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Deteriorating Items Inventory Model with Different Deterioration Rates for Imperfect Quality Items and Shortages

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Abstract

Units produced or ordered are not of 100% good quality items in general. In this paper, a deterministic inventory model is developed for imperfect quality items when deterioration rate is different during a cycle. Demand is considered as linear function of time. Shortages are allowed and completely backlogged. Numerical example and sensitivity analysis is taken to support the model.

Keywords: Inventory model, Defective items, Varying Deterioration, Linear demand, Shortages, Time varying holding cost

1. INTRODUCTION:

Many existing inventory models in the literature assume that items can be stored indefinitely to meet the future demand. However, certain types of commodities either deteriorate or become obsolete in the course of time and hence are unstable. Therefore, if the rate of deterioration is not sufficiently low, its impact on modeling of such an inventory system cannot be ignored. Ghare and Schrader [4] considered inventory model with constant rate of deterioration. Covert and Philip [3] extended the model by considering variable rate of deterioration. Hariga [6] developed EOQ model for deteriorating items with log-concave time varying demand. An order level inventory model for deteriorating items with ramp type demand was developed by

Manna and Chaudhuri [9].The related works are found in (Nahmias [11], Raffat [14], Ruxian, et al [16]).

Generally traditional economic order quantity model assumes that products are all perfect. But, in reality, it happens that units ordered are not of 100% good quality. Rosenblat and Lee [15] were the first to focus on defective items. Cheng [2] developed a model of imperfect production quantity by establishing relationship between demand dependent unit production cost and imperfect production process. Salman and Jaber [17] developed an inventory model in which items received are of defective quality and after 100% screening, imperfect items are withdrawn from the inventory and sold at a discounted price. An EPQ model under simple approach to determine economic production quantity for production systems that produces imperfect quality items was developed by Goyal and Cardenas-Barron [5]. Chang [1] studied an inventory model to investigate the effects of imperfect products on the total inventory cost associated with an EPQ model. Patel and Patel [13] developed an EOQ model for deteriorating items with imperfect quantity items. An optimal inventory model for items with imperfect quality and shortages were considered by Hsu and Hsu [8]. Hauck and Voros [7] considered inventory model in which percentage of defective items as a random variable and defined the speed of the quality checking as a variable. Mukhopadyay and Goswami [10] developed an inventory model for one way substitutions of imperfect quality items to cope up with lost sales and shortages. Vishkaei et al. [18] extended the model developed by Hsu and Hsu [8] to determine the optimal order quantity of product batches that contain defective items with percentage nonconforming following a known probability density function. Patel and Sheikh [12] developed an inventory model with different deterioration rates and time varying holding cost.

In this paper we have developed an inventory model for imperfect quality items with different deterioration rates for the cycle time. Shortages are allowed. Numerical example is provided to illustrate the model. Sensitivity analysis for major parameters is also carried out.

2. ASSUMPTIONS AND NOTATIONS:

NOTATIONS:

The following notations are used for the development of the model:

D(t) : Demand rate is a linear function of time t ($a+bt$, $a>0$, $0<b<1$)

c : Purchasing cost per unit

p : Selling price per unit

d : defective items (%)

1-d : good items (%)

λ : Screening rate

SR : Sales revenue

A : Replenishment cost per order for

z : Screening cost per unit

p_d : Price of defective items per unit

h(t) : Variable Holding cost (x + yt)

c_2 : Shortage cost per unit

t_1 : Screening time

T : Length of inventory cycle

I(t) : Inventory level at any instant of time t, $0 \leq t \leq T$

Q_1 : Order quantity initially

Q_2 : Quantity of shortages

Q : Order quantity

θ : Deterioration rate during $\mu_1 \leq t \leq \mu_2$, $0< \theta_1<1$

θt : Deterioration rate during , $\mu_2 \leq t \leq T$, $0< \theta_2<1$

π : Total relevant profit per unit time.

ASSUMPTIONS:

The following assumptions are considered for the development of the model.

- The demand of the product is declining as a linear function of time.
- Replenishment rate is infinite and instantaneous.
- Lead time is zero.
- Shortages are allowed and completely backlogged.
- The screening process and demand proceeds simultaneously but screening rate (λ) is greater than the demand rate i.e. $\lambda > (a+bt)$.
- The defective items are independent of deterioration.
- Deteriorated units can neither be repaired nor replaced during the cycle time.
- A single product is considered.
- The screening rate (λ) is sufficiently large. In general, this assumption should be acceptable since the automatic screening machine usually takes only little time to inspect all items produced or purchased.

3. THE MATHEMATICAL MODEL AND ANALYSIS:

At time t=0, a lot size of Q units enters the system. Each lot having a d % defective items. The nature of the inventory level is shown in the given figure, where screening process is done for all the received quantity at the rate of λ units per unit time which is greater than demand rate. After screening, a portion is used to meet the backlogging items towards previous shortages and initial inventory for period is Q_1. During the screening process the demand occurs parallel to the screening process and is fulfilled from goods which are found to be of perfect quality by the screening process. The defective items are sold immediately after the screening process at time t_1 as a single batch at a discounted price. After the screening process at time t_1 the inventory level will be $I(t_1)$ and at time t_0, inventory level will become zero due to demand and partially due to deterioration. Shortages occur during the period t_0 to T and of size Q_2.

Also here $t_1 = \frac{Q}{\lambda}$ (1)

and defective percentage (d) is restricted to $d \le 1 - \frac{(x+yt)}{\lambda}$ (2)

Let I(t) be the inventory at time t ($0 \le t \le T$) as shown in figure.

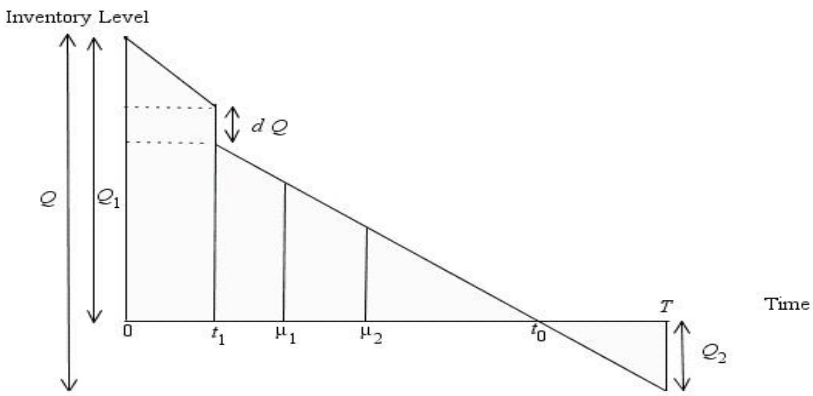

Figure 1

The differential equations which describes the instantaneous states of I(t) over the period (0, T) is given by

$$\frac{dI(t)}{dt} = -(a + bt), \qquad 0 \le t \le \mu_1 \tag{3}$$

$$\frac{dI(t)}{dt} + \theta I(t) = -(a + bt), \qquad \mu_1 \le t \le \mu_2 \tag{4}$$

$$\frac{dI(t)}{dt} + \theta t I(t) = -(a + bt), \qquad \mu_2 \le t \le t_0 \tag{5}$$

$$\frac{dI(t)}{dt} = -(a + bt), \qquad t_0 \le t \le T \tag{6}$$

with initial conditions $I(0) = Q_1$, $I(\mu_1) = S_1$, $I(t_0) = 0$ and $I(T) = -Q_2$.

Solutions of these equations are given by

$$I(t) = Q_1 - (at + \frac{1}{2}bt^2), \tag{7}$$

$$I(t) = \begin{bmatrix} a(\mu_1 - t) + \frac{1}{2}b(\mu_1^2 - t^2) + \frac{1}{2}a\theta(\mu_1^2 - t^2) \\ + \frac{1}{3}b\theta(\mu_1^3 - t^3) - a\theta t(\mu_1 - t) - \frac{1}{2}b\theta t(\mu_1^2 - t^2) \end{bmatrix} + S_1[1 + \theta(\mu_1 - t)] \tag{8}$$

$$I(t) = \begin{bmatrix} a(t_0 - t) + \frac{1}{2}b(t_0^2 - t^2) + \frac{1}{6}a\theta(t_0^3 - t^3) \\ + \frac{1}{8}b\theta(t_0^4 - t^4) - \frac{1}{2}a\theta t^2(t_0 - t) - \frac{1}{4}b\theta t^2(t_0^2 - t^2) \end{bmatrix}. \tag{9}$$

$$I(t) = \left[-at - \frac{1}{2}bt^2 + at_0 + \frac{1}{2}bt_0^2 \right]. \tag{10}$$

(by neglecting higher powers of θ)

After screening process, the number of defective items at time t_1 is dQ.

So effective inventory level during $t_1 \le t \le T$ is given by

$$I(t) = -(at + \frac{1}{2}bt^2) + Q_1 - Qd. \tag{11}$$

From equation (7), putting $t = \mu_1$, we have

$$Q_1 = S_1 + \left(a\mu_1 + \frac{1}{2}b\mu_1^2 \right). \tag{12}$$

From equations (8) and (9), putting $t = \mu_2$, we have

$$I(\mu_2) = \begin{bmatrix} a(\mu_1 - \mu_2) + \frac{1}{2}b(\mu_1^2 - \mu_2^2) + \frac{1}{2}a\theta(\mu_1^2 - \mu_2^2) \\ + \frac{1}{3}b\theta(\mu_1^3 - \mu_2^3) - a\theta\mu_2(\mu_1 - \mu_2) - \frac{1}{2}b\theta\mu_2(\mu_1^2 - \mu_2^2) \end{bmatrix} + S_1[1 + \theta(\mu_1 - \mu_2)] \tag{13}$$

$$I(\mu_2) = \left[\begin{array}{l} a\left(t_0 - \mu_2\right) + \frac{1}{2}b\left(t_0^2 - \mu_2^2\right) + \frac{1}{6}a\theta\left(t_0^3 - \mu_2^3\right) \\ + \frac{1}{8}b\theta\left(t_0^4 - \mu_2^4\right) - \frac{1}{2}a\theta\mu_2^2\left(t_0 - \mu_2\right) - \frac{1}{4}b\theta\mu_2^2\left(t_0^2 - \mu_2^2\right) \end{array}\right]. \tag{14}$$

So from equations (13) and (14), we get

$$S_1 = \frac{1}{\left[1+\theta\left(\mu_1 - \mu_2\right)\right]}\left[\begin{array}{l} a\left(t_0 - \mu_2\right) + \frac{1}{2}b\left(t_0^2 - \mu_2^2\right) + \frac{1}{6}a\theta\left(t_0^3 - \mu_2^3\right) \\ + \frac{1}{8}b\theta\left(t_0^4 - \mu_2^4\right) - \frac{1}{2}a\theta\mu_2^2\left(t_0 - \mu_2\right) - \frac{1}{4}b\theta\mu_2^2\left(t_0^2 - \mu_2^2\right) \\ - a\left(\mu_1 - \mu_2\right) - \frac{1}{2}b\left(\mu_1^2 - \mu_2^2\right) - \frac{1}{2}a\theta\left(\mu_1^2 - \mu_2^2\right) \\ - \frac{1}{3}b\theta\left(\mu_1^3 - \mu_2^3\right) + a\theta\mu_2\left(\mu_1 - \mu_2\right) + \frac{1}{2}b\theta\mu_2\left(\mu_1^2 - \mu_2^2\right) \end{array}\right] \tag{15}$$

Putting value of S_1 from equation (15) into equation (12), we have

$$Q_1 = \frac{1}{\left[1+\theta\left(\mu_1-\mu_2\right)\right]}\left[\begin{array}{l} a\left(t_0 - \mu_2\right) + \frac{1}{2}b\left(t_0^2 - \mu_2^2\right) + \frac{1}{6}a\theta\left(t_0^3 - \mu_2^3\right) \\ +\frac{1}{8}b\theta\left(t_0^4-\mu_2^4\right)-\frac{1}{2}a\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right) \\ -a\left(\mu_1-\mu_2\right)-\frac{1}{2}b\left(\mu_1^2 - \mu_2^2\right) - \frac{1}{2}a\theta\left(\mu_1^2 - \mu_2^2\right) \\ -\frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)+a\theta\mu_2\left(\mu_1-\mu_2\right)+\frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right) \end{array}\right] + \left(a\mu_1+\frac{1}{2}b\mu_1^2\right). \tag{16}$$

Using (16) in (7), we have

$$I(t) = \frac{1}{\left[1+\theta\left(\mu_1-\mu_2\right)\right]}\left[\begin{array}{l} a\left(t_0-\mu_2\right) + \frac{1}{2}b\left(t_0^2 - \mu_2^2\right) + \frac{1}{6}a\theta\left(t_0^3 - \mu_2^3\right) \\ +\frac{1}{8}b\theta\left(t_0^4-\mu_2^4\right)-\frac{1}{2}a\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right) \\ -a\left(\mu_1-\mu_2\right)-\frac{1}{2}b\left(\mu_1^2 - \mu_2^2\right) - \frac{1}{2}a\theta\left(\mu_1^2 - \mu_2^2\right) \\ -\frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)+a\theta\mu_2\left(\mu_1-\mu_2\right)+\frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right) \end{array}\right] + a\left(\mu_1-t\right)+\frac{1}{2}b\left(\mu_1^2-t^2\right). \tag{17}$$

Similarly, using (15) in (8), we have

$$I(t) = \frac{\left[1+\theta\left(\mu_1 - t\right)\right]}{\left[1+\theta\left(\mu_1 - \mu_2\right)\right]}\left[\begin{array}{l} a\left(t_0 - \mu_2\right) + \frac{1}{2}b\left(t_0^2 - \mu_2^2\right) + \frac{1}{6}a\theta\left(t_0^3 - \mu_2^3\right) \\ + \frac{1}{8}b\theta\left(t_0^4 - \mu_2^4\right) - \frac{1}{2}a\theta\mu_2^2\left(t_0 - \mu_2\right) - \frac{1}{4}b\theta\mu_2^2\left(t_0^2 - \mu_2^2\right) \\ - a\left(\mu_1 - \mu_2\right) - \frac{1}{2}b\left(\mu_1^2 - \mu_2^2\right) - \frac{1}{2}a\theta\left(\mu_1^2 - \mu_2^2\right) \\ - \frac{1}{3}b\theta\left(\mu_1^3 - \mu_2^3\right) + a\theta\mu_2\left(\mu_1 - \mu_2\right) + \frac{1}{2}b\theta\mu_2\left(\mu_1^2 - \mu_2^2\right) \end{array}\right] + \left[\begin{array}{l} a\left(\mu_1 - t\right) + \frac{1}{2}b\left(\mu_1^2 - t^2\right) + \frac{1}{2}a\theta\left(\mu_1^2 - t^2\right) \\ + \frac{1}{3}b\theta\left(\mu_1^3 - t^3\right) - a\theta t\left(\mu_1 - t\right) - \frac{1}{2}b\theta t\left(\mu_1^2 - t^2\right) \end{array}\right] \quad (18)$$

Putting t = T in equation (10), we have

$$Q_2 = \left[aT + \frac{1}{2}bT^2 - at_0 - \frac{1}{2}bt_0^2\right]. \quad (19)$$

Putting values of Q_1 and Q_2 from equations (16) and (19), we get Q.

Similarly putting values of Q_1 and Q in equation (11), we get

$$I(t) = \frac{(1-d)}{\left[1+\theta\left(\mu_1-\mu_2\right)\right]}\left[\begin{array}{l} a\left(t_0 - \mu_2\right) + \frac{1}{2}b\left(t_0^2 - \mu_2^2\right) + \frac{1}{6}a\theta\left(t_0^3 - \mu_2^3\right) \\ +\frac{1}{8}b\theta\left(t_0^4-\mu_2^4\right)-\frac{1}{2}a\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right) \\ -a\left(\mu_1-\mu_2\right)-\frac{1}{2}b\left(\mu_1^2 - \mu_2^2\right) - \frac{1}{2}a\theta\left(\mu_1^2 - \mu_2^2\right) \\ -\frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)+a\theta\mu_2\left(\mu_1-\mu_2\right)+\frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right) \end{array}\right] + (1-d)\left(a\mu_1+\frac{1}{2}b\mu_1^2\right) - d\left[aT + \frac{1}{2}bT^2 - at_0 - \frac{1}{2}bt_0^2\right] - \left(at + \frac{1}{2}bt^2\right) \quad (20)$$

Based on the assumptions and descriptions of the model, the total annual relevant profit (μ), include the following elements:

(i) Ordering cost (OC) = A (21)

(ii) Screening cost (SrC) = zQ (22)

(iii) $HC = \int_0^{t_0} (x+yt)I(t)dt$

$$= \int_0^{t_1} (x+yt)I(t)dt + \int_{t_1}^{\mu_1} (x+yt)I(t)dt + \int_{\mu_1}^{\mu_2} (x+yt)I(t)dt + \int_{\mu_2}^{t_0} (x+yt)I(t)dt \quad (23)$$

(iv) $DC = c\left(\int_{\mu_1}^{\mu_2} \theta I(t)dt + \int_{\mu_2}^{t_0} \theta tI(t)dt\right)$ (24)

(v) Shortage cost (SC) is given by

$$SC = -c_2\left(\int_{t_0}^{T} I(t)dt\right) \quad (25)$$

(vi) $SR = \left(p\int_0^T (a+bt)dt + p_d dQ\right)$ (26)

The total profit (π) during a cycle consisted of the following:

$$\pi = \frac{1}{T}\left[SR - OC - PC - SrC - HC - DC\right] \quad (27)$$

Substituting values from equations (21) to (26) in equation (27), we get total profit per unit. Putting $\mu_1 = v_1T$ and $\mu_2 = v_2T$ and value of t_1 and Q in equation (27), we get profit in terms of t_0 and T. The optimal value of $t_0 = t_0^*$ and $T = T^*$ (say), which maximizes profit $\pi(t_0,T)$ can be obtained by differentiating it with respect to t_0 and T and equate it to zero

$$\text{i.e. } \frac{\partial \pi(t_0, T)}{\partial t_0} = 0, \ \frac{\partial \pi(t_0, T)}{\partial T} = 0 \quad (28)$$

provided it satisfies the condition

$$\begin{vmatrix} \frac{\partial^2 \pi(t_0, T)}{\partial t_0^2} & \frac{\partial^2 \pi(t_0, T)}{\partial t_0 \partial T} \\ \frac{\partial^2 \pi(t_0, T)}{\partial T \partial t_0} & \frac{d^2 \pi(t_0, T)}{\partial T^2} \end{vmatrix} > 0. \quad (29)$$

4. NUMERICAL EXAMPLE:

Considering A= Rs.100, a = 500, b=0.05, c=Rs. 25, p= Rs. 40, p_d = 15, d= 0.02, z = 0.40, λ= 10000, θ=0.05, x = Rs. 5, y=0.05, c_2= Rs. 8, v_1=0.30, v_2 = 0.50, in appropriate units. The optimal value of t_0* = 0.2122, T* =0.3536, Profit*= Rs. 19386.2460 and optimum order quantity Q* = 176.9584.

The second order conditions given in equation (30) are also satisfied. The graphical representation of the concavity of the profit function is also given.

t_0 and Profit

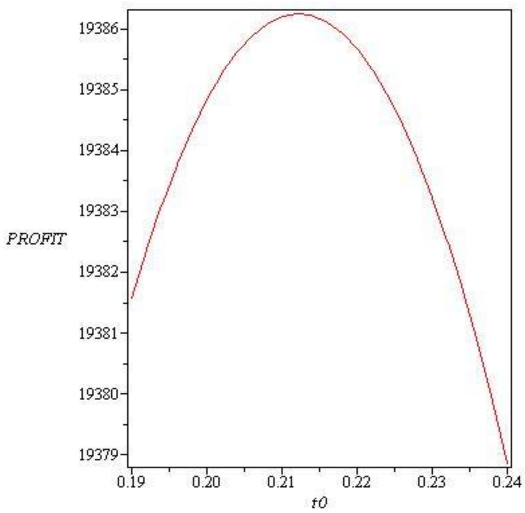

Figure 1

T and Profit

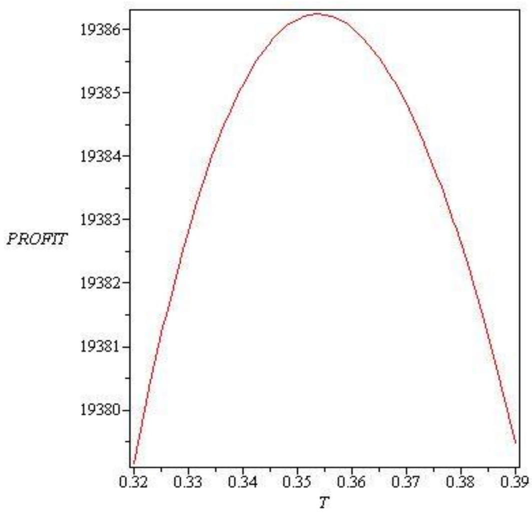

Figure 2

5. SENSITIVITY ANALYSIS:

On the basis of the data given in example above we have studied the sensitivity analysis by changing the following parameters one at a time and keeping the rest fixed.

Table 1: Sensitivity Analysis

Parameter	%	t_0	T	Profit	Q
a	+20%	0.1938	0.3229	23322.3513	193.8963
	+10%	0.2023	0.3372	21353.6910	185.6172
	-10%	0.2235	0.3727	17420.2048	167.8746
	- 20%	0.2370	0.3952	15455.8101	158.2413

x	+20%	0.1883	0.3369	19357.7051	168.5733
	+10%	0.1995	0.3447	19371.2954	172.4892
	-10%	0.2267	0.3641	19402.7877	182.2322
	- 20%	0.2437	0.3764	19421.2114	188.4124
θ	+20%	0.2100	0.3521	19384.0195	176.2355
	+10%	0.2111	0.3528	19385.1281	176.5721
	-10%	0.2132	0.3544	19387.3732	177.3443
	- 20%	0.2143	0.3552	19388.5100	177.7300
A	+20%	0.2321	0.3871	19332.2513	193.7418
	+10%	0.2224	0.3708	19358.6382	185.5751
	-10%	2014	0.3356	19415.2630	167.9418
	- 20%	0.1900	0.3165	19445.9317	158.3753
λ	+20%	0.2122	0.3537	19386.3936	177.0084
	+10%	0.2122	0.3537	19386.3265	177.0084
	-10%	0.2121	0.3535	19386.1476	176.9083
	- 20%	0.2121	0.3535	19386.0245	176.9082
c_2	+20%	0.2194	0.3414	19366.3738	170.8697
	+10%	0.2160	0.3470	19375.6647	173.6643
	-10%	0.2077	0.3615	19398.4137	180.9016
	- 20%	0.2026	0.3711	19412.5649	185.6942

From the table we observe that as parameter a increases/ decreases average total profit and optimum order quantity also increases/ decreases.

Also, we observe that with increase and decrease in the value of θ, x and c_2, there is corresponding decrease/ increase in total profit and optimum order quantity.

From the table we observe that as parameter A increases/ decreases average total profit decreases/ increases and optimum order quantity increases/ decreases.

Also, we observe that with increase and decrease in the value of λ, there is almost no change in average total profit and optimum order quantity.

6. CONCLUSION:

In this paper, we have developed an inventory model for deteriorating items with linear demand with different deterioration rates. Sensitivity with respect to parameters have been carried out. The results show that with the increase/ decrease in the parameter values there is corresponding increase/ decrease in the value of profit.

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