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Iterative Method For Approximating a Common Fixed Point of Infinite Family of Strictly Pseudo Contractive Mappings in Real Hilbert Spaces

Mollalgn Haile Takele[1*] and B. Krishna Reddy[2]
[1]Department of Mathematics, College of science Osmania University, Hyderabad, India & Bahir Dar University, Bahir Dar, Ethiopia

**Corresponding Author*

[2]Department of Mathematics, University College of Science Osmania University, Hyderabad, India

Abstract

Let $\{T_n\}_{n=1}^{\infty} : K \to H$ be a countable infinite family of k-strictly Pseudo contractive, uniformly weakly closed and inward mappings on a non empty, closed and strictly convex subset K of a real Hilbert space H in to H with $F = \bigcap_{k=1}^{\infty} F(T_k)$ is non empty. Let $\alpha \in (k,1)$ and for each n, $h_n : K \to \Re$ be defined by $h_n(x) = \inf\{\lambda \geq 0 : \lambda x + (1-\lambda)T_{n_\alpha} x \in K\}$.Then for each $x_1 \in K$, $\beta_1 = \max\{\beta, h_1(x_1)\}, \beta > k$, we define the Krasnoselskii-Mann type algorithm by $x_{n+1} = \beta_n x_n + (1-\beta_n)T_{n_\alpha} x_n$, where $\beta_{n+1} = \max\{\beta_n, h_{n+1}(x_{n+1})\}, n = 1,2,...$ and we prove the weak and strong convergence of the sequence $\{x_n\}$ to a common fixed point of the family $\{T_n\}_{n=1}^{\infty}$. Also we prove the weak and strong convergence theorems for the algorithm to the family of nonexpansive mappings in uniformly convex Banach space, which is more general than Hilbert space.

Keywords and phrases: Common fixed point; strictly Pseudo contractive mapping; nonself mapping; Krasnoselskii-Mann's iterative method; infinite family of mappings.

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1. INTRODUCTION

Finding fixed point or common fixed point (if it exists) is important in the study of many real world problems, such as; inverse problems; the split feasibility problem and the convex feasibility problem in signal processing and image reconstruction can both be formulated as a problem of finding fixed points of certain operators(mappings), respectively. In particular, k-strictly pseudo contractive mappings are more applicable than nonexpansive mappings in solving various problems .Therefore, it is desirable to develop the algorithms for the class of strictly pseudo contractive mappings, which is an intermediate between the class of non expansive mappings and that of the class of pseudo contractive mappings in which the exact solution of the nonlinear problem may not be possible.

The class of κ-strictly pseudo contractive mappings was first introduced by Browder and Petryshyn in Hilbert spaces (see, for example [1]). Since then many research efforts have been made for the study of fixed point and common fixed point for family of such mappings.

Here, we study the fixed point iterative method for approximating a common fixed point of countable infinite family of k-strictly pseudo contractive mappings in Hilbert space setting and its extensions. Let K be a nonempty, closed and convex subset of a Hilbert space H and let $T: K \to H$ be a mapping. Then T is said to be nonexpansive if $\|Tx - Ty\| \leq \|x - y\|$ for arbitrary $x, y \in K$.

Whereas $T: K \to H$ is called *k*-strictly pseudo contractive if there exists $k \in [0,1)$ such that

$$\|Tx - Ty\|^2 \leq \|x - y\|^2 + k\|(I - T)x - (I - T)y\|^2 \; x, y \in K. \tag{1.1}$$

We see that every nonexpansive mapping is strictly pseudo contractive, hence the class of k-strictly pseudo contractive mappings is more general than the class of nonexpansive mappings.

If the fixed point set $F(T) = \{x \in K : Tx = x\}$ is nonempty and T is self ($T: K \to K$) and nonexpansive mapping, Mann in [9] introduced an iterative method of the form

$$x_{n+1} = \alpha_n x_n + (1 - \alpha_n) T x_n \text{ for any } x_1 \in K \text{ and } \{\alpha_n\} \subset [0,1) \tag{1.2}$$

Since then, a number of extensive research works have been made (See, for example [3&6] and their references). Reich in [12] studied the weak convergence of the Mann's algorithm in [3].Several attempts have been made in lowering the requirement for the mapping to be self-mapping by assuming T to be non-self at the cost of additional requirements on the sequence $\{\alpha_n\}$and on the domain K (see, for example, [10,13,15,18,19&21] and their references). However the study was using the calculation of metric projection $P: H \to K$ which is costly and in many cases it requires approximation technique. However, Colao and Marino in [4] introduced a new technique for the coefficients $\{\alpha_n\}$ and they have proved that the Krasnoselskii–Mann algorithm (1.2) is well-defined for choice of the sequence $\{\alpha_n\}$. They also have proved both weak and strong convergence results for the algorithm (1.2) when *K* is a strictly convex subset of H and T is inward. To be precise, we put their result as follow.

They define inward mapping as;

Definition 1.1 A mapping $T: K \to H$ is said to be inward (or to satisfy the inward condition) if for any $x \in K$, $Tx \in IK(x) = \{x + c(u - x) : c \geq 1 \,\&\, u \in K\}$ and T is said to satisfy weakly inward condition if $Tx \in \overline{IK(x)}$ (*the closure of* $IK(x)$.

Theorem 1.1 CM [13]) Let K be a convex, closed and nonempty subset of a Hilbert space H and $T: K \to H$ be a mapping and let for any given $x \in K$, $h: K \to \Re$ be defined by $h(x) = \inf\{\lambda \geq 0 : \lambda x + (1-\lambda)Tx \in K\}$.Then the algorithm defined by

$$\begin{cases} x_0 \in K \\ \alpha_0 = \max\left\{\frac{1}{2}, h(x_0)\right\}, \\ x_{n+1} = \alpha_n x_n + (1-\alpha_n)Tx_n \\ \alpha_{n+1} = \max\{\alpha_n, h(x_{n+1})\} \end{cases}$$

is well-defined and assume that; K is strictly convex set ,T is nonexpansive, nonself and Inward mapping with F(T) is non empty , then $\{x_n\}$ converges weakly to $p \in F = F(T)$. Moreover, if $\sum_{n=1}^{\infty}(1-\alpha_n) < \infty$, then the convergence is strong.

Meanwhile, they proposed an open question to approximation for a common fixed point of a countable family of nonself and nonexpansive mappings.

Motivated by the work of Colao and Marino and their open question, several attempts have been made to generalize the theorem of CM to approximation for finite or infinite family of nonself and nonexpansive mappings (see for example ,[8 and 16] respectively and their references) . To mention a few, Haile and Reddy in [8] constructed Krasnoselskii–Mann type algorithm and prove weak and strong convergence theorems for approximating a common fixed point of the finite family of nonself, nonexpansive and inward mappings. Moreover, it was earlier Gao ***et al*** in [16] constructed Krasnoselskii–Mann type algorithm for approximating a common fixed point of countable infinite family of nonself, nonexpansive and inward mappings. They also proved weak and strong convergence theorem by imposing additional conditions such as;

(i) $\{T_n\}_{n=1}^{\infty} : K \to H$ to be uniformly weakly closed (ii) The pair (F, K) satisfies S-condition.

where the conditions (i) and (ii) can be redefined as;

Definition 1.2 Let *F*, *K* be two closed and convex nonempty sets in a Hilbert space *H* and $F \subset K$. For any sequence $\{x_n\} \subset K$, if $\{x_n\}$ converges strongly to an element $x \in K \setminus F, x_n \neq x$, implies that $\{x_n\}$ is not Fejer-monotone with respect to the set $F \subset K$, we called that, the pair (*F*, *K*) satisfies *S*-condition.

Definition 1.3 Let $\{T_n\}_{n=1}^{\infty} : K \to H$ be sequence of mappings with nonempty common fixed point set $F = \bigcap_{n=1}^{\infty} F(T_n)$. The the family $\{T_n\}_{n=1}^{\infty}$ is said to be uniformly weakly closed if for any convergent sequence $\{x_n\} \subset K$ such that $\lim_{n\to\infty}\|x_n - T_n x_n\| = 0$ the weak cluster Points of $\{x_n\} \subset K$ belong to *F*.

To be precise we put the result of Guo ***et al*** in [16] in the following theorem.

Theorem 1.2 GLY Let K be a convex, closed and nonempty subset of a Hilbert space H and let $\{T_n\}_{n=1}^{\infty} : K \to H$ be a uniformly weakly closed countable family of non self nonexpansive

Mappings. For any $x \in K$, $h_n : K \to \Re$ be defined by $h_n(x) = \inf\{\lambda \geq 0 : \lambda x + (1-\lambda)T_n x \in K\}$. Then, the sequence $\{x_n\}$ define by the algorithm,

$$\begin{cases} x_1 \in K \\ \alpha_1 = \max\left\{\frac{1}{2}, h_1(x_1)\right\}, \\ x_{n+1} = \alpha_n x_n + (1-\alpha_n)T_n x_n; \\ \alpha_{n+1} = \max\{\alpha_n, h_{n+1}(x_{n+1})\}. \end{cases}$$

is well-defined. Let that K be strictly convex and each T_n satisfies the inward condition and such that $F = \bigcap_{n=1}^{\infty} F(T_n)$ is nonempty. Then if there exist $a, b \in (0,1)$ such that $\{\alpha_n\} \subset [a,b]$ for all $n \geq 1$, the $\{x_n\}$ weakly converges to a common fixed point $p \in F$. Moreover, if $\sum_{n=1}^{\infty}(1-\alpha_n) < \infty$ and (F, K) satisfies S-condition, the convergence is strong.

We raise open question that, is it possible to extend the theorem of GLY for the class of k-strictly pseudo contractive mappings which is more general class than that of the class of nonexpansive mappings which has not been studied?. Approximating a common fixed point for the class of k-strictly strictly pseudo contractive mappings has been extensively studied for finite and infinite family as well. (See, for example [2,7, 8,11,14&20] and their references). However, all the studies was for self mappings. Consequently, Haile and Reddy in [17] constructed Krasnoselskii–Mann type algorithm for approximating a common fixed point for finite family of k-strictly pseudo contractive mappings. They also proved weak and strong convergence theorem and proposed an open question for the possibility of approximating common fixed point for the countable infinite family.

Thus, it is the purpose of this paper to approximate a common fixed point for the countable infinite family of k-strictly pseudo contractive, nonself, inward mappings in Hilbert spaces which is a positive answer to our question.

2. PRELIMINARY CONCEPTS

Let K be a non empty subset of a real Hilbert space and $\{T_k\}_{k=1}^{\infty} : K \to H$ be family of mappings, then we shall need the following assumptions**;**

Lemma 2.1 ***(***See, for example [4] lemma 3.1& [16] lemma 3.1) Let for each $k \in \{1,2,...\}$,$T_K : K \to H$ be non self mappings. If we define $h_k : C \to \Re$ by $h_k(x) = \inf\{\lambda \in [0,1] : \lambda x + (1-\lambda)T_k x \in K\}$, then

a) for any $x \in K, h_k(x) \in [0,1]$ and $h_k(x) = 0$ if and only if $T_k(x) \in K$;
b) for any $x \in K$ and $\alpha_k \in [h_k(x),1], \alpha_k x + (1-\alpha_k)T_k(x) \in K$;
c) If T_k is inward mapping ,then $h_k(x) < 1$ for any $x \in K$;
d) If $T_k x \notin K$, then $h_k(x)x + (1-h_k(x))T_k x \in \partial K$

Lemma 2.2 (see, for example, Reich [12]) Let $\{x_n\},\{y_n\}$ in a uniformly convex Banach space E be two sequences, if there exists a constant $r \geq 0$ such that $\limsup_{n\to\infty}\|x_n\| \leq r$, $\limsup_{n\to\infty}\|y_n\| \leq r$ and $\lim_{n\to\infty}\|\lambda_n x_n + (1-\lambda_n)y_n\| = r$, where $\{\lambda_n\} \subset [\varepsilon, 1-\varepsilon] \subset (0,1)$ for some $\varepsilon \in (0,1)$, then $\|x_n - y_n\| \to 0$.

Lemma 2.3 (Extension of lemma 2.1 for nonself mapping [17]). Let K be a closed and convex subset of a Hilbert space H. Let $T : K \to H$ be a mapping on K. Then

if T is a κ-strict-pseudo contractive, then T satisfies the Lipschitz condition

$$\|Tx - Ty\| \leq \frac{k+1}{1-k}\|x - y\|, \text{ for all } x, y \in K.$$

Lemma 2.4 (See, for example [17]) Let $T : K \to H$ be k-strictly pseudo contractive for some $k \in (0,1)$ and $\alpha \in (k,1)$, then $T_\alpha : K \to H$ defined by $T_\alpha x = \alpha x + (1-\alpha)Tx$ is non expansive and $F(T_\alpha) = F(T)$.

Definition 2.1 (see, for example [3] pp 61) A uniformly convex space E is a normed space E for which for every $0<\varepsilon<2$, there is a $\delta>0$, such that for every $x,y\in S=\{x\in E:\|x\|=1\}$, if $\|x-y\|>\varepsilon(x\neq y)$, then $\left\|\frac{x+y}{2}\right\|\leq 1-\delta$ .For each $x,y\in E$ the modulus of convexity of E is defined by $\delta_E(t)=\inf\left\{1-\frac{\|x+y\|}{2},\|x\|=\|y\|=1\,\&\,\|x-y\|=t\right\},0\leq t\leq 2$ and E is said to be uniformly convex if $\delta_E(t)>0$ for all $0<t\leq 2$.

Hilbert spaces, the Lebesgue L_p, the sequence l_p, for $p\in(1,\infty)$ are examples of uniformly convex Banach spaces. For $p\neq 2$ L_p and l_p are not Hilbert spaces.

3. MAIN RESULT

Let $\{T_k\}_{k=1}^{\infty}:K\to H$ be k-strictly pseudo contractive, nonself, inward mappings, then our objective is to construct iterative method for approximating a common fixed point of the family. We will have the following main theorem.

Lemma 3.1 Let $\{T_k\}_{k=1}^{\infty}:K\to H$ be uniformly weakly closed. If for each $k\in\{1,2,...\}$ and $\alpha\in(0,1)$, $T_{k\alpha}$ is defined by $T_{k\alpha}=\alpha+(1-\alpha)$, then $\{T_{k\alpha}\}_{k=1}^{\infty}$ is uniformly weakly closed.

Proof: Suppose $\{T_k\}_{k=1}^{\infty}:K\to H$ is uniformly weakly closed.

Thus, for any sequence $\{x_n\}$ in K such that $x_n\to x$ weakly and $\|x_n-T_nx_n\|\to 0$ strongly, then $x\in F=\bigcap_{n=1}^{\infty}F(T_n)$.

Suppose $\|x_n-T_{n\alpha}x_n\|\to 0$ strongly, then, since

$$\begin{aligned}\|x_n-T_{n\alpha}x_n\|&=\|x_n-(\alpha+(1-\alpha)T_n)x_n\|\\&=(1-\alpha)\|x_n-T_nx_n\|\end{aligned}$$

and $\alpha\in(0,1)$ we have $\|x_n-T_nx_n\|\to 0$ strongly, hence $x\in F=\bigcap_{n=1}^{\infty}F(T_n)$.

Since, $F(T_n)=F(T_{n\alpha})$, we have $x\in F=\bigcap_{n=1}^{\infty}F(T_{n\alpha})$.

This completes the proof of the lemma.

Theorem. 3.2 Let K be a non empty, closed and convex subset of a real Hilbert space H and let $\{T_i\}_{i=1}^{\infty} : K \to H$ be a uniformly weakly closed countable family of nonself ,k-strictly pseudo contractive and inward mappings with $F = \bigcap_{i=1}^{\infty} F(T_i)$ is non empty. Let $\alpha \in (k,1)$ and for $i = 1,2,...,$ let $T_{i\alpha} = \alpha + (1-\alpha)T_i$ and we define $h_i(x) = \inf\{\lambda \geq 0 : \lambda x + (1-\lambda)T_{i\alpha}\, x \in K\}$. Let $F = \bigcap_{i=1}^{N} F(T_i)$ is nonempty. Then the sequence $\{x_n\}$ given by

$$\begin{cases} x_1 \in K, \beta > k \\ \beta_1 = \max\{\beta, h_1(x_1)\}, \\ x_{n+1} = \beta_n x_n + (1-\beta_n) T_{\alpha_n} x_n \\ \beta_{n+1} = \max\{\beta_n, h_{n+}(x_{n+1})\} \end{cases}$$

is well-defined and if $\{\beta_n\} \subset [\varepsilon, 1-\varepsilon] \subset (0,1)$ for some $\varepsilon \in (0,1)$ $\{x_n\}$ converges weakly to some element p of $F = \bigcap_{k=1}^{N} F(T_k)$. Moreover, if $\sum_{n=1}^{`}(1-\beta_n) < \infty$ and (F, K) satisfies S-condition, then the convergence is strong.

Proof: For each $\alpha \in (0,1)$, and for each n, T_n is k- strictly pseudo contractive, inward mapping, then by lemma 3.6 and theorem 3.7 in [17] $T_{n\alpha} = \alpha + (1-\alpha)T_n$ is inward and nonexpansive mappings.

Thus $\{T_{k\alpha}\}_{k=1}^{\infty}$ is uniformly weakly closed, nonself, non expansive and inward mappings.

Hence by theorem 1.2 in [20] and $F = \bigcap_{n=1}^{\infty} F(T_{n\alpha}) = \bigcap_{n=1}^{\infty} F(T_n)$, we complete the proof.

The result can be extended in to more general spaces such as real uniformly convex Banach spaces with the assumptions of opial's condition ;

Let K be a non empty subset of a real Banach space E. Then we shall have the following definition;

Definition 3.1 A mapping $T : K \to E$ is said to be inward (or to satisfy the inward condition) if for any $x \in K$, $Tx \in IK(x) = \{x + c(u - x) : c \geq 1 \,\&\, u \in K\}$ and T is said to satisfy weakly inward condition if $Tx \in \overline{IK(x)}$ (*the closure of* $IK(x)$.

Let $\{T_n\}_{n=1}^{\infty} : K \to E$ be family of mappings and $h_n(x) = \inf\{\lambda \in [0,1] : \lambda x + (1-\lambda)T_n x \in K\}$. Then, we will have the following theorem.

Theorem 3.3 Let K be a convex, closed and nonempty subset of a real uniformly convex Banach space E and let $\{T_n\}_{n=1}^{\infty} : K \to E$ be a uniformly weakly closed countable family of non-self and nonexpansive mappings. Then the algorithm defined in theorem 1.2 is well-defined. Let that K be strictly convex and each T_n satisfies the inward condition and such that $F = \bigcap_{n=1}^{\infty} F(T_n)$ is nonempty. Then if there exist $a, b \in (0,1)$ such that $\{\alpha_n\} \subset [a,b]$ for all $n \geq 1$, the $\{x_n\}$ weakly converges to a common fixed point $p \in F$ provided that E satisfies opial's condition. Moreover, if $\sum_{n=1}^{\infty}(1-\alpha_n) < \infty$ and (F, K) satisfies S-condition, the convergence is strong.

Proof. Let $p \in F = \bigcap_{n=1}^{\infty} F(T_n) \neq \emptyset$. Then

$$\begin{aligned} \|x_n - p\| &= \|\alpha_n x_n + (1-\alpha_n) T_n x_n - p\| \\ &\leq \|x_n - p\| \end{aligned}$$

Thus $\{\|x_n - p\|\}$ is decreasing and bounded below, and hence converges to some $r \geq 0$.

Thus, $\lim_{n\to\infty}\|x_n - p\| = r = \lim_{n\to\infty}\|x_{n+1} - p\| = \|\alpha_n(x_n - p) + (1-\alpha_n)(T_n x_n - p)\|$.

Since $\|T_n x_n - p\| = \|T_n x_n - T_n p\| \leq \|x_n - p\|$.

Thus, $\limsup_{n\to\infty}\|T_n x_n - p\| \leq r$, hence by the lemma 2.2 we have $\lim_{n\to\infty}\|x_n - T_n x_n\| = 0$.

Moreover, the sequence $\{x_n\}$ is bounded, hence has a weakly convergent subsequence $\{x_{n_k}\}$ which converges weakly to $x \in K$, since K is closed. Since $\{T_n\}_{n=1}^{\infty}$ is uniformly weakly closed, $x \in F$.

It remains to show $x_n \to x$ weakly.

Suppose not, there is $\{x_{n_j}\}$ of $\{x_n\}$ such that $x_{n_i} \to q$, similarly $q \in F$.

Suppose $p \neq q$.

Since E satisfies opial's condition

$$\begin{aligned} \lim_{n\to\infty}\|x_n - p\| &= \lim_{k\to\infty}\|x_{n_k} - p\| \\ &< \lim_{k\to\infty}\|x_{n_k} - q\| = \lim_{j\to\infty}\|x_{n_j} - q\| \\ &< \lim_{j\to\infty}\|x_{n_j} - p\| = \lim_{n\to\infty}\|x_n - p\| \end{aligned}$$

which is a contradiction.

Thus, $x_n \to x \in F$ weakly.

Moreover, if $\sum_{n=1}^{\infty}(1-\alpha_n) < \infty$, then $\lim_{n\to\infty} \alpha_n = 1$.Since the sequences $\{x_n\}$ and $\{T_n x_n\}$ are bounded and $\|x_{n+1} - x_n\| = (1-\alpha_n)\|x_n - T_n x_n\|$, we have $\lim_{n\to\infty}\|x_{n+1} - x_n\| = 0$.

Thus, $\{x_n\}$ is Cauchy sequence in E, since K is closed subset of E and $\{x_n\}$ is in K, $\{x_n\}$ Converges in norm to some $x \in K$.

It suffices to show that $x \in F$.

For each n, T_n is inward implies that $h_n(x) < 1$, thus for $\beta_n \in [h_n(x),1)$ we have

$$\beta_n x + (1-\beta_n)T_n x \in K .$$

Since $\lim_{n\to\infty} \alpha_n = 1$ and $\alpha_{n+1} = \max\{\alpha_n, h_{n+1}(x_{n+1)}\}$, there is a subsequence $\{x_{n_j}\}$ of $\{x_n\}$ such that $\lim_{j\to\infty} h_{n_j}(x_{n_j}) = 1$.

Since $\frac{j}{j+1} h_{n_j}(x_{n_j}) < h_{n_j}(x_{n_j})$, $\frac{j}{j+1} h_{n_j}(x_{n_j})x_{n_j} + (1-\frac{j}{j+1} h_{n_j}(x_{n_j}))T_{n_j}x_{n_j} \notin K$ and $\lim_{j\to\infty}(\frac{j}{j+1} h_{n_j}(x_{n_j})x_{n_j} + (1-\frac{j}{j+1} h_{n_j}(x_{n_j}))T_{n_j}x) = x$, thus by lemma 2.1 $x \in \partial K$.

Since $F \subset K$, the sequence $\{x_n\}$ in K is fejer monotone with respect to F and (F,K) satisfies S-condition , $x \in F$.

Therefore, $x_n \to x \in F$ strongly, which completes the proof.

4. CONCLUSION

Our theorems generalize many results such as our theorem 3.2 generalize theorem 1.2 to the class of k-strictly pseudo contractive mappings ,which is more general class than the class of non expansive mappings . Theorem 3.3 generalizes theorem 1.2 to uniformly convex Banach space, which is more general than Hilbert space.

Meanwhile, we raise open questions;

Question 1 Is it possible to extend theorem 3.2 and 3.3 to uniformly smooth Banach spaces, reflexive Banach spaces and general Banach spaces? If so under what conditions?

Question 2 Is it possible to extend Theorem 3.2 and 3.3 to the class of Pseudo contractive mappings? If so under what conditions?

AUTHORS' CONTRIBUTIONS

Both authors contributed equally and significantly in writing this article. Both authors read and approved the final manuscript.

COMPETING INTERESTS

The authors declare that they have no competing interests.

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