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Bounds For The Status connectivity Index of Line Graph

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Abstract

A recently studied,distance based graph invariant or say a topological invariant have been a great subject of interest fot those researcher studying QSAR and QSPR properties of chemical graphs. In this paper we have studied the first status connectivity index for a specific derived class of graph called to be the line graphs. Here we begin with finding the first status connectivity index of line graphs of $diam \leq 2$ and later giving the close class of bounds for first status connectivity index for line graph of any graph G.

AMS subject classification:
Keywords: status connectivity Index, first zagreb index, second zagreb index, forgotton topological index, line graph, complement of a graph.

1. Introduction

As of late, the idea of Status Connectivity Indices and Coindices has advanced, pulling in much consideration of scientists in numerical science by presenting its application towards finding the boiling point of benzenoid hydrocarbons.The purpose of the present work is to set up a few limits for the status connectivity indices for the line graph.

In this paper we are concerned with simple graphs, having no directed or weighted edges, and no self loops. Let G be such a graph and let $V(G)$ and $E(G)$ be its vertex and edge sets, respectively. The number of vertices and edges of G will be denoted by $n = n(G)$ and $m = m(G)$, respectively. In addition, the edge connecting the vertices u and v will be denoted by uv. The line graph, [2] of the graph G, written $L(G)$, is the simple graph whose vertices are the edges of G, with $ef \in L(G)$ when e and f have a common end point in G. The complement $\overline{G}$ of a graph G is the graph with vertex set $V(G)$,in which two vertices are adjacent if and only if they are not adjacent in G.

In view to obtain the first status connectivity index of line graphs we recall upon certain definitions like The first and second *Zagreb indices* of a graph G are defined as [8]

$$M_1(G) = \sum_{uv \in E(G)} [d(u) + d(v)] \quad \text{and} \quad M_2(G) = \sum_{uv \in E(G)} d(u)d(v).$$

The Zagreb indices were used in the structure property model [7, 14]. Recent results on the Zagreb indices can be found in [1, 5, 6, 10, 11, 16]. The Forgotton topological Index, this vertex degree based graph invariant was studied at the earliest in [8] and defined as,

$$F = F(G) = \sum_{v \in V(G)} d_G(v)^3 = \sum_{uv \in E(G)} d_G(u)^2 + d_G(v)^2$$

This index was sighted off during the past and hence very less literature is availaible for the readers. One can go through [3]. *First status connectivity index* $S_1(G)$ and *Second status connectivity index* $S_2(G)$ of a connected graph G as:

$$S_1(G) = \sum_{uv \in E(G)} [\sigma(u) + \sigma(v)] \quad \text{and} \quad S_2(G) = \sum_{uv \in E(G)} \sigma(u)\sigma(v).$$

We need the following Results

Theorem 1.1. [6] Let G be a graph with n-vertices and m-edges. Then,

$$M_1(\overline{G}) = M_1(G) + n(n-1)^2 - 4m(n-1)$$

Proposition 1.2. [6] Let L be the line graph of the graph G. Then

$$M_1(L) = F - 4M_1 + 2M_2 + 4m$$

where, M_1, M_2, F are the first Zagreb index, second Zagreb index, and forgotten topological index of the parent graph G respectively.

Theorem 1.3. [13] Let G be a connected graph with n vertices, m edges and let $diam(G) = D$. Then

$$4m(n-1) - M_1(G) \leq S_1(G) \leq 2mD(n-1) - (D-1)M_1(G)$$

2. First Status Connectivity Index of Line Graphs

Theorem 2.1. Let $L(G)$ be the line graph of the graph G with n- vertices and m- edges and $diam(G) \leq 2$. Then

$$S_1(L(G)) = 2(m+1)M_1(G) - 2M_2(G) - 4m^2 - F$$

Proof. From the definition of *line graphs* [2], the number of vertices of $L(G)$ is $n_1 = m$ and the number of edges of $L(G)$ is $m_1 = \frac{1}{2}\sum_{i=1}^{n} d_i^2 - m$ [9]. Since from [12], if

$diam(G) \leq 2$ and none of the graphs F_1, F_2, F_3 (refer [12]) is an induced subgraph of G then $diam(L(G)) \leq 2$, from [13]

$$S_1(G) = 4m(n-1) - M_1(G)$$

Therefore, *Status connectivity index of line graph* can be written as,

$$\begin{aligned} S_1(L(G)) &= 4m_1(n_1 - 1) - M_1(L(G)) \\ &= 4\big(\frac{1}{2}\sum_{i=1}^{n} d_i^2 - m\big)(m-1) - M_1(L(G)) \end{aligned}$$

From Prop. 1.2 and definition of Zagreb index [4]

$$M_1(G) = \sum_{i=1}^{n} d_i^2$$

. Hence,

$$S_1(L(G)) = 2(m+1)M_1(G) - 2M_2(G) - 4m^2 - F$$

■

The following corollary directly follows from the theorem 2.1.

Proposition 2.2. The first status connectivity index of line graph of complete bipartite graph $K_{p,q}$,

$$S_1(K_{p,q}) = pq\big[2(pq+1)(p+q) - (p^2+q^2)\big] - 6p^2q^2$$

Proof. The graph $K_{p,q}$ has $n = p + q$ vertices and $m = pq$ edges. Also $diam(K_{p,q})$ ≤ 2. The vertex set $V(K_{p,q})$ can be partitioned into two sets V_1 and V_2 such that for every edge uv of $K_{p,q}$, the vertex $u \in V_1$ and $v \in V_2$, where $|V_1| = p$ and $|V_2| = q$. Therefore $d(u) = q$ and $d(v) = p$ and hence $M_1(K_{p,q}) = pq(p+q)$ and $M_2(K_{p,q}) = (pq)^2$ and $F = pq(p^2+q^2)$. Therefore by the Theorem 2.1. ■

Corollary 2.3. Let G be a connected regular graph of degree r on n- vertices and m-edges.Let $diam(G) \leq 2$. Then

$$S_1(L(G)) = 4m(m-r)(r-1)$$

Proof. Since for regular graphs, $di = r$ for all $i \in V(G)$, followed by $M_1(G) = 2mr$, $M_2(G) = mr^2$ and $F = 2mr^2$. ■

Corollary 2.4. The first status connectivity index of line graph of a cycle C_n on n -vertices is

$$S_1(L(C_n)) = S_1(C_n) = 4n(n-2)$$

.

Corollary 2.5. he first status connectivity index of line graph of a complete graph K_n on n - vertices is

$$S_1(L(K_n)) = S_1(C_n) = n((n-1)^2(n-2)^2$$

.

Theorem 2.6. If G is any connected graph with n- vertices, m- edges, maximum degree = $\triangle(G)$ and $diam(G) = D$. Then the first status connectivity index of its *line graph*

$$\begin{aligned} S_1(L(G)) \quad \leq \quad & \big(m^2 - 2(m+3)\triangle(G) + 6m + 5\big)M_1(G) + (2M_2(G) + F + 4m) \\ & \big(2(\triangle(G)-1) - m\big) + 2m(m-1)\big(2\triangle(G) - m - 3\big) \end{aligned}$$

Proof. We know from Theorem 1.3 the first status connectivity of any graph G,

$$S_1(G) \leq 2mD(n-1) - (D-1)M_1(G) \tag{2.1}$$

Now for any graph G, the number of vertices of its *line graph*,$L(G)$ is $n_1 = m$ and the number of edges is $m_1 = \frac{1}{2}\sum_{i=1}^{n} d_i^2 - m$ [9]. Therefore, from 2.1 we have

$$S_1(L(G)) \leq 2\big[\frac{1}{2}\sum_{i=1}^{n} d_i^2 - m\big]D(m-1) - (D-1)M_1(L(G))$$

where, D is now $diam$ of $L(G)$ and for any graph G of order n, $diam(G) \leq n - \triangle(G) + 1$ [15] and hence

$$diam(L(G)) \leq m - \triangle(L(G)) + 1 \tag{2.2}$$

Also for any Graph G

$$\triangle(L(G)) \leq 2(\triangle(G) - 1) \tag{2.3}$$

Substituting these two properties, 2.6 and 2.3 into 2.1 we obtain

$$\begin{aligned} S_1(L(G)) \quad \leq \quad & \big(m^2 - 2(m+3)\triangle(G) + 6m + 5\big)M_1(G) + (2M_2(G) + F + 4m) \\ & \big(2(\triangle(G)-1) - m\big) + 2m(m-1)\big(2\triangle(G) - m - 3\big) \end{aligned}$$

Equallity holds only for $L(P_4)$ i,e P_3. ■

Theorem 2.7. Let G be a graph whose *line graph* $L(G)$ has $diam(L(G)) > 3$, then

$$S(\overline{L(G)}) = m(m-1)^2 - 4m + 4M_1(G) - 2M_2(G) - F$$

Proof. Let G be any graph with n- vertices and m- edges whose line graph $L(G)$ has $diam(L(G)) > 3$.If $\overline{L(G)}$ be the complement of line graph. We know from the definition of first status connectivity index for any graph G 1.3.

$$4m(n-1) - M_1(G) \leq S_1(G)$$

Equality holds for graphs with $diam(G) \leq 2$.Since there exist a fact that for any graph G, If $diam(G) > 3$, then $diam(\overline{(G)}) \leq 2$.[15]. Therefore,if n_1, m_1 are the number of vertices and edges of $\overline{L(G)}$ respectively. Then,

$$\begin{aligned} S_1(\overline{L(G)}) &= 4m_1(n_1 - 1) - M_1(\overline{L(G)}) \\ &= 2m(m-1)^2 - 2(m-1)(M_1(G) - 2m) - M_1(\overline{L(G)}) \end{aligned}$$

Now, from the theorem 1.1 we have,

$$\begin{aligned} S_1(\overline{L(G)}) &= 2m(m-1)^2 - 2(m-1)(M_1(G) - 2m) - M_1(\overline{L(G)}) \\ &= 2m(m-1)^2 - 2(m-1)(M_1(G) - 2m) - m(m-1)(3+m) + \\ &\quad 2M_1(G)(m-1) - M_1(L(G)) \\ &= m(m-1)^2 + 4M_1(G) - 2M_2(G) - 4m - F \end{aligned}$$

■

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References

[1] K.C. Das, K. Xu, J. Nam, Zagreb indices of graphs, Front. Math. China, 10, 567–582 (2015).

[2] B. W. Douglas, Introduction to Graph Theory, Prentice Hall, second ed., 2001.

[3] B. Furtula, I. Gutman, A forgotten topological index, J. Math. Chem. 53 1184–1190 (2015).

[4] I. Gutman, N. Trinajstic, Chem. Phys. Lett. 17 (1972) 535–538.

[5] I. Gutman,K.C. Das, The first Zagreb index 30 years after, MATCH Commun. Math. Comput. Chem., 50, 83–92 (2004).

[6] I.Gutman, B. Furtula,Ž. K. Vukićević and G. Popivoda, On Zagreb Indices and Coindices, MATCH Commun. Math. Comput. Chem. 74 (2015) 5–16.

[7] I.Gutman, B. Ruščić, N. Trinajstić, C.F. Wilcox, Graph theory and molecular orbitals, XII, acyclic polyenes, J. Chem. Phys., 62, 3399–3405 (1975).

[8] I.Gutman, N. Trinajstić, Graph theory and molecular orbitals. Total π-electron energy of alternant hydrocarbons, Chem. Phys. Lett., 17, 535–538 (1972).

[9] F. Harary, Graph Teory, Narosa Publishing House, New Delhi, 1999.

[10] M.H. Khalifeh, H. Yousefi-Azari, A.R. Ashrafi, The first and second Zagreb indices of some graph operations, Discrete Appl. Math., 157, 804–811 (2009).

[11] S. Nikolić, G. Kovačević, A. Miličević, N. Trinajstić, The Zagreb indices 30 years after, Croat. Chem. Acta, 76, 113–124 (2003).

[12] H. S. Ramane, D.S. Revankar, I. Gutman and H. B. walikar, Distances Spectra and Distance Energy of Iterated Line Graphs of Regular Graphs, Publ. de L' Inst. Math. 85(99) (2009), 39–46.

[13] H. S. Ramane, A. S. Yalnaik, Status connectivity indices of graphs and its applications to the boiling point of benzenoid hydrocarbons, J. Appl. Math. Comput., DOI 10.1007/s12190-016-1052-5.

[14] Todeschini, R., Consonni, V. Handbook of Molecular Descriptors, Wiley-VCH, Weinheim (2000).

[15] W. Goddard and O. R. Oellermann, Distance in Graphs.

[16] B. Zhou, I. Gutman, Further properties of Zagreb indices, MATCH Commun. Math. Comput. Chem., 54, 233–239 (2005).