An EOQ Model with Certain Uncertainties When Payment Periods are Offered

Dr. Jayjayanti Ray

*Department of Mathematics, Gokhale Memorial Girls' College
(Calcutta University affiliation), Kolkata, India.*

Abstract

An economic order quantity model has been developed for a constantly deteriorating item for which the supplier permits a fixed delay in payments or in other words trade credit, the demand rate being time-dependent. This inventory model with no shortage is also considered under fuzzy environment. The costs involved – ordering cost and holding cost, being imprecise in nature, are considered as fuzzy parameters. Different decision making situations are illustrated with the help of numerical examples.

Keywords: inventory; trade credit; EOQ; fuzziness

INTRODUCTION

The concept of *uncertainty* is a notable change among the various formal changes in Science and Mathematics in this century. According to the traditional view, Science should strive for certainty in all its manifestations (precision, specificity, sharpness, consistency, etc.); hence, uncertainty (imprecision, non-specificity, vagueness, inconsistency, etc.) is regarded as unscientific. But according to the modern view, uncertainty which is unavoidable is considered essential to Science and has great utility.

An important point in the evolution of the modern concept of uncertainty was the publication of a seminar paper by Lofti A. Zadeh [1], where he introduced a theory whose objects – *fuzzy sets* – are sets with boundaries that are not precise. The membership in a fuzzy set is not a matter of affirmation or denial, but rather a matter

of a *degree.* Zadeh's paper challenged not only probability theory as the sole agent for uncertainty, but the very foundations upon which probability theory is based: Aristotelian two-valued logic. A fuzzy set can be defined mathematically by assigning to each possible individual in the Universe of discourse a value representing its grade of membership in the fuzzy set. Individuals may belong in the fuzzy set to a greater or lesser degree as indicated by a larger or smaller membership grade. Following Zadeh a membership grade allows finer detail, such that the transition from membership to non-membership is gradual rather than abrupt. Given a collection of objects U, a fuzzy set A in U is defined as a set of ordered pairs $A \equiv \{\langle x, \mu_A(x)\rangle \mid x \epsilon \text{ U}\}$ where $\mu_A(x)$ is called the membership function for the set of all objects x in U. The membership function relates to each x a membership grade $\mu_A(x)$, a real number in the closed interval$[0,1]$. So here we see that it is necessary to work with pairs $\langle x, \mu_A(x)\rangle$ whereas for classical sets a list of objects suffices, as their membership is understood. Thus the definition of a fuzzy set is the extension of the definition of a classical set, since membership values μ are permitted in the interval $0 \leq \mu \leq 1$, higher the value, the higher the membership; whereas in a classical set the membership values are restricted to $\mu \in \{0,1\}$. Since full membership or full non-membership in the fuzzy set can still be indicated by the values of 1 and 0, respectively, we can consider crisp set to be a restricted case of the more general concept of a fuzzy set, for which only these two grades of membership are allowed.

As almost every business must carry out some inventory for smooth and efficient running of its operation, inventory control plays an important role. The problem is to take decisions that how much should be stocked and when should be stocked for un-interrupted production. There are various types of uncertainties that cannot be appropriately treated by the usual probabilistic models. Now the question is how to define inventory optimization tasks in such environment and how to interpret optimal solutions. Bellman and Zadeh [3] first introduced fuzzy set theory in fuzzy decision making process. Kaufmann and Gupta [4] explained the fuzzy arithmetic and their applications. Zadeh ([1], [5]) showed that for the new products and seasonal items it is better to use fuzzy numbers rather than probabilistic approaches.

For solving various EOQ (Economic Order Quantity) models which arise in the real life situation, the values of the quantities such as demand rate, production rate, deterioration rate, etc. are not crisp but uncertain in nature. Hence these variables should be treated as fuzzy variables. Park [6] and Vujosevic et. al. [7] developed the inventory models in fuzzy sense where ordering cost and holding cost are represented by fuzzy numbers. Park has represented costs as trapezoidal fuzzy numbers. Whereas Vujosevic et. al. represented ordering cost by triangular fuzzy number and holding

cost by trapezoidal fuzzy number. Centroid of fuzzy total cost was taken as the estimate of fuzzy total cost. Yao and Lee [8] developed an EOQ model by considering order quantity as fuzzy and allowing shortages. The same authors Yao and Lee [9-10] developed another inventory model with fuzzy demand quantity and fuzzy production quantity. T. K Roy and M. Maiti [11] presented a fuzzy EOQ model with demand dependent unit cost under limited storage capacity. Chang et. al. [12] presented a fuzzy inventory model with backorder where the backorder quantity was fuzzified as the triangular fuzzy number. S. De and A. Goswami [13] presented the EOQ model with fuzzy deterioration rate. G.C. Mahata and A. Goswami [14] has used fuzzy concepts to develop a fuzzy EOQ model with stock-dependent demand rate and non-linear holding cost by taking rate of deterioration to be a triangular fuzzy number. An EOQ model for perishable items with fuzzy partial backlogging factor and fuzzy deterioration rate was developed by Halim et. al. [15]. A. Roy and G. Samanta [16] have developed an inventory model considering that the cycle time is uncertain and described it by a triangular fuzzy number (symmetric).

After the introduction of fuzzy set theory in 1965 by Zadeh, extensive research work has been done on *defuzzification* of fuzzy numbers. Among these, centroid method [17] , weighted average method [18], graded mean value method [19], nearest interval approximation method [20], graded mean integration value method [21], etc., have drawn more attention. All these techniques replace the fuzzy parameters by their nearest crisp number / interval and the reduced crisp objective function is optimized. Study shows that among the various methods, the Signed Distance Method is better for *defuzzification* [22]. Syed and Aziz [23] in their paper developed an inventory model without shortages, representing both the ordering and holding costs by fuzzy triangular number and calculating the optimal order quantity using Signed Distance Method for *defuzzification*. Research on the fuzzy sets has been growing steadily since the inception of the theory in the mid-1960s and is quite impressive. Research on a broad variety of applications in inventory management using fuzzy concepts has also been very active and has produced results that are perhaps even more impressive.

Many studies in inventory management assumed the payment for the goods will be made to the vendor immediately after receiving the consignment, which is not quite practical in the real world. In most marketing situations, a vendor often provides buyers with a trade credit period to stimulate the demand, boost market share or decrease inventories of certain items.

The problem of determining the economic order quantity under the condition of a permissible delay in payment has drawn the attention of researchers in recent times. It is assumed that the supplier (whole-saler) allows a delay of a fixed period, that is the

trade credit period, for settling the amount owed to him. There is no interest charged on the outstanding amount if it is paid within the permissible delay period. Beyond this period, interest is charged. During this fixed period of permissible delay in payments, the customer (a retailer) can sell the items, invest the revenues in an interest-earning account and earn interest instead of paying off the over-draft which is necessary if the supplier requires settlement of the account immediately after replenishment. The customer finds it economically beneficial to delay the settlement to the last moment of the permissible period of delay. The effect of supplier credit policies on optimal order quantity has received the attention of many researchers such as Aggarwal and Jaggi [24], Chang and Dye [25], Chang et al.[26], Chen and Chuang [27], Chu et al. [28], Chung [29,30,31], Goyal [32], Jamal et al.[33, 34], Khouja and Mehrez [35], Liao et al.[36], Sarkar et al.[37, 38] and Shah and Shah [39].

THE MODEL

In the model [41], the *optimal order quantity* and *optimal retail price* of a retailer was obtained when the supplier permits a fixed delay in payments or in other words *trade credit*, for an order of a product whose *demand rate is linear and time dependent*. This type of demand pattern is observed in the market in the case of many products. The inventory is depleted not only due to customers' demand but also by *constant deterioration*. This model has been reconsidered here in fuzzy sense. The *ordering (set-up) cost* and *holding cost* per rupee of unit purchase cost per unit time, imprecise in nature, are considered as *fuzzy parameters*. Both the costs are represented by fuzzy triangular number, the optimum cost and economic order quantity is calculated using Signed Distance Method for defuzzification. The results are illustrated with the help of numerical examples for both crisp & fuzzy models, comparing the two.

Notations and Assumptions in the model:

The following *notations* are used in this model:-

$D(t)$ = demand rate which is linearly dependent on time i.e. $D(t) = a + bt$ where $a > 0,\ b > 0$

h_p= holding cost (excluding interest charges) per rupee of unit purchase cost per unit time

p = unit purchase cost of an item in rupees

s = cost of placing an order in rupees (set-up cost)

I_p = interest charges per rupee investment in stocks per year

I_e = interest earned per rupee in a year

t_1 = permissible period (in years) of delay in settling the accounts with the supplier

T = time interval (in years) between two successive orders

θ = constant rate of deterioration of an item

$\hat{s}$ = fuzzy cost of placing an order

$\hat{h}_p$ = fuzzy holding cost (excluding interest charges) per unit purchase cost per unit time

The following *assumptions* are made in the development of the model:

(i)   The demand rate for the item is represented by a linear and continuous function of time.
(ii)   Shortages are not allowed.
(iii) Depletion of inventory over time due to both demand and deterioration occurs simultaneously.
(iv)   During the time the account is not settled, generated sales revenue is deposited in an interest-bearing account. At the end of this period, the account is settled and the customer starts paying for the interest charges on the items in stock.
(v)   Time horizon is infinite.
(vi)   The inventory system involves only one type of item.
(vii) Replenishments are instantaneous with a known and constant lead time.

The ***total annual variable cost*** consists of the following elements:

(1) Cost of placing orders.
(2) Cost of stock holding (excluding interest charges) per year.
(3) The interest earned during the permissible settlement period for the cases when $T \geq t_1$ and $T < t_1$.
(4) Cost of interest charges for the items kept in stock.
    As items are sold before settlement of the replenishment account, the sales revenue is used to earn interest. When the replenishment account is settled, the situation is reversed, and effectively the items still in stock have to be financed at interest rate I_p. The interest is payable during the time
    $(T - t_1)$.
(5) The cost of deteriorated units.

Costs with uncertain parameters:

Practically, ordering and holding costs are often not precisely known. Fuzzy numbers can represent these imprecise parameters [40]. So considering these costs to be uncertain, it is possible to describe them with triangular fuzzy numbers.

Since in practical problems, it is not easy to decide the ordering cost for long period due to some uncountable factors, it becomes reasonable to locate the *ordering cost s* in an interval such that it is given by $\tilde{s} = (s - \Delta_1, \ s, \ s + \Delta_2)$ where $0 < \Delta_1 < s$ *and* $\Delta_1\Delta_2 > 0$, where $\Delta_1, \ \Delta_2$ are chosen appropriately.

Similarly, as we know that in a competitive market, the cost of storing a unit per day in a plan period T may fluctuate a little from its actual value, we can find a fuzzy triangular number to represent the vagueness in holding cost as well. So then taking h_p as fuzzy we have

$$\widetilde{h_p} = \left(h_p - \Delta_3, \ h_p, h_p + \Delta_4 \ \right) \text{ where } 0 < \Delta_3 < h_p \ and \ \Delta_3\Delta_4 > 0$$

The signed distance method is then used to defuzzify the fuzzy cost function with the fuzzy set-up cost and holding cost.

The signed distance of $\tilde{s}$ as given by

$$d(\tilde{s}, \tilde{0}) = s + (\Delta_2 - \Delta_1)\frac{1}{4}$$

where $d(\tilde{s}, \tilde{0}) > 0$ *and* $d(\tilde{s}, \tilde{0}) \in (s - \Delta_1, s + \Delta_2)$, $\qquad\qquad$ (A)

can be taken as the estimate of the total ordering cost during the plan period $[0, T]$ in the fuzzy sense.

Similarly, the estimate of the storage cost per unit purchase cost per day based on the signed distance is given by

$$d(\widetilde{h_p}, \tilde{0}) = h_p + (\Delta_4 - \Delta_3)\frac{1}{4}$$

where $d(\widetilde{h_p}, \tilde{0}) > 0$ *and* $d(\widetilde{h_p}, \tilde{0}) \in \left(h_p - \Delta_3, h_p + \Delta_4\right)$ $\qquad\qquad$ (B)

Formulation of the Model:

The instantaneous state of the stock level q (t) at any time t when the stock deteriorates at a constant rate $\theta, 0 < \theta < 1$, is governed by the differential equation

$$\frac{dq(t)}{dt} + \theta q(t) = -D(t), \ 0 \le t \le T \qquad\qquad (1)$$

where $q(0) = q_0, q(T) = 0$ $\qquad\qquad$ (2)

and $D(t) = a + bt$ where $a > 0, \ b > 0$.   (3)

The solution of (1) using (2) and (3) is

$$q(t) = \frac{1}{\theta}\left[e^{\theta(T-t)}\left(a - \frac{b}{\theta} + bT\right) - \left(a - \frac{b}{\theta} + bt\right)\right].$$ (4)

The initial order quantity at t = 0 is

$$q_0 = q(0) = \frac{1}{\theta}\left[e^{\theta T}\left(a - \frac{b}{\theta} + bT\right) - \left(a - \frac{b}{\theta}\right)\right].$$ (5)

The total demand during one cycle is

$$\int_0^T (a + bt)dt = aT + \frac{bT^2}{2}.$$

Number of deteriorated units $= q_0 - \left(aT + \frac{bT^2}{2}\right)$

$$= \frac{1}{\theta}\left[e^{\theta T}\left(a - \frac{b}{\theta} + bT\right) - \left(a - \frac{b}{\theta}\right)\right] - \frac{T}{2}(2a + bT) \quad (6)$$

The cost of stock holding for one cycle $= h\int_0^T q(t)dt$, where $h = ph_p$

$$= \frac{h}{\theta}\int_0^T \left[e^{\theta(T-t)}\left(a - \frac{b}{\theta} + bT\right) - \left(a - \frac{b}{\theta} + bt\right)\right]dt$$

$$= \frac{h}{\theta}\left[\frac{\left(a - \frac{b}{\theta} + bT\right)}{\theta}\left(e^{\theta T} - 1\right) - T\left(a - \frac{b}{\theta} + \frac{bT}{2}\right)\right].$$

Hence, the holding cost per unit time is

$$\frac{h}{T\theta}\left[\frac{\left(a - \frac{b}{\theta} + bT\right)}{\theta}\left(e^{\theta T} - 1\right) - T\left(a - \frac{b}{\theta} + \frac{bT}{2}\right)\right],$$ (7)

where $h = ph_p$.

We now consider the three cases of the crisp model and also write the cost functions with the fuzzy parameters and obtain the optimal solution for both crisp as well as fuzzy model.

Case I: Let $T > t_1$.

Since the interest is payable during the time $(T - t_1)$, the interest payable in one cycle is

$$pI_p \int_{t_1}^T q(t)dt = \frac{pI_p}{\theta}\int_{t_1}^T \left[e^{\theta(T-t)}\left(a - \frac{b}{\theta} + bT\right) - \left(a - \frac{b}{\theta} + bt\right)\right]dt$$

$$= \frac{pI_p}{\theta}\left[\frac{1}{\theta}\left(a - \frac{b}{\theta} + bT\right)\left\{e^{\theta(T-t_1)} - 1\right\} - (T - t_1)\left\{a - \frac{b}{\theta} + \frac{b}{2}(T + t_1)\right\}\right].$$

Hence, interest payable per unit time is

$$\frac{pI_p}{\theta T}\left[\frac{1}{\theta}\left(a-\frac{b}{\theta}+bT\right)\left\{e^{\theta(T-t_1)}-1\right\}-(T-t_1)\left\{a-\frac{b}{\theta}+\frac{b}{2}(T+t_1)\right\}\right] \tag{8}$$

Interest earned per unit time is

$$\frac{pI_e}{T}\int_0^T tD(t)dt = pI_eT\left(\frac{a}{2}+\frac{bT^2}{3}\right). \tag{9}$$

Total variable cost per cycle = Ordering cost + cost of deteriorated units + inventory holding cost + interest payable beyond the permissible period – interest earned during the cycle.

Hence, the *total variable cost per unit time* in this case is given by

$$z_1(T) = \frac{s}{T}+\frac{p}{T}\left[\frac{1}{\theta}\left\{e^{\theta T}\left(a-\frac{b}{\theta}+bT\right)-\left(a-\frac{b}{\theta}\right)\right\}-\frac{T}{2}(2a+bT)\right]$$

$$+\frac{h}{T\theta}\left[\frac{\left(a-\frac{b}{\theta}+bT\right)}{\theta}\left(e^{\theta T}-1\right)-T\left(a-\frac{b}{\theta}+\frac{bT}{2}\right)\right]$$

$$+\frac{pI_p}{T\theta}\left[\frac{\left(a-\frac{b}{\theta}+bT\right)}{\theta}\left\{e^{\theta(T-t_1)}-1\right\}-\left\{\left(a-\frac{b}{\theta}\right)(T-t_1)+\frac{b}{2}(T^2-t_1^2)\right\}\right]$$

$$-pI_eT\left(\frac{a}{2}+\frac{bT}{3}\right). \tag{10}$$

We have now to minimize $z_1(T)$ for a given value of t_1.

The necessary and sufficient conditions to minimize $z_1(T)$ for a given value of t_1 are respectively $\qquad\qquad\dfrac{dz_1(T)}{dT}=0 \tag{11}$

and $\qquad\dfrac{d^2z_1(T)}{dT^2}>0 \tag{12}$

After simplification, $\dfrac{dz_1(T)}{dT}=0$ yields the following *non-linear equation* in T:

$$2b\theta\left[p\theta+h+pI_pe^{-\theta t_1}\right]T^2e^{\theta T}+2\theta\left[p\theta\left(a-\frac{b}{\theta}\right)+h\left(a-\frac{b}{\theta}\right)+pI_p\left(a-\frac{b}{\theta}\right)e^{-\theta t_1}\right]Te^{\theta T}-2\left(a-\frac{b}{\theta}\right)\left[p\theta+h+pI_pe^{-\theta t_1}\right]e^{\theta T}-\frac{4}{3}pI_eb\theta^2T^3-\theta\left[b(h+p\theta)+paI_e\theta+pbI_p\right]T^2+2\left(a-\frac{b}{\theta}\right)\left[h+p\theta-pI_p\theta t_1+pI_p\right]-2s\theta^2-pbI_p\theta t_1^2=0. \tag{13}$$

By solving (13) for T, using Newton–Raphson method we obtain the optimal cycle length T_1^*, provided it satisfies (12).

The EOQ q_0^* for this case is given by

$$q_0{}^*(T_1^*) = q(0) = \frac{1}{\theta}\left[e^{\theta T_1^*}\left(a - \frac{b}{\theta} + bT_1^*\right) - \left(a - \frac{b}{\theta}\right)\right].$$

The minimum annual variable cost for the crisp model, $z_1\left(T_1^*\right)$ is then obtained from (10) for $T = T_1^*$.

Now from (10), we get the fuzzy *total variable cost per unit time* as

$$\widetilde{z_1}(T) = \frac{\tilde{s}}{T} + \frac{p}{T}\left[\frac{1}{\theta}\left\{e^{\theta T}\left(a - \frac{b}{\theta} + bT\right) - \left(a - \frac{b}{\theta}\right)\right\} - \frac{T}{2}(2a + bT)\right] +$$

$$\frac{p\widetilde{h_p}}{T\theta}\left[\frac{\left(a - \frac{b}{\theta} + bT\right)}{\theta}(e^{\theta T} - 1) - T\left(a - \frac{b}{\theta} + \frac{bT}{2}\right)\right] + \frac{pI_p}{T\theta}\left[\frac{\left(a - \frac{b}{\theta} + bT\right)}{\theta}\left\{e^{\theta(T-t_1)} - 1\right\} -$$

$$\left\{\left(a - \frac{b}{\theta}\right)(T - t_1) + \frac{b}{2}(T^2 - t_1^2)\right\}\right] - pI_e T\left(\frac{a}{2} + \frac{bT}{3}\right).$$

Defuzzification of fuzzy number $\widetilde{z_1}(T)$ by signed distance method [using (A) & (B)], we write the defuzzified total variable cost per unit time as

$$F_d\{(z_1(T)\} = d(\widetilde{z_1}, \tilde{0}) = z_1(T) + \frac{1}{4T}\left[(\Delta_2 - \Delta_1) + \frac{p}{\theta}(\Delta_4 - \Delta_3)\left\{\frac{\left(a - \frac{b}{\theta} + bT\right)}{\theta}(e^{\theta T} - \right.\right.$$

$$1) - T\left(a - \frac{b}{\theta} + \frac{bT}{2}\right)\right\}\right] \qquad\qquad \text{-- (14)}$$

This is an estimate of the total variable cost per unit time in the fuzzy sense based on signed distance.

Now differentiating (14) w.r.t. variable T and equating it to zero, we get

$$\frac{dF_d(z_1)}{dT} = \frac{dz_1}{dT} - \frac{1}{4T^2}(\Delta_2 - \Delta_1)$$

$$+ \frac{p}{4\theta}(\Delta_4 - \Delta_3)\left[\left\{\frac{\left(a - \frac{b}{\theta}\right)}{\theta T} + \frac{b}{\theta}\right\}\theta e^{\theta T} - (e^{\theta T} - 1)\frac{\left(a - \frac{b}{\theta}\right)}{\theta T^2}\right] = 0$$

$$\Rightarrow 2T^2\theta^2\left[2b\theta\left(p\theta + h + pi_p e^{-\theta t_1}\right)T^2 e^{\theta T} + 2\theta\left(a - \frac{b}{\theta}\right)(p\theta + h + \right.$$

$$pi_p e^{-\theta t_1})Te^{\theta T} - 2\left(a - \frac{b}{\theta}\right)(p\theta + h + pi_p e^{-\theta t_1})e^{\theta T} - 4pi_e b\theta^2 T^3/3 -$$

$$\theta\{b(h + p\theta) + pai_e\theta + pbi_p\}T^2 + (a - b/\theta)(2h + 2pi_p + 2p\theta - 2pi_p\theta t_1) -$$

$$2s\theta^2 - pbi_p\theta t_1{}^2\Big] - \theta^2(\Delta_2 - \Delta_1)/2 + p\theta(\Delta_4 - \Delta_3)/2\left[\left\{\frac{\left(a-\frac{b}{\theta}\right)T}{\theta} + bT^2\right\}\theta e^{\theta T} - \right.$$

$$\left.(a - b/\theta)(e^{\theta T} - 1)/\theta\right] = 0 \qquad\qquad\qquad \text{-- (15)}$$

Also we see that $F_d\{(z_1(T)\},\,given\;by$ (14), is strictly convex in T as $\frac{d^2 F_d(z_1)}{dT^2} > 0$.

Hence $F_d(z_1)$ is minimum by the principle of minima and maxima.

So there exists a unique optimal cycle length $T = T_1{}^*$ which minimizes $F_d(z_1(T))$ and it is the solution of equation (15).

Case II: T < t₁.

In this case, the customer earns interest on the sales revenue up to the permissible delay period and no interest is payable during this period for the items kept in stock.

Interest earned up to T = $pI_e \int_0^T tD(t)dt = pI_e T^2\left(\frac{a}{2} + \frac{bT^3}{3}\right)$

and interest earned during ($t_1 - T$) i.e. up to the permissible delay period is

$$pI_e(t_1 - T)\int_0^T D(t)dt = pI_e\left(a + \frac{b}{2}T\right)(t_1 - T)T.$$

Hence the total interest earned during the cycle

$$= pI_e T^2\left(\frac{a}{2} + \frac{bT}{3}\right) + pI_e\left(a + \frac{b}{2}T\right)(t_1 - T)T.$$

$$= pI_e T\left\{\frac{(bt_1 - a)T}{2} - \frac{1}{6}bT^2 + at_1\right\}. \qquad\qquad \text{-- (16)}$$

Total variable cost per cycle = Ordering cost + cost of deteriorated units + inventory holding cost – interest earned during the cycle.

Hence, the total variable cost per unit time in this case is

$$z_2(T) = \frac{s}{T} + \frac{p}{T}\left[\frac{1}{\theta}\left\{e^{\theta T}\left(a - \frac{b}{\theta} + bT\right) - \left(a - \frac{b}{\theta}\right)\right\} - \frac{T}{2}(2a + bT)\right] +$$

$$\frac{h}{T\theta}\left[\frac{\left(a - \frac{b}{\theta} + bT\right)}{\theta}(e^{\theta T} - 1) - T\left(a - \frac{b}{\theta} + \frac{bT}{2}\right)\right] - pI_e\left[(bt_1 - a)\frac{T}{2} - \frac{1}{6}bT^2 + at_1\right]. \quad \text{-- (17)}$$

We have now to minimize $z_2(T)$ as before for a given value of t_1.

After some simplification, $\frac{dz_2(T)}{dT} = 0$ yields the result

$$-s\theta - p\theta \left[\frac{1}{\theta}\left\{e^{\theta T}\left(a - \frac{b}{\theta} + bT\right) - \left(a - \frac{b}{\theta}\right)\right\} - \frac{T}{2}(2a + bT)\right] + p\theta T\left[\frac{1}{\theta}\left\{be^{\theta T} + \left(a - \frac{b}{\theta} + bT\right)\theta e^{\theta T}\right\} - (a + bT)\right] - h\left[\frac{1}{\theta}\left(a - \frac{b}{\theta} + bT\right)(e^{\theta T} - 1) - T\left(a - \frac{b}{\theta} + \frac{1}{2}bT\right)\right] + hT\left[\left(a - \frac{b}{\theta} + bT\right)e^{\theta T} + \frac{1}{\theta}be^{\theta T} - (a + bT)\right] - pI_e\theta T^2\left[\frac{1}{2}(bt_1 - a) - \frac{1}{3}bT\right] = 0.$$
$$\text{-- (18)}$$

The optimal cycle length $T = T_2^*$ which minimizes $z_2(T)$ is obtained by solving the equation (18) for T using Newton-Raphson method, provided $\frac{d^2 z_2(T)}{dT^2} > 0$.

The EOQ in this case is given by

$$q_0{}^*(T_2^*) = \frac{1}{\theta}\left[e^{\theta T_2^*}\left(a - \frac{b}{\theta} + bT_2^*\right) - \left(a - \frac{b}{\theta}\right)\right]$$

and the minimum annual crisp variable cost $z_2(T_2^*)$ is obtained from (17) for $T = T_2^*$.

Now from (17), we get the fuzzy total variable cost per unit time as

$$\tilde{z}_2(T) = \frac{\tilde{s}}{T} + \frac{p}{T}\left[\frac{1}{\theta}\left\{e^{\theta T}\left(a - \frac{b}{\theta} + bT\right) - \left(a - \frac{b}{\theta}\right)\right\} - \frac{T}{2}(2a + bT)\right] + \frac{p\tilde{h}_p}{T\theta}\left[\frac{\left(a - \frac{b}{\theta} + bT\right)}{\theta}(e^{\theta T} - 1) - T\left(a - \frac{b}{\theta} + \frac{bT}{2}\right)\right] - pI_e\left[(bt_1 - a)\frac{T}{2} - \frac{1}{6}bT^2 + at_1\right].$$

$$(19)$$

Defuzzification of fuzzy number $\tilde{z}_2(T)$ by signed distance method [using (A) & (B)], we write the defuzzified total variable cost per unit time in this case as

$$F_d\{(z_2(T)\} = d(\tilde{z}_2, \tilde{0})$$

$$= z_2(T) + \frac{1}{4T}(\Delta_2 - \Delta_1) + \frac{p}{4T\theta}(\Delta_4 - \Delta_3)\left[\frac{\left(a - \frac{b}{\theta} + bT\right)}{\theta}(e^{\theta T} - 1) - T\left(a - \frac{b}{\theta} + \frac{bT}{2}\right)\right].$$

$$\text{- (20)}$$

$$\frac{dF_d(z_2)}{dT} = \frac{dz_2}{dT} - \frac{1}{4T^2}(\Delta_2 - \Delta_1)$$

$$-\frac{p}{4\theta T^2}(\Delta_4 - \Delta_3)\left[\frac{\left(a - \frac{b}{\theta} + bT\right)}{\theta}(e^{\theta T} - 1) - T\left(a - \frac{b}{\theta} + \frac{bT}{2}\right)\right]$$

$$+\frac{p}{4T\theta}(\Delta_4 - \Delta_3)\left[\left(a - \frac{b}{\theta} + bT\right)e^{\theta T} + \frac{(e^{\theta T} - 1)b}{\theta} - \left(a - \frac{b}{\theta} + \frac{bT}{2}\right)\right.$$

$$\left.- \frac{Tb}{2}\right] = 0$$

$$\Rightarrow -s\theta - p\theta\{e^{\theta T}(a - b/\theta + bT) - (a - b/\theta))/\theta - T(2a + bT)/2\} +$$
$$p\theta T\left\{\left(a - \frac{b}{\theta} + bT\right)e^{\theta T} + \frac{b}{\theta}e^{\theta T} - a - bT\right\} - h\{(a - b/\theta + bT)(e^{\theta T} - 1)/\theta -$$
$$T(a - b/\theta + bT/2)\} + hT(a + bT)(e^{\theta T} - 1) - pi_e\theta T^2\{(bt_1 - a)/2 - bT/3\} -$$
$$\frac{p}{4}(\Delta_4 - \Delta_3)\left\{(a - b/\theta + bT)(e^{\theta T} - 1)/\theta - T(a - b/\theta + bT/2)\right\} +$$
$$\frac{pT(\Delta_4 - \Delta_3)}{4}(a + bT)(e^{\theta T} - 1) = 0. \qquad\qquad \text{-- (21)}$$

The solution of equation (21) which minimizes $F_d(z_2(T))$ is the optimal cycle length

$T = T_2^*$ in case of fuzziness.

Case – III: $T = t_1$.

For $T = t_1$, both the cost functions $z_1(T)$ and $z_2(T)$ become identical and the cost
function is then denoted by $z(t_1)$, say. $z(t_1)$ is obtained on substituting $T = t_1$ either in
(10) or in (17) .

Thus

$$z(t_1) = \frac{s}{t_1} + \frac{p}{t_1}\left[\frac{1}{\theta}\left\{e^{\theta t_1}\left(a - \frac{b}{\theta} + bt_1\right) - \left(a - \frac{b}{\theta}\right)\right\} - \frac{t_1}{2}(2a + bt_1)\right] + \frac{h}{\theta t_1}\left[\frac{1}{\theta}\left(a - \frac{b}{\theta} + \right.\right.$$
$$\left.\left. bt_1\right)\left(e^{\theta t_1} - 1\right) - t_1\left(a - \frac{b}{\theta} + \frac{1}{2}bt_1\right)\right] - \frac{1}{6}pI_e t_1(3a + 2bt_1). \qquad \text{-- (22)}$$

In case of fuzziness, for $T = t_1$, both the defuzzified cost functions $F_d(z_1(T))$
and $F_d(z_2(T))$ become identical and the cost function is then denoted by
$F_d(z(t_1))$, say, is obtained on substituting $T = t_1$ either in (14) or in (20). Thus

$$F_d(z(t_1)) = s/t_1 + \frac{p}{t_1}(\frac{1}{\theta}(e^{\theta t_1}(a - b/\theta + bt_1) - (a - b/\theta)) - t_1(2a + bt_1)/2)$$

$$+ph_p/(t_1\theta)((a - b/\theta + bt_1)(e^{\theta t_1} - 1)/\theta - t_1(a - b/\theta + bt_1/2))$$

$$-pi_e(t_1(bt_1 - a)/2 + at_1 - bt_1{}^2/6)$$

$$+\frac{\Delta_2 - \Delta_1}{4T} + p(\Delta_4 - \Delta_3)(a - b/\theta + bt_1)(e^{\theta t_1} - 1)/\theta - t_1(a - b/\theta +$$

$$bt_1/2)))/(4\theta t_1) \qquad\qquad\qquad -- (23)$$

Here the EOQ for both crisp and fuzzy case is

$$q_0{}^*(t_1) = \frac{1}{\theta}\left[e^{\theta t_1}\left(a - \frac{b}{\theta} + bt_1\right) - \left(a - \frac{b}{\theta}\right)\right].$$

To obtain the economic ordering policy (considering all the three cases):

Now in order to obtain the *economic ordering policy*, the following steps are to be followed:

Step 1: Determine T_1^* from (13) [crisp] or from (15) [fuzzy]. If $T_1^* \geq t_1$, obtain the respective costs as $z_1(T_1^*)$ from (10) or $F_d\{z_1(T_1^*)\}$ from (14).

Step 2: Determine T_2^* from (18) [crisp] or from (21) [fuzzy]. If $T_2^* < t_1$, obtain $z_2(T_2^*)$ from (17) or $F_d\{z_2(T_2^*)\}$ from (20), respectively.

Step 3: If $T_1^* < t_1$ and $T_2^* \geq t_1$, then evaluate $z(t_1)$ from (22) [crisp] or $F_d\{z(t_1)\}$ from (23) [fuzzy].

Step 4: Compare $z_1(T_1^*)$, $z_2(T_2^*)$ and $z(t_1)$ and take the minimum [crisp].

Compare $F_d\{z_1(T_1^*)\}$, $F_d\{z_2(T_2^*)\}$ and $F_d\{z(t_1)\}$ and take the minimum [fuzzy].

Numerical Illustrations

The solution procedure involving different decision making situations are illustrated by the following examples [41], covering all the three cases that arise in the model for both crisp and fuzzy situations:

Example-I

Let a = 1000 units per year, b = 150 units per year, I_p= 0.15 per year, I_e= 0.13 per year,   s = Rs. 200 per order, h_p= Rs. 0.12 per year, p = Rs. 20 per unit, t_1= 0.25 year, θ = 0.20.

Crisp model: (Case I & II)

Solving (13) for T, we obtain the optimal value $T_1^* = 0.284$ and the minimum average cost is $z_1(T_1^*) = 1283.53$.

Again solving (18), we have $T_2^* = 0.206$ and the minimum average cost is $z_2(T_2^*) = 1263.53$.

Here $T_1^* > t_1$ and $T_2^* < t_1$ both hold and this implies that both the cases I and II hold.

Now $z_2(T_2^*) < z_1(T_1^*)$.

Hence the *minimum average cost* in this case is $z_2(T_2^*) = \text{Rs. } 1263.53$, where the *optimal cycle length* is $T_2^* = 0.206 \text{ year} < t_1$. The EOQ is given by $q_0^*(T_2^*) = 213.82$ units.

Fuzzy model: (Case I)

Solving (15) for T, we get $T_1^* = 1.946$ and the minimum average cost is $f_d z_1(T_1^*) = \text{Rs. } 1958.9$.

Again solving (21), we have $T_2^* = 0.311$ and the corresponding minimum average cost is $f_d z_2(T_2^*) = 618.53$. Here $T_2^* > t_1$ this contradicts case-II.

Hence in this case $T_1^* > t_1$ which is case I.

Therefore, the *minimum average cost* in this case is $f_d z_1(T_1^*) = \text{Rs. } 1958.9$, the EOQ is $q_0^*(T_1^*) = 2748.52$ units and the *optimal cycle length* is $T_1^* = 1.946 \text{ year} > t_1$.

Here fuzziness increases optimal cost & the optimal cycle length slightly. The EOQ increases considerably in comparison to the crisp case.

Example-II

Let a = 1000 units per year, b = 150 units per year, I_p= 0.15 per year, I_e= 0.13 per year, s = Rs. 200 per order, h_p= Rs. 0.12 per year, p = Rs. 20 per unit, t_1= 0.25 year, θ = 0.01.

Crisp model: (Case I)

Solving (13) for T, we get $T_1^* = 0.432$ and the minimum average cost is $z_1(T_1^*) = 585.31$.

Again solving (18), we have $T_2^* = 0.274$ and the corresponding minimum average cost is $z_2(T_2^*) = 793.94$. Here $T_2^* > t_1$ this contradicts case-II.

In this case $T_1^* > t_1$ which is case I.

Therefore, the *minimum average cost* in this case is $z_1(T_1^*) = \text{Rs. } 585.31$, the EOQ is $q_0^*(T_1^*) = 447.23$ units and the *optimal cycle length* is $T_1^* = 0.432 \text{ year} > t_1 = 0.25$.

Fuzzy model: (Case II)

The same parameter values as above & $\Delta_1 = 4, \Delta_2 = 5, \Delta_3 = 2, \Delta_4 = 1$, the equation (15) gives the root as $T_1^* = 0.003$ and the corresponding minimum average cost is $f_d z_1(T_1^*) = 108473.242$.

Here we see that $T_1^* < t_1 = 0.250$, which contradicts case-I.

Again we see that the root of the equation (21) is $T_2^* = -129.46 < t_1$, which case II is and the corresponding minimum average cost is $f_d z_2(T_2^*) = -265714.031$ and $q_0^*(T_2^*) = 484310.125$. These results are practically impossible.

We now slightly modify some of these parameter values from [41] in the following Example-III.

Example-III

a=1200 units per year, b=200 units per year, I_p= 0.15 per year, I_e= 0.13 per year, s=350 per order, h_p= Rs. 0.12 per year, p = Rs. 20 per unit, t_1= 0.25 year, θ = 0.14
Crisp model: (Case I)

Solving (13) for T, we get $T_1^* = 0.356$ and the minimum average cost is $z_1(T_1^*) = 1640.134$.

Again solving (18), we have $T_2^* = 0.266$ and the corresponding minimum average cost is $z_2(T_2^*) = 1804.78$. Here $T_2^* > t_1 = 0.25$ this contradicts case-II.

In this case $T_1^* > t_1$ which is case I.

Therefore, the *minimum average cost* in this case is $z_1(T_1^*) = \text{Rs. } 1640.134$, the EOQ is $q_0^*(T_1^*) = 450.548$ units and the *optimal cycle length* is $T_1^* = 0.356 \text{ year} > t_1 = 0.25$.

Fuzzy model: (Case I)

With the same parameter values as above & $\Delta_1 = 4, \Delta_2 = 5, \Delta_3 = 2, \Delta_4 = 1$ (for fuzzy parameters s & h_p), **w**e get the output as follows:

It is seen that $T_2^* = 0.452$ such that $T_2^* > t_1 = 0.250$, which is a contradiction. Again it is seen that the *optimal cycle length* of fuzzy model is $T_1^* = 1.6803$ such that $T_1^* > t_1 = 0.250$, which is case I. Thus, the *optimal cost of fuzzy model* is $\mathbf{f_d z_1(T_1^*) = Rs.}$ **176.3825**. The EOQ of fuzzy model is q0*=2603.998 units.

The optimal cost decreases considerably due to fuzziness whereas the EOQ & the optimal cycle length increase considerably.

But on decreasing the deterioration rate to $\boldsymbol{\theta = 0.05}$, keeping the other parameter values unchanged we get $T_1^* = .006 < t_1 = 0.25$, which contradicts case-I and $T_2^* = 0.864 > t_1$ which contradicts case-II. In this case, $T^* = t_1 = 0.25$ which is case III. The *optimal fuzzy cost* in this case is $\boldsymbol{f_d z (t_1) = Rs. 752.466}$ and the EOQ is $q_0^* = 308.185$ units for *optimal cycle lengt□ $t_1 = 0.25$* year.

Thus on decreasing the deterioration rate in the fuzzy model the optimal cost increases slightly whereas the EOQ & the optimal cycle length decrease considerably.

Again on increasing the deterioration rate to $\boldsymbol{\theta = 0.2}$, keeping the other parameter values unchanged we get $T_1^* = 1.859 > t_1 = 0.25$, which is case-I and $T_2^* = 0.372 > t_1$ which contradicts case-II. In this case, *optimal cycle length* is $T_1^* = 1.859$, the *optimal cost* is $\mathbf{f_d\, z_1(T_1^*) = 2259.291}$ and the EOQ of is q0*=3147.135.

Whereas on increasing the deterioration rate in the fuzzy model the optimal cost increases considerably, the EOQ & the optimal cycle length increase slightly.

Now for the further increase of deterioration rate to $\boldsymbol{\theta = 0.427}$, it is seen that $T_2^* = 0.250 = t_1$. The optimal cost in this case is $f_d z_2(T_2^*) = f_d z (t_1) = 1949.506$ and the EOQ is $q_0^* = 323.311$ units for $t_1 = 0.25$ year. Again the optimal value $T_1^* = 1.003 > t1 = 0.250$, which is case I and the corresponding minimum average cost is $f_d z_1(T_1^*) = 4505.461$ and the EOQ is $q_0^* = 1637.164$.

Here $T_1^* > t_1$ and $T_2^* = t_1$ both hold and this implies that both the cases I and III hold.

Now $f_d z_2(T_2^*) < f_d z_1(T_1^*)$. Hence the *minimum average cost* in this case is $\mathbf{f_d z_2(T_2^*) = \boldsymbol{f_d z} (t_1) = 1949.506}$, where the *optimal cycle length* is $T_2^* = 0.25 = t_1$. The EOQ is given by $q_0^*(T_2^*) = 323.311$.

Thus, for much greater deterioration rate in the fuzzy model the optimal cost increases considerably. Whereas the EOQ and the optimal cycle length decrease considerably, results being similar to the case of very small deterioration rate, the optimal cycle length being equal to the permissible delay period.

Example-IV

Let a = 1000 units per year, b = 150 units per year, I_p= 0.15 per year, I_e= 0.13 per year,   s = Rs. 200 per order, h_p= Rs. 0.12 per year, p = Rs. 40 per unit, t_1= 0.25 year, θ = 0.20.

Crisp model: (Case II)

Solving (13) for T, we obtain the optimal value $T_1^* = 0.232$ and the optimal cost $z_1(T_1^*) = 1792.29$. Here $T_1^* < t_1$ which contradicts case I.

Again solving (18), we have $T_2^* = 0.147$ and the minimum average cost is $z_2(T_2^*) = 1395.29$. In this case, $T_2^* < t_1$ which is case II.

Therefore, *the minimum average cost* in this case is $z_2(T_2^*) = 1395.29$, the EOQ is $q_0^*(T_2^*) = 150.81$ and *the optimal cycle length* is $T_2^* = 0.147 < t_1$.

Fuzzy model: (Case I & II)

With the same parameter values as above & $\Delta_1 = 4, \Delta_2 = 5, \Delta_3 = 2, \Delta_4 = 1$, solving equation (21) gives the optimal cycle length of fuzzy model as $T_2^* = 0.222$ where $T_2^* < t_1 = 0.250$, which is case II. The optimal cost of fuzzy model is is $f_d z_2(T_2^*) = 486.457$ is less than in the crisp model. The EOQ of fuzzy model is $q_0^*(T_2^*) = 230.563$.

Again the equation (15) gives the root as $T_1^* = 1.942 > t_1 = 0.250$, and the corresponding minimum average cost is $f_d z_1(T_1^*) = 3801.472$ & $q_0^*(T_1^*) = 2741.762$.

Here $T_1^* > t_1$ and $T_2^* < t_1$ both hold and this implies that both the cases I and II hold.

Now $f_d z_2(T_2^*) < f_d z_1(T_1^*)$.

Hence the *minimum average cost* in this case is $f_d z_2(T_2^*) = Rs. 486.457$, where the *optimal cycle length* is $T_2^* = 0.222$ year $< t_1$. The EOQ is given by $q_0^*(T_2^*) = 230.563$ units.

Here we see that fuzziness gives cost effective results. The optimal cost decreases considerably.

The EOQ & the optimal cycle length increase slightly.

Example-V

Let a = 1300 units per year, b = 100 units per year, I_p= 0.5 per year, I_e= 0.01 per year, s = Rs. 97 per order, h_p= Rs. 0.12 per year, p = Rs. 40 per unit, t_1= 0.09 year, θ = 0.3.

Crisp model: (Case III)

In this case, $T_1^* = T_2^* = 0.09 = t_1$, this is case III.

The *optimal cost* in this case is $z(t_1) = $ Rs.2050.56 and the EOQ is $q_0^* = 119.01$ *units* for $t_1 = .09$ *year*.

Fuzzy model: (Case I)

Solving (15) for T, we get $T_1^* = 1.132 > t_1$ and the minimum average cost is $f_d\,z_1(T_1^*) = 20451.732$.

Again solving (21), we have $T_2^* = 0.141$ and the corresponding minimum average cost is

$f_d z_2(T_2^*) = 1316.133$. Here $T_2^* > t_1$ this contradicts case-II.

Hence in this case $T_1^* > t_1$ which is case I.

Therefore, the *minimum average cost* in this case is $f_d\,z_1(T_1^*) = $ Rs. 20451.732, the EOQ is

$q_0^*(T_1^*) = 1833.015$ units and the *optimal cycle length* is $T_1^* = 1.132$ year $> t_1$.

Here it is seen that fuzziness increases the optimal cost & EOQ considerably. The optimal cycle length also increases to a large extent.

An Analysis

On taking the parameter values as (as in the numerical Example-IV), the following nature of the optimal solution is observed on varying t_1 and θ for a *less expensive item* (p = Rs. 40 per unit) [shown in Table-I and III] to a *more expensive item* (p = Rs. 200 per unit) [shown in Table-II & IV]:

Crisp model:

For a *less expensive item* (p = Rs. 40 per unit) the following observations are made [shown in Table-I]:

- The cycle length increases marginally, the order quantity increases slightly and the cost decreases slightly as the **credit period (t_1) increases keeping θ fixed.**

- **As the value of θ increases**, **keeping the credit period t_1 fixed,** there is significant reduction in both the cycle length and the order quantity while the cost increases considerably.

However for a *very expensive item* (p = Rs. 200 / unit) [shown in Table-II] it is seen that the cycle length, the order quantity and the average system cost all undergo considerable changes.

Fuzzy model:

- In this case for a *less expensive item* (p = Rs. 40 per unit) the following observations are made [shown in Table-III]:

- The cycle length & the order quantity decreases considerably and the cost increases considerably **if the credit period (t_1) increases in $0 \leq t_1 \leq .05$, keeping θ fixed and small**. Beyond this range there is no significant change in any of these quantities.

- The above situation is completely different when the **deterioration rate θ is high**. Then the cycle length increases marginally, the order quantity increases slightly and the cost decreases slightly as the credit period (t_1) increases keeping θ fixed. **This is similar to the crisp model**

- **As the value of θ increases, keeping the credit period t_1 fixed**, there is significant increase in both the cycle length and the order quantity while the cost decreases considerably. **This is completely opposite to that in the crisp model.**

Same as the crisp model, for a *very expensive item* (p = Rs. 200 / unit) [shown in Table-IV] it is seen that the cycle length, the order quantity and the average system cost all undergo considerable changes.

Table-I: (*Crisp model*) Results for $p = Rs. 40$ per unit

t_1 Yrs.	T_1^* or T_2^* (yrs)	$z_1(T_1^*)$ or $z_2(T_2^*)(Rs.)$	q_0^* units	Optimal cycle length
		$\theta = .01$		
0	0.252	1569.92	256.62	T_1^*
0.05	0.257	1293.44	260.36	T_1^*
0.10	0.271	1072.94	277.27	T_1^*
		$\theta = 0.10$		
0	0.198	1989.25	203.15	T_1^*
0.05	0.207	1719.89	207.68	T_1^*
0.10	0.214	1522.66	219.44	T_1^*
		$\theta = 0.20$		
0	0.166	2370.09	171.08	T_1^*
0.05	0.170	2107.29	174.90	T_1^*
0.10	0.180	1931.33	184.79	T_1^*

Table-II: (*Crisp model*) Results for $p = Rs. 200$ per unit

t_1 Yrs.	T_1^* or T_2^* (yrs)	$z_1(T_1^*)$ or $z_2(T_2^*)(Rs.)$	q_0^* units	Optimal cycle length
		$\theta = .01$		
0	0.114	3485.11	115.14	T_1^*
0.05	0.125	2285.97	125.95	T_1^*
0.10	0.152	1589.08	153.52	T_1^*
		$\theta = 0.10$		
0	0.090	4411.78	91.08	T_1^*
0.05	0.098	3296.12	99.62	T_1^*
0.10	0.075	2697.22	75.93	T_2^*
		$\theta = 0.20$		
0	0.076	5253.46	76.60	T_1^*
0.05	0.083	4213.92	83.78	T_1^*
0.10	0.066	3413.56	67.00	T_2^*

Table-III: *(Fuzzy model)* Results for $p = Rs.\,40$ per unit

t_1 Yrs.	T_1^* or T_2^* (yrs)	$fz_1(T_1^*)$ or $fz_2(T_2^*)(Rs.)$	q_0^* units	Optimal cycle length
		$\theta = .01$		
0	0.002	104816.69	1.9109	T_1^*
0.05	0.00001	20024740	.01	T_2^*
0.10	0.00001	20024480	.01	T_2^*
		$\theta = 0.10$		
0	0.016	12638.212	15.872	T_1^*
0.05	0.00001	20024740	.01	T_2^*
0.10	0.00001	20024480	.01	T_2^*
		$\theta = 0.20$		
0	1.915	5655.118	2689.349	T_1^*
0.05	1.920	5260.297	2698.854	T_1^*
0.10	1.925	4877.692	2708.845	T_1^*

Table-IV: *(Fuzzy model)* Results for $p = Rs.\,200$ per unit

t_1 Yrs.	T_1^* or T_2^* (yrs)	$z_1(T_1^*)$ or $z_2(T_2^*)(Rs.)$	q_0^* units	Optimal cycle length
		$\theta = .01$		
0	0.0009	234260.594	0.855	T_1^*
0.05	0.00001	20023700	0.01	T_2^*
0.10	0.0001	1999900.125	0.100001	T_2^*
		$\theta = 0.10$		
0	0.0071	28302.752	7.08	T_1^*
0.05	0.00001	20023700	.01	T_2^*
0.10	0.00001	20022400	.01	T_2^*
		$\theta = 0.20$		
0	1.912	27797.604	2683.984	T_1^*
0.05	1.917	25825.514	2693.474	T_1^*
0.10	0.0996	1401.217	101.365	T_2^*

CONCLUDING REMARKS

This model is a modification of [41] with introduction of fuzziness. Comparisons with the crisp model show that fuzziness in the setup cost and holding cost changes the optimal cost & EOQ sometimes considerably or sometimes to a small extent compared to the crisp model, as shown by the examples. In some cases fuzziness gives cost- effective results.

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