}^4, S_4)$ and similarly, $d(v_{11}^4, S_2) = d(v_{12}^4, S_2)$ but $d(v_{11}^4, S_4) = 1 + d(v_{12}^4, S_4)$. For $v_{16}^4 \in S_4$, $d(v_{16}^4, S_3) = 1 + d(v_{15}^4, S_3) = 2 + d(v_{14}^4, S_3) = 3 + d(v_{15}^4, S_3)$. Thus, we find that $r(v|\Pi)$ are distinct for all $v \in V(X(4))$ and conclude that $pd(X(4)) = 4$.

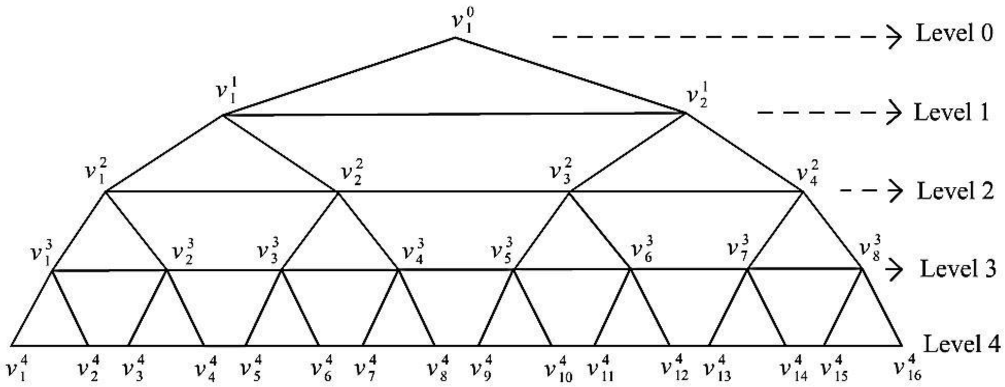

Figure 1: $X(4)$

Lemma 1. $pd(X(5)) = 4$.
Proof. The vertices of $X(5)$ are partitioned as follows.
$S_1 = \{x;\ x \in V_I\}$, $S_2 = \{v_1^4, v_2^4, v_3^4, v_4^4\}$, $S_3 = \{v_i^4;\ 5 \leq i \leq 12\} \cup \{v_i^4;\ 21 \leq i \leq 28\}$, S_4

= $\{v_i^4; 13 \leq i \leq 20\} \cup \{v_i^4; 29 \leq i \leq 32\}$. Let Π = $\{S_i; 1 \leq i \leq 4\}$. It is easy to verify that $r(v|\Pi)$ are distinct for all $v \in X(5)$.

Theorem 2. For $n \geq 5, pd(X(n)) = n - 1$.
Proof. Consider the siblings v_j^i, v_{j+1}^i in the Level i. For the vertices v_{j+r+1}^i, v_{j-r}^i, $1 \leq r \leq 3$ in the Level i, $d(v_j^i, v_{j+r+1}^i) \neq d(v_{j+1}^i, v_{j+r+1}^i)$ and $d(v_j^i, v_{j-r}^i)$ neq $d(v_{j+1}^i, v_{j-r}^i)$. But for $r \geq 4$, we find that $d(v_j^i, v_{j+r+1}^i) = d(v_{j+1}^i, v_{j+r+1}^i)$ and $d(v_j^i, v_{j-r}^i) = d(v_{j+1}^i, v_{j-r}^i)$. For the vertices v_r^m, $v_{r'}^m$ in the Level $m \geq i + 1$, there exist some k_i and $k_{i'}$ $(k_{i'} < k_i)$ such that $d(v_j^i, v_r^m) \neq d(v_{j+1}^i, v_r^m)$, $r < k_i$ and $d(v_j^i, v_{r'}^m) \neq d(v_{j+1}^i, v_{r'}^m)$, $r' < k_{i'}$. For $r \geq k_i$, $r' \geq k_{i'}$, $d(v_j^i, v_r^m) = d(v_{j+1}^i, v_r^m)$, $d(v_j^i, v_{r'}^m) \neq d(v_{j+1}^i, v_{r'}^m)$. Thus, the representation of the vertices v_j^i and v_{j+1}^i becomes distinct with respect to some Π based on the partition of the vertices v_{j+r+1}^i, v_{j-r}^i, $1 \leq r \leq 3$ in the Level i and v_r^m, $v_{r'}^m$, $r < k_i, r' < k_{i'}$ in the Level $m \geq i + 1$.

Let $\{v_1^n, v_2^n, v_3^n, v_4^n, v_5^n\} \subseteq S_i$. It is clear that $d(v_1^n, x) = d(v_2^n, x)$ for all $x \in V(X(n)) \backslash\{v_1^n, v_2^n, v_3^n, v_4^n, v_5^n\}$. Thus, only four vertices $\{v_1^n, v_2^n, v_3^n, v_4^n\}$ can be the members of the set S_i. Similarly, the last four vertices $\{v_{2^n}^n, v_{2^n-1}^n, v_{2^n-2}^n, v_{2^n-3}^n\}$ can be the members of the set S_j. Suppose $i \neq j$ and $S_i = \{v_1^n, v_2^n, v_3^n, v_4^n\}$, $S_j = \{v_{2^n}^n, v_{2^n-1}^n, v_{2^n-2}^n, v_{2^n-3}^n\}$, then for any $x \in V(\text{Level } (i \geq 1))$, $d(x, S_i) \neq d(x, S_j)$.

Let $S_l = \{V(\text{Level } i); 0 \leq i \leq n-2\}$ and $\{v_{k+j}^{n-1}; 1 \leq j \leq 8\} \subseteq S_i$. Consider the vertices $\{v_{k'+j}^n; 1 \leq j \leq 16\}$ in Level n whose ancestors are $\{v_{k+j}^{n-1}; 1 \leq j \leq 8\}$. Suppose the vertices $\{v_{k''+j}^n; 1 \leq j \leq 9\} \subset \{v_{k'+j}^n; 1 \leq j \leq 16\}$ be the members of the set S_t. In this case, $d(v_{k''+5}^n, S_k) = d(v_{k''+6}^n, S_k)$ or $d(v_{k''+4}^n, S_k) = d(v_{k''+5}^n, S_k)$ for all k. Thus, at most eight vertices which are in a path can be members of the set S_t. Without loss of generality, let $\{v_{k'+j}^n; 1 \leq j \leq 8\} \subseteq S_t$ and $\{v_{k'+j}^n; 9 \leq j \leq q16\} \subseteq S_i$. Here, $d(v_{k'+7}^n, S_i) = d(v_{k'+8}^n, S_i) = 1$ and $d(v_{k'+7}^n, S_k) = d(v_{k'+8}^n, S_k)$ for all k. So the vertices $\{v_{k'+j}^n; 1 \leq j \leq 16\}$ can not be the member of the set S_i. Therefore, let the vertices $\{v_{k'+j}^n; 9 \leq j \leq 16\}$ be members of a different set $S_{t'}$.

Now, consider the vertices $\{v_{k+j}^{n-1}; 9 \leq j \leq 16\}$ in Level $n-1$ and its descendants $\{v_{k'+j}^n; 17 \leq j \leq 32\}$ in Level n. Suppose $\{v_{k+j}^{n-1}; 9 \leq j \leq 16\} \subseteq S_i$, as discussed above the vertices $\{v_{k'+j}^n; 17 \leq j \leq 32\}$ must be partitioned into two sets. If $\{v_{k'+j}^n; 17 \leq j \leq 24\} \subseteq S_{t'}$, then the sixteen vertices $\{v_{k'+j}^n; 9 \leq j \leq 24\} \subseteq S_{t'}$. By our earlier discussion this option is ruled out. Therefore, let $\{v_{k'+j}^n; 17 \leq j \leq 24\} \subseteq S_t$ and $\{v_{k'+j}^n; 25 \leq j \leq 32\} \subseteq S_{t'}$. It is clear that $d(v_{k'+j}^n, S_i) = 1$, $9 \leq j \leq 16$ and $d(v_{k'+8+j}^n, S_t) = d(v_{k'+17-j}^n, S_t)$, $1 \leq j \leq 4$. Also, $d(v_{k'+j}^n, v_{k'+r}^n) = d(v_{k'+j'}^n, v_{k'+r}^n)$, $9 \leq j \neq j' \leq 16$, $k' - 4 \leq r \geq k' + 33$. Further, $d(v_{k'+8+j}^n, v_{k'+r}^n) \neq d(v_{k'+17-j}^n, v_{k'+r}^n)$, $1 \leq j \leq 4$, $-3 \leq r \leq 0$, by including the vertices

$v^n_{k'}, v^n_{k'-1}, v^n_{k'-2}, v^n_{k'-3}$ into $S_{t''}$, the representation of $v^n_{k'+8+j}$ and $v^n_{k'+17-j}$ can be made distinct. In this process, we find that $V(\text{Level } n)$ is partitioned into $(n-2)$-sets.

Let $\Pi_1 = \{S_1, S_2 \ldots S_n\}$, where $S_1 = \{V(\text{Level } i);\ 0 \le i \le n-2\}$, $S_2 = \{V(\text{Level } n-1)\}$, $S_3 = \{v^n_1, v^n_2, v^n_3, v^n_4\}$, $S_4 = \{v^n_{16i-12+k}\}$ for $1 \le i \le 2^{n-4}$ and $1 \le k \le 8$, $S_5 = \{v^n_{(16i-13)4+k}, v^n_{(16i-9)4+k}, v^n_{(16i-5)4+k}\}$ for $1 \le i \le 2^{n-6}$ and $1 \le k \le 8$. For $6 \le i \le n-1$ and $1 \le k \le 2^{(n-1)-i}$, $S_i = \{v_{2^i(2k-1)-(2+m)^n};\ -6 \le m \le 1\}$, $S_n = \{v^n_{2^n-3}, v^n_{2^n-2}, v^n_{2^n-1}, v^n_{2^n}\}$. It is clear that for each vertex in $X(n)$ has distinct representation with respect to Π_1 and Π_1 is a resolving partition.

Suppose $X_i = S_i \cup S_j$, $2 \le i \ne j \le n$, then $\Pi_1 = n-1$. In this case, there are two possibilities. First, if $S_2 \cup S_j$, $j \ge 3$, then this possibility is ruled out by our above discussion. Secondly, if $S_i \cup S_j$, $3 \le i \ne j \le n$, then the number of sets partitioned from the vertices of Level n might be reduced to $n-3$, which is not possible by our above discussion.

Let $\Pi = \{S_1, S_2 \ldots S_{n-1}\}$, where $S_1 = \{V(\text{Level } i);\ 0 \le i \le n-1\}$, $S_2 = \{v^n_1, v^n_2, v^n_3, v^n_4\}$, $S_3 = \{v^n_{16i-12+k}\}$ for $1 \le i \le 2^{n-4}$ and $1 \le k \le 8$, $S_4 = \{v^n_{(16i-13)4+k}, v^n_{(16i-9)4+k}, v^n_{(16i-5)4+k}\}$ for $1 \le i \le 2^{n-6}$ and $1 \le k \le 8$. For $5 \le i \le n-2$, $S_i = \{v_{2^{i+1}(2k-1)-(2+m)^n};\ -6 \le m \le 1\}$, $1 \le k \le 2^{(n-2)-i}$, $S_{n-1} = \{v^n_{2^n-3}, v^n_{2^n-2}, v^n_{2^n-1}, v^n_{2^n}\}$. Since partition of $V(\text{Level } n)$ can not be reduced to $n-3$ sets, Π is a minimum resolving partition of $X(n)$.

3. SLIM TREE

A slim tree $ST(n) = (V, E, u, l, r)$, where V is the vertex set, E is the edge set, $u \in V$ is the root vertex, $l \in V$ is the left vertex and $r \in V$ is the right vertex, $n \ge 2$ is an integer. The n^{th} slim tree $ST(n)$ is recursively defined as follows.

- **(i)** $ST(2)$ is the complete graph K_3 with its vertices labeled with u, l and r.
- **(ii)** The n^{th} slim tree $ST(n)$, $n \ge 3$ is composed of a root vertex u and two disjoint copies of $(n-1)^{th}$ slim tree as the left subtree and right subtree, denoted by $ST^l(n-1) = (V_1, E_1, u_1, l_1, r_1)$ and $ST^r(n-1) = (V_2, E_2, u_2, l_2, r_2)$, respectively, where in particular $u \in V_1 \cup V_2$. To be specific, $ST(n) = (V, E, u, l, r)$ is given by $V = V_1 \cup V_2 \cup \{u\}$, $E = E_1 \cup E_2 \cup \{(u, u_1), (u, u_2), (r_1, l_2)\}$, $l = l_1$, $r = l_2$ [7].

The left subtree $ST^l(n-1)$ and the right subtree $ST^r(n-1)$ are isomorphic. This property is referred as symmetry property of $ST(n)$. The vertices at level n are called as *leaf vertices*. The root of $ST(n)$ is considered to be at Level 0. The vertices of $ST(n)$ other than the root and the leaf vertices are called *internal vertices*. Let V_L and V_I denote the set of leaf and internal vertices of $ST(n)$ respectively. The vertices of Level i in $ST(n)$ is labeled from left to right as v^i_j, $1 \le j \le 2^i$. A slim tree of dimension 4 is shown in Figure 2.

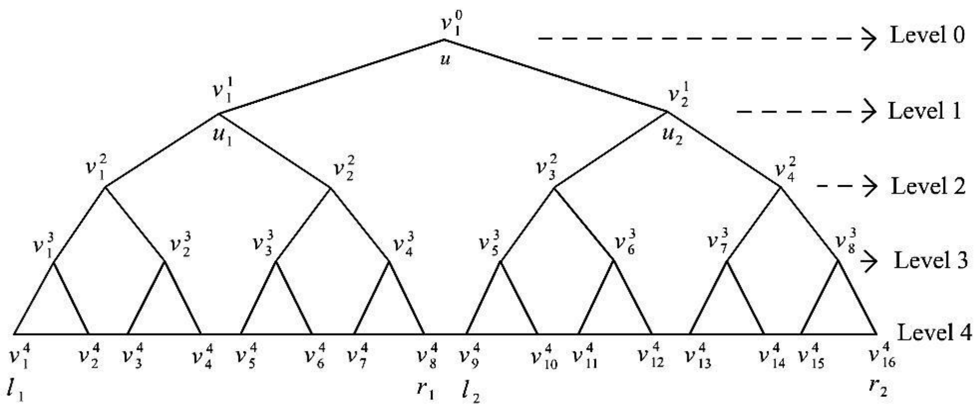

Figure 2: $ST(4)$

The partition dimension of $ST(1)$, $ST(2)$ and $ST(3)$ are 3. The minimum resolving partition of $ST(4)$ is 3 whose vertices are partitioned as $S_1 = \{x;\ x \in V_I\} \cup \{v_{13}^n, v_{14}^n, v_{15}^n, v_{16}^n\}$, $S_2 = \{v_1^n, v_2^n, v_3^n, v_4^n\}$, $S_3 = \{v_i^n;\ 5 \le i \le 12\}$.

Theorem 1. For $n \ge 4$, $pd(ST(n)) = n - 1$.
Proof. Consider the siblings v_j^l, $v_{j+1}^l \in V(\text{Level } i)$, $0 \le i \le n-1$. It is observed that $d(v_j^l, V(\text{Level } n)) = d(v_{j+1}^l, V(\text{Level } n)) = n - i$ but there exist a vertex v_m^n in Level n such that $d(v_j^l, v_m^n) \ne d(v_{j+1}^l, v_m^n)$. Suppose $\{V(\text{ Level } i);\ 0 \le i \le n-1\} = S_t$, then the distinct representation for the vertices of S_t depends on the proper partitioning of the vertices of Level n.
Let $\{V(\text{ Level } i);\ 0 \le i \le n-1\} = S_t$. Then there exist a vertex v_j^n in Level n such that $d(v_j^n, v_{j+k}^n) = k$ for $1 \le k \le 8$. For $k > 8$, there exist at least two vertices v_{j+k}^n, v_{j+k+1}^n such that $d(v_j^n, v_{j+k}^n) = d(v_j^n, v_{j+k+1}^n)$. Thus, eight vertices which are in a path of Level n can be in the same set S_i.

Let $\{v_{j+k}^n;\ 1 \le k \le 8\} \subseteq S_i$ and the vertex $v_j^n \in S_{i'}$, then $d(v_j^n, v_{j+k}^n) = k$ for $1 \le k \le 8$. Including the vertex v_{j+9}^n into $S_{i'}$, we get $d(v_j^n, v_{j+k}^n) = d(v_j^n, v_{j+9-k}^n)$ for $1 \le k \le 4$. In this case, the vertices v_{j+k}^n, v_{j+9-k}^n, $1 \le k \le 4$ get distinct representation only when there exist a vertex v_m^n such that $d(v_m^n, v_{j+k}^n) \ne d(v_m^n, v_{j+9-k}^n)$ and $v_m^n \in S_r$, $i \ne i' \ne r$.

If there exist another set of ten vertices, where $v_{j'}^n$, $v_{j'+9}^n \in S_{i'}$ and $\{v_{j'+k}^n;\ 1 \le k \le 8\} \subseteq S_i$, then the vertices $v_{j'+k}^n$ get distinct representation when $d(v_m^n, v_{j+k}^n) \ne d(v_m^n, v_{j'+k}^n)$. In case, $d(v_m^n, v_{j+k}^n) = d(v_m^n, v_{j'+k}^n)$, then there must be a vertex $v_{m'}^n$ in Level n such that $d(v_{m'}^n, v_{j+k}^n) \ne d(v_{m'}^n, v_{j'+k}^n)$ and $v_{m'}^n \in S_{r'}$, $i \ne i' \ne r \ne r'$.

Let $\{v_{j+k}^{n};\ 1 \le k \le 16\} \subseteq S_i$. For $7 \le k \le 16$, there exist at least two vertices v_{j+k}^{n}, v_{j+k+1}^{n} such that $d(v_j^n, v_{j+k}^n) = d(v_j^n, v_{j+k+1}^n)$. If $v_j^n \in S_{i'}$ and $v_{j+17}^n \in S_r$, then $d(v_{j+17}^n, v_{j+k}^n) \neq d(v_{j+17}^n, v_{j+k+1}^n)$. Thus, $V($ Level $n)$ must be partitioned into $n-1$ sets.

Let $\Pi_1 = \{S_1, S_2 \dots S_n\}$, where $S_1 = \{V(\text{Level } i);\ 0 \le i \le n-1\}$, $S_2 = \{v_{32i-(20-k)}^{n};\ 1 \le i \le 2^{n-4}, 1 \le k \le 8\}$, $S_3 = \{v_1^n, v_2^n, v_3^n, v_4^n\}$, $S_4 = \{v_{16i-12+k}\}$ for $1 \le i \le 2^{n-4}$ and $1 \le k \le 8$, For $5 \le i \le n-1$ and $1 \le k \le 2^{(n-1)-i}$, $S_i = \{v_{2^i(2k-1)-(2+m)};\ -6 \le m \le 1\}$. $S_n = \{v_{2^n-3}^n, v_{2^n-2}^n, v_{2^n-1}^n, v_{2^n}^n\}$. By the above discussion Π_1 is a resolving partition.

Let $\Pi = \{S_1, S_2 \dots S_{n-1}\}$, where $S_1 = \{V(\text{Level } i);\ 0 \le i \le n-1\} \cup \{v_{32i-(20-k)}^{n};\ 1 \le i \le 2^{n-4}, 1 \le k \le 8\}$, $S_2 = \{v_1^n, v_2^n, v_3^n, v_4^n\}$, $S_3 = \{v_{16i-12+k}\}$ for $1 \le i \le 2^{n-4}$ and $1 \le k \le 8$, For $4 \le i \le n-2$ and $1 \le k \le 2^{(n-2)-i}$, $S_i = \{v_{2^{i+1}(2k-1)-(2+m)};\ -6 \le m \le 1\}$. $S_{n-1} = \{v_{2^n-3}^n, v_{2^n-2}^n, v_{2^n-1}^n, v_{2^n}^n\}$. Consider any two vertices v_j^l, $v_{j'}^l \in S_i$, there exist a set S_r such that $d(v_j^l, S_r) \neq d(v_{j'}^l, S_r)$.

If $X_i = S_i \cup S_j$, $1 \le i \ne j \le n-1$, then $|\Pi| = n-2$ which implies that there exist two vertices in $ST(n)$ with identical representation. Thus, $\Pi = \{S_1, S_2 \dots S_{n-1}\}$ is a minimum resolving partition of $ST(n)$.

4. CONCLUSION

In this paper, we have established the exact value for partition dimension of X-tree and slim tree. This problem is open for other binary tree related architectures.

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