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A New Innovative method For Solving Fuzzy Electrical Circuit Analysis

B.Srinivas[1] and B.Sankara Rao[2]

1. Department of Mathematics, ISTS Women's Engineering College, Rajahmundry, Andhra Pradesh, India.
E-mail: srinivas _ssmist@yahoo.com

2.Department of Mathematics, Adikavi Nannaya University, Rajahmundry, Andhra Pradesh, India.
E-mail: sankararao.b@gmail.com

Abstract

This paper proposes a new method for solving fuzzy system of linear equations with fuzzy coefficients on left side matrix and fuzzy or interval on the right side of linear system of equation. Some conditions for the existence of a fuzzy or interval solution of $n\times n$ linear system are derived and also a practical algorithm is introduced in detail. The method is based on linear programming problem. Finally, the applicability of the proposed method is illustrated by some numerical examples

1. INTRODUCTION

In recent years, there has been a substantial amount of research related to the fuzzy applied linear programming problems. Over the last few years, more and more manufacturers had applied the optimization technique most frequently in linear programming to solve real-world problems and there it is important to introduce new tools in the approach that allow the model to fit into the real world as much as possible. Any linear programming model representing real-world situation involves a lot of parameters whose values are assigned by experts, and in the conventional approach, they are required to fix an exact value to the aforementioned parameters. However, both experts and the decision maker frequently do not precisely know the value of those parameters. If exact values are suggested these are only statistical inference from past data and their stability doubtful, so the parameters of the Systems of simulations linear equations play a major role in various areas such as mathematics, physics, statistics, engineering, and social sciences. Since in many applications at least some of the system's parameters and measurements are represented by fuzzy rather than crisp numbers, it is important to develop mathematical models and numerical procedures that would appropriately treat general fuzzy linear systems. A general model for solving an $m \times n$ fuzzy system of linear equation (FSLE) whose coefficients' matrix is crisp and right hand side column is an arbitrary fuzzy number vector was first proposed by Friedman et al. [1]. Different authors [2–5] have investigated numerical methods for solving such FSLE. Most of mentioned methods in different articles are based on numerical methods such as matrices decomposition and iterative solutions. Previously mentioned papers do not discuss a lot the possibility of solutions. In addition they cannot find alternative solutions. But the proposed method does not include these defects. Allahviranloo et al. [6] have presented that the abovementioned method is not applicable and does not have solution generally. This paper sets out to investigate the solution of the fuzzy linear system using a linear programming to solve the problems of interval data in [7–9]. The structure of this paper is organized as follows

In Section 2, we provide some basic definitions and results which will be used later.

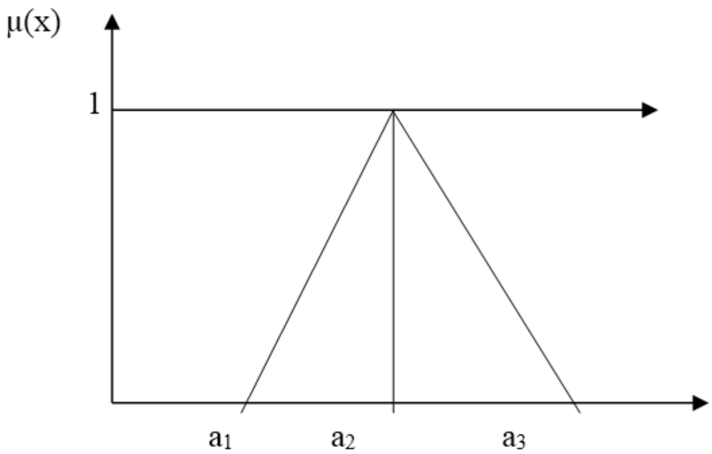
In Section 3, we prove some theorems which are used for proposed method and present a practical procedure. The introduced method is illustrated by solving some examples in Section 4 and conclusions are drawn in Section 5.

2. PRELIMINARIES

2 Triangular Fuzzy number: A triangular fuzzy number A is a fuzzy fully specified by 3-tuples (a_1, a_2 , a_3) such that with membership function defined as

$$\mu_A = \begin{cases} 0 & x < a_1 \\ \dfrac{x-a_1}{a_2-a_1} & a_1 \leq x \leq a_2 \\ \dfrac{a_3-x}{a_3-a_2} & a_2 \leq x \leq a_3 \\ 0 & a_3 > 1 \end{cases}$$

This representation is interpreted as membership functions diagrammatically as



Note. If α = c, then TFN is defined

$$\mu_A = \begin{cases} \dfrac{a_3 - x}{a_3 - a_1} & a_1 \le x \le a_3 \\ 0 & otherwise \end{cases}$$

And finally if a_1 =a_2 = a_3 then TFN is defined:

$$\mu_A = \begin{cases} 1 & a_2 = a_3 \\ 0 & \text{otherwise} \end{cases}$$

Definition: - Let $\mu_A = (a_{1,}a_2, a_3)$ be a triangular fuzzy number; Then one defines

Supp ($\widetilde{\mu_A}$) = [a_1, a_3] Core ($\widetilde{\mu_A}$)= a_2

For arbitrary interval, I=$[\underline{x} \quad \overline{x}]$ and J= $[\underline{y} \quad \overline{y}]$ the following properties hold

(i) $[\underline{x} \quad \overline{x}] + [\underline{y} \quad \overline{y}] = [\underline{x} + \underline{y}, \ \overline{x} + \overline{y}]$

(ii) $[\underline{x} \quad \overline{x}] - [\underline{y} \quad \overline{y}] = [\underline{x} - \underline{y}, \ \overline{x} = \overline{y}]$

(iii) for each $k \in R$,

$$k\,[\underline{x} \quad \overline{x}] = \begin{cases} [k\underline{x} \quad k\overline{x}] \text{ if } k \ge 0 \\ [k\overline{x} \quad k\underline{x}] \ k < 0 \end{cases}$$

Definition: - Let $F(R)$ be a set of all fuzzy numbers on r and $I(R)$ a set of intervals on R. The n × n

Linear system of equations is as follows:

$a_{11}x_1 + a_{12}x_2 + \ldots\ldots\ldots\ldots a_{1n}x_n = y_1$

$a_{21}x_1 + a_{22}x_2 + \ldots\ldots\ldots\ldots a_{2n}x_n = y_2$

…………………………………

………………………………….

$a_{n1}x_1 + m_{12}x_2 + \ldots\ldots\ldots\ldots a_{nn}x_n = y_m$

where the coefficient matrix $A = (a_{ij})$, $1 \le i \le n$, $1 \le j \le n$, is a crisp $n \times n$ matrix and $y \in F(R)\ I(R)$, $1 \le i \le m$, is called a FSLE

Kirchoff's Laws:

Kirchhoff's Current Law: The algebraic sum of current in to any junction in a circuit is equal to zero

i.e $i_1 - i_2 + i_3 - i_4 + i_5 - i_6 = 0$

Kirchhoff's Voltage Law: At The constant temperature, the voltage in the circuit is the Product of the current (I) and Resistance(R) = Ri

$$I = \frac{V}{R} \qquad V = IR$$

Electrical Circuits: A simple Electric Circuit is closed connection of Batteries, Resisters, and wires. An electrical circuit consist of voltage loops and current nodes.

First of all the following definitions and theorems are introduced. $\overline{x}$

Definition:- let I= $[\underline{x} \quad \overline{x}]$ it can be written as I= $[-L_I, L_I] + [M_I, M_I]$ such that

$$L_I = \frac{\overline{x} - x}{2} \qquad M_I = \frac{\overline{x} + x}{2}$$

Now one can have the following theorems

3. PROPOSED METHOD

First of all the following definitions and theorems are introduced

Theorem:-Let A be $m \times n$ matrix and b an m vector. The system $Ax = b$ with condition $m \le x \le M$ has a solution if and only if the optimal solution of the below system is zero:

Min z = 1xa

s. t. Ax + xa = b,

m ≤ x ≤ M,

xa ≥ 0.

as fuzzy numerical data which can be represented by means of fuzzy sets of the real line known as fuzzy numbers.

Theorem 2:- let I=$[\underline{x} \quad \overline{x}]$ and J= $[\underline{y} \quad \overline{y}]$ be intervals and $k \in R$; then

i. $I = J$ if and only if $L_I = L_J$ and $M_I = M_J$

ii. if I+J =$[\underline{x} \quad \overline{x}] + [\underline{y} \quad \overline{y}]$ then $L_{I+J} = L_I + L_J$ and $M_{I+J} = M_I + M_J$

iii. iii. let $S = kI$ such that $k \in$ R; then L = |k |L and $M = kM$

Proofs of parts (i) and (ii) are obvious so we prove part (iii). Let $k < 0$ such that

$$S = k[\underline{x} \quad \overline{x}] = [k\underline{x} \quad K\overline{x}] = ([\,-L_{I,}\, L_J\,] + [\,M_I, M_J\,])$$

$$L_S = \frac{\underline{\underline{kx}} \quad -K\overline{x}}{2} \qquad M_S = \frac{\underline{\underline{kx}} \quad +K\overline{x}}{2}$$

For $k \geq 0$ the proof is the same.

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Theorem 3: If A is a $m \times n$ matrix and X is an interval vector, then

If A is a $m \times n$ matrix and X is an interval vector, then

$$AX = [-A^+L_X, A^+L_X] + [ML_X, ML_X]$$

where $A^+{}_{ij} = |A_{ij}|$ and L_{X_j}= =$(\overline{x}_j - \underline{x}_j)/2$ and $M_{\widetilde{x_j}}$ =$(\overline{x}_j + \underline{x}_j)/2$

Proof:- According to Theorem 1, for i = 1,2,…, m, we have

$$A^i{}_X = \sum_{j=1}^{j=n} A_{ij}\, \underline{x}_j \quad \overline{x}_j$$

$$\sum_{j=1}^{j=n} A_{ij} \left([-L_{X_j} \quad L_{X_j}] + [-M_{X_j} \quad M_{X_j}] \right.$$

$$\sum_{j=1}^{j=n} |A_{ij}| \left([-L_{X_j} \quad L_{X_j}] + A_{ij}[-M_{X_j} \quad M_{X_j}] \right)$$

$$\sum_{j=1}^{j=n} |A_{ij}| \left([-L_{X_j} \quad L_{X_j}] + \sum_{j=1}^{j=n} |A_{ij}| [-M_{X_j} \quad M_{X_j}] \right)$$

$$=[-A^+L_X, A^+L_X] + [M^+L_X, M^+L_X]$$

Theorem 4:- If A is a $m \times n$ matrix and X, b are two interval vectors, then the system $AX = b$ has solution(s) if and only if the following systems have solution:

$A^{+}L_X$=L_b $L_X \geq 0$

AM_X=M_b

Proof:- Let X be an interval solution of $AX = b$; then according to Definition 7

$X = [-L, L]$+$[M, M]$ with $L \geq 0$ and according to Theorem 2, we have

AX =[-$A^{+}L_X$, $A^{+}L_X$] + [ML_X, ML_X]

but $AX = b$ and by using part (i) of Theorem 1 this means (18)

$A^{+}L_X$=L_b $L_X \geq 0$

AM_X=M_b

The converse holds obviously.

Theorem5:- The system with $A^{+}L_X$=L_b condition $L_X \geq 0$ has a solution if and only if optimized value of the below linear programming is zero:

Z*=Min $1x_n$

s.t $A^{+}L_X + x_n$ =L_b

$L_{X,}$ $x_n \geq 0$

Theorem6:- This is proved by using Theorem 6.

Now we are going to apply the same method for solving A$\tilde{X} = \tilde{B}$, where is $\tilde{B}$ TFN.

Theorem7:- If A is a $m \times n$ matrix with crisp coefficients and is $\tilde{B}$ a TFN vector the same as $\tilde{B}$ = ($\alpha_{i,}$ $c_{i,}$ β_i) then the A$\tilde{X} = \tilde{B}$ system has TFN solution(s) $\tilde{X}$ the same as $\widetilde{x_\iota}$ =

($\underline{x}_i$, x_i $\overline{x}_i$) if

and only if the systems (20) have solution:

A($S_{\tilde{X}}$)=$S_{\tilde{B}}$ A($C_{\tilde{X}}$)=$C_{\tilde{B}}$

$C_{\widetilde{X_J}} \in$ Supp ($\widetilde{X_J}$) j-1,2,3……..n

where S($\widetilde{B_\iota}$) = Supp($\widetilde{B_\iota}$) as $\tilde{B}$ = [$\alpha_{i,}$ β_i], i=1,2,3 ……m

S $(\widetilde{X_j})$ =Supp $(\widetilde{X_j})$ $[\ \underline{x}_j\ ,\ \overline{x}_j]$ j=1,2,3 ……n

C $(\widetilde{B_i})$ =Core $(\widetilde{B_i})$ =C_i i=1,2,3 ……m

C $(\widetilde{X_j})$ =Core $(\widetilde{X_j})$ =X_j j=1,2,3 ……n

Proof:- From part (i) of Theorem 5 is$\tilde{X}$ a TFN solution of system A$\tilde{X}$ =$\tilde{B}$ if and only if

Supp $(A^i\tilde{X})$= $\widetilde{B_i}$ i=1,2,3 ……m

Core $(A^i\tilde{X})$= $\widetilde{B_i}$ i=1,2,3 ……m

So according to part (ii) of Theorem 5 for *i* = 1,2,…, *m* we have

S$(\widetilde{B_i})$ = Supp$(\widetilde{B_i})$

= Supp $(A^i\tilde{X})$

= Supp $(\sum_{j=1}^{n} A^i\tilde{X})$

= $(\sum_{j=1}^{n}$ Supp$(A^i\tilde{X})$

= $A^i S_{\tilde{X}}$

Also according to part (iii) of Theorem 5 for *i* = 1,2,…, *m* we have

C$(\widetilde{B_i})$ = C$(\widetilde{B_i})$

= C$(A^i\tilde{X})$

= C$(\sum_{j=1}^{n} A^i\tilde{X})$

= $(\sum_{j=1}^{n}$ C$(A^i\tilde{X})$

= $A^i C_{\tilde{X}}$

but C$(\widetilde{X_j})$∈ $Supp\widetilde{X_j}$ j-1,2,…..n

Theorem8:- The system A$C_{\tilde{X}}$ = $C_{\tilde{B}}$ with condition $C_{\tilde{X}} \in Supp\ (\tilde{X_i})$ has a solution if and only if optimal solution of the following linear programming is zero:

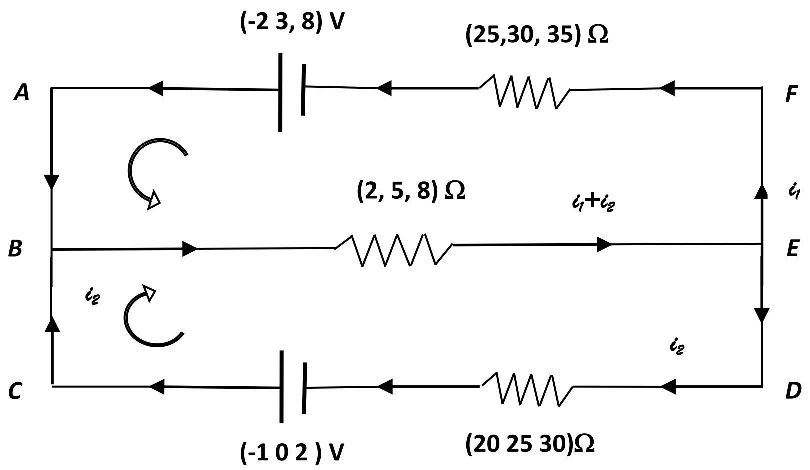
$$Z^* = \text{Min } 1x_n$$

$$\text{s.t } A^{+}L_X + x_n = L_b$$

$$L_{X,}\ x_n \geq 0$$

Proof : This is proved by using **Theorem1**

Example: - Consider the following circuit to find the currents in following net work



Solution:- According to Kirchhoff's Voltages laws in the Loop ABEFA

-(-2,3,8) +(2,5,8)(i_1+i_2) +(25,30,35)i_1=(0,0,0)

In the loop BCDEB:

(-1, 0,2)+ (2, 5, 8) (i_1+i_2) + (20, 25, 30)i_2=(0,0,0)

we get the equations

(20,25,30) $\tilde{\imath}_1$ +(2,5,8) $\tilde{\imath}_2$=(-2,3,8) (1)

+(2,5,8) $\tilde{\imath}_1$+(25,30,35) $\tilde{\imath}_2$=(-1,1,3)

We change the left elements in to crisp form and right elements into interval form

Now we calculate R(1,5,9) by applying Robust ranking method.

For which ($a^L_{\alpha,}$ a^U_{α})= { (b-a) α + a, c-(c-b)α}

R ($\tilde{a}_{11}$)=$\int_0^1 0.5(5 + 5\alpha + 30 - 5\alpha)d\alpha$=25 similarly

R ($\tilde{a}_{12}$) = 5 R($\tilde{a}_{21}$)= 5 R($\tilde{a}_{22}$)=30

To solve this system, we proceed in two successive stages according to **Theorem 7.**

Stage1: Find Supp ($\tilde{X}$) where Supp ($\tilde{x}_j$) is the interval.[$\underline{x}_j$ $\overline{x}_j$] .Therefore, the following system must be solved

$$\begin{bmatrix} 25 & 5 \\ 5 & 30 \end{bmatrix} = \begin{bmatrix} [\underline{x}_1 & \overline{x}_1] \\ [\underline{x}_2 & \overline{x}_2] \end{bmatrix} = \begin{bmatrix} [-2 & 8] \\ [-1 & 3] \end{bmatrix} \quad (2)$$

Stage 2: After calculating the intervals[$\underline{x}_j$ $\overline{x}_j$] in the first stage, search for core ($\tilde{X}$) =$\tilde{x}_j$ that satisfies $\underline{x}_j \leq x_j \leq \overline{x}_j$. Therefore, the following system must be solved:

$$\begin{bmatrix} 25 & 5 \\ 5 & 30 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \quad (3)$$

$\underline{x}_j \leq x_j \leq \overline{x}_j$ for j=1,2

Here, the system of the first stage (i.e., system (2)) is solved. It is an interval system, so it must be solved in two sub stages according to **Theorem 3**. The first sub stage is finding the interval length, that is L_X . Thus, the following system is solved

$$\begin{bmatrix} 25 & 5 \\ 5 & 30 \end{bmatrix} \begin{bmatrix} L\tilde{x}_1 \\ L\tilde{x}_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix} \quad (4)$$

$0 \leq L\tilde{x}_j$ j=1,2

Here right hand side is $L_{supp(\widetilde{B})}$ and $L_{\widetilde{x_j}}=(\overline{x}_j - \underline{x}_j)/2$ But according to **Theorem 5,** the presence of solution (29) is equivalent to the following LP problem

Z*=Min $x_{a1}+x_{a2}$

$$\text{s.t } \begin{bmatrix} 25 & 5 \\ 5 & 30 \end{bmatrix} \begin{bmatrix} L\tilde{x}_1 \\ L\tilde{x}_2 \end{bmatrix} + \begin{bmatrix} x_{a1} \\ x_{a2} \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix} \quad (5)$$

$0 \leq L\tilde{x}_j$ j=1,2

$0 \leq \tilde{x}_j$ j=1,2

We solve it with the Simple*x* method . Since the optimal value (Z^*) is zero, system (4) has the following solution

		1	1	-M	-M		
CV	BV	I1	I2	A1	A2	CB	Min. ratio
-M	A1	25	5	1	0	5	5/30
-M	A2	5	30*	0	1	2	2/30 ←
	Cj-Zj	1+30M	1+35M	-M	-M		
-M	A1	145/6	0	1	-1/6	14/3	←
1	I2	1/6	1	0	1/30	1/15	-
	Cj-Zj	(145M+5)/6 ↑	0	0	-(35M+1)/30		
1	I1	1	0	6/145	-1/145	28/145	
1	I2	0	1	-1/145	1/29	14/435	
	Cj-Zj	0	0				

$L_{\tilde{x}}$=[0.1931034, 0.03218]^t (6)

The second sub stage is to find the center of $[\underline{x}_j,\ \overline{x}_j]$. So, the solution of the following system is calculated by common methods in linear algebra:

$$\begin{bmatrix} 25 & 5 \\ 5 & 30 \end{bmatrix}\begin{bmatrix} M\tilde{x}_1 \\ M\tilde{x}_2 \end{bmatrix}=\begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

Here right hand side is $M_{supp(\widetilde{B})}$ and $M_{\widetilde{x_j}} =(\overline{x}_j + \underline{x}_j)/2$. Therefore, the solution of the center of is as follows:

$M_{\tilde{x}}$=[0.11948. 0.01379]^t (7)

Finally, a solution for Supp ($\tilde{X}$) is as follows according to Theorem 10 and solutions (6) and (7)

$$\text{Supp } (\tilde{X}) = \begin{bmatrix} S\tilde{x}_1 \\ S\tilde{x}_2 \end{bmatrix}\begin{bmatrix} [\underline{x}_1 & \overline{x}_1] \\ [\underline{x}_2 & \overline{x}_2] \end{bmatrix}=\begin{bmatrix} [-0.07362, & 0.31258] \\ [-0.01839, & 0.04597] \end{bmatrix} \quad (8)$$

Now we come back to the second stage. According to **Theorem 13**, solving system (28) is equivalent to the presence of a solution for the following system:

$$Z^*=\text{Min } 1x_{a1}+1x_{a2}$$

$$\text{s.t } \begin{bmatrix} 25 & 5 \\ 5 & 30 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} x_{a1} \\ x_{a2} \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \qquad (9)$$

$0 \le L\tilde{x}_j \quad j=1,2$

$0 \le \tilde{x}_j \quad j=1,2$

Again, we solve this problem with the Simplex method. Since the optimal value is zero, system (28) has the following solution

		1	1	-M	-M		
CV	BV	I1	I2	A1	A2	CB	Min. ratio
-M	A_1	25	5	1	0	3	3/30 ←
-M	A_2	5	30*	0	1	1	1/30
	C_j-Z_j	1+30M	1+35M ↑	-M	-M		
-M	A_1	145/6	0	1	-1/6	17/6	-
1	I_2	1/6	1	0	1/30	1/30	←
	C_j-Z_j	(145M+5)/6 ↑	0	0	-(35M+1)/30		
1	I_1	1	0	6/145	-1/145	17/145	
1	I_2	0	1	-1/145	1/29	12/870	
	C_j-Z_j	0	0				

$$\text{Core }(\tilde{X}) = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0.11724 \\ 0.01379 \end{bmatrix} \qquad (10)$$

According to Theorem 7 and solutions (8) and (10) a final solution for system (2) is as follows

$$\tilde{X} = \begin{bmatrix} [\underline{x}_1 \; x_1 & \overline{x}_1] \\ [\underline{x}_2 \; x_2 & \overline{x}_2] \end{bmatrix} = \begin{bmatrix} [-0.07362, \; 0.11724 & 0.31258] \\ [-0.01839, \; 0.01379 & 0.04597] \end{bmatrix}$$

4. CONCLUSION

In this paper, we presented a method which is novel for transformed fuzzy system of linear equations. The proposed method is applicable than any other existing methods. Because the base of this method is linear programming, it can express clearly the presence or the absence of a solution. In addition, if a solution exists, it expresses the possibility of other solutions. This is not possible in numerical methods based on the embedding method. With a slight change, this method can be used for systems whose right hand side is triangular fuzzy numbers; in previous methods, they did not have this ability. Finding the optimal solutions of fuzzy LP problems is one important application of solving fuzzy linear systems. With this new method, these problems can easily be solved. The presented method can introduce fundamental change in operation research with interval data. It also changes and modifies some application of linear programming with fuzzy data as some method in [12–15]. In the numerical solution of fuzzy differential equations, this method provides an explicit method so, it is extremely efficient. This method can analytically present the algebraic structure of fuzzy polyhedrons in the same way the representation theorem provides the algebraic structure for crisp polyhedrons. The previous methods are slacking this capability.

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