International Journal of Computational and Applied Mathematics.
ISSN 1819-4966 Volume 12, Number 2 (2017), pp. 441-448

http://www.ripublication.com

MHD Boundary Layer Flow Past Over a Shrinking Sheet with Heat Transfer And Mass Suction

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Abstract

This paper considers the problem of MHD boundary layer flow and heat transfer past a shrinking with wall mass suction. The governing boundary layer equations for fluid flow and energy are reduced into ordinary differential equations by means of similarity transformations. Numerical solutions of the resulting similarity equations are obtained and the effects of various parameters are presented and discussed.

Keywords: MHD boundary layer flow, similarity transformation, heat transfer, shrinking sheet.

NOMENCLATURE

B_0	Constant applied magnetic field
c	Shrinking constant
M	Magnetic parameter
c_p	Specific heat of the fluid
f	Dimensionless stream function
Q	Volumetric rate of heat generation
S	Suction parameter
Pr	Prandtl number
T	Temperature of the fluid
T_w	Temperature at the wall

T_∞	Free stream temperature
u, v	Velocity component of the fluid along the x and y directions, respectively
x, y	Cartesian coordinates along the surface and normal to it, respectively

Greek symbols

ρ	Density of the fluid
μ	Viscosity of the fluid
σ_e	Electrical conductivity
η	Dimensionless similarity variable
κ	Thermal conductivity
υ	Kinematic viscosity
Ψ	Stream function
λ	Heat source/sink parameter
θ	Dimensionless temperature

Superscript

$'$	Derivative with respect to η

Subscripts

w	Properties at the plate
∞	Free stream condition

INTRODUCTION

The study of boundary layer flow over a stretching sheet has generated much interest in recent years in view of its significant applications in industrial manufacturing such as glass-fibber and paper production, hot rolling, wire drawing, drawing of plastic films, metal and polymer extrusion and metal spinning. Both the Kinematics of stretching and simultaneous heating or cooling during such processes has a decisive influence on the quality of the final products (Magyari and Keller [1]). In recent years, MHD flow problem have become more important industrially. Indeed, MHD laminar boundary layer behavior over a stretching surface is significant type of flow having considerable practical application in chemical engineering electrochemistry and polymer processing. In his pioneering work, Sakiadis [2] developed the flow field due to a flat surface, which is moving with a constant velocity in a quiescent fluid. Crane [3] extended the work of Sakiadis [2] for the two-dimensional problem where the surface velocity is proportional to the distance from the flat surface. As many natural phenomena and engineering problems are worth being subjected to MHD analysis, the effect of transverse magnetic field on the laminar flow over a stretching surface was studied by number of researchers [4-8].

The boundary layer flow of an incompressible viscous fluid over a shrinking sheet has received considerable attention of modern day researchers because of its increasing application to many engineering systems. Wang [9] first pointed out the flow over a shrinking sheet when he was working on the flow of a liquid film over an unsteady stretching sheet. Later, Miklavcic and Wang [10] obtained an analytical solution for steady viscous hydrodynamic flow over a permeable shrinking sheet. Then, Hayat et al. [11] derived both exact and series solution describing the magnetohydrodynamic boundary layer flow of a second grade fluid over a shrinking sheet. The problem of stagnation flow towards a shrinking sheet was studied by Wang [12]. Nadeem and Awais [13] studied thin film flow of an unsteady shrinking sheet through porous medium with variable viscosity. Viscous flow over an unsteady shrinking sheet with mass transfer was studied by Fang and Zhang [14]. Fang and Zhang [15] solved the Full N-S equation analytically for two dimensional MHD viscous flow due to a shrinking sheet. Fang and Zhang [16] investigated the heat transfer characteristics of the shrinking sheet problem with a linear velocity. Later on, Noor et al. [17] studied the MHD viscous flow due to shrinking sheet using Adomian decomposition Method (ADM) and they obtained a series solution. Sajid and Hayat [18] applied homotopy analysis method for the MHD viscous flow due to a shrinking sheet. Midya [19] studied the magnetohydrodynamic viscous flow and heat transfer over a linearly shrinking porous sheet. Effect of chemical reaction, heat and mass transfer on nonlinear boundary layer past a porous shrinking sheet in the presence of suction was studied numerically by Muhaimin et al. [20]. Midya [21] obtained a closed form analytical solution for the distribution of reactant solute in a MHD boundary layer flow over a shrinking sheet.

Motivated by works mentioned above and practical applications, the main concern of the present paper is to study the problem of MHD boundary layer flow and heat transfer past a shrinking with wall mass suction.

FORMULATION OF THE PROBLEM

Let us consider two dimensional laminar boundary layer flows over a permeable shrinking sheet in an incompressible electrically conducting fluid, where x-axis is along the sheet and y-axis perpendicular to it, the applied magnetic field B_0 is transversely to x-axis. The magnetic Reynolds number of the flow is taken to be small enough so that the induced magnetic field can be neglected. Under the usual boundary layer approximations, the governing equation of continuity, momentum and energy under the influence of externally imposed transverse magnetic field are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \dots(1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma_e B_0^2}{\rho}u \qquad \dots(2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\kappa}{\rho C_p}(T - T_\infty) + \frac{Q}{\rho c_p}(T - T_\infty) \qquad \dots(3)$$

Along with the boundary conditions are:

$$y = 0: u = U_w(x) = -cx, v = v_w, T = T_w$$

$$y \to \infty: u \to 0, T \to T_\infty \qquad \dots(4)$$

The continuity equation (1) is satisfied by introducing a stream function Ψ such that $u = \frac{\partial \Psi}{\partial y}$ and $v = -\frac{\partial \Psi}{\partial x}$.

The momentum equation can be transformed into the corresponding ordinary nonlinear differential equation by using the following similarity transformations:

$$\eta = y\left(\frac{c}{\nu}\right)^{1/2}, \Psi = \sqrt{c\nu}xf(\eta) \quad \text{and } T = T_\infty + (T_w - T_\infty)\theta(\eta) \qquad \dots(5)$$

Where η is the independent similarity variable. The transformed nonlinear ordinary differential equations are:

$$f''' + ff'' - f'^2 - Mf' = 0 \qquad \dots(6)$$

$$\theta'' + Pr(f\theta' - \lambda\theta) = 0 \qquad \dots(7)$$

The transformed boundary conditions are:

$$f(0) = S, \ \ f'(0) = -1, \theta(0) = 1$$

$$\text{and } f'(\infty) \to 0, \theta(\infty) \to 0. \qquad \dots(8)$$

Where prime denotes differentiation with respect to η , $M = \frac{\sigma_e B_0^2}{c\rho}$ is the magnetic parameter, $Pr = \frac{\mu c_p}{\kappa}$ is the Prandtl number, $\lambda = \frac{Q}{\rho c C_p}$ is the heat source ($\lambda<0$) or sink ($\lambda>0$) parameter and $S = \frac{v_w}{(c\nu)^{1/2}} (> 0)$ is the mass suction parameter.

NUMERICAL SOLUTION AND DISCUSSION

The non-linear differential equations (6) and (7) subject to the boundary conditions (8) are solved numerically using Runge-Kutta-Fehlberg Forth-Fifth order method. To solve this equation we adopted symbolic algebra software Maple. Maple uses the well known Runge-Kutta-Feulberg Forth-Fifth (RKF45) order method to generate the numerical solution of boundary value problem.

Figure 1 shows the effect of magnetic parameter (M) on the velocity profile. From this plot it is observed that the effect of increasing values of M is to increases the velocity distribution in the flow region i.e. momentum boundary layer thickness decreases. This is because of Lorentz force arising from the interaction of magnetic and electric fields during the motion of the fluid.

Figure 2 which illustrate the effect of magnetic parameter (M) on the temperature profiles. We infer from this figure that the temperature decreases with an increase in the M. In this case temperature asymptotically approaches to zero in free stream region. This is in contrast to the effects of other parameters on heat transfer.

Figure 3 is plotted for the velocity profiles for different values of suction parameter (S). We observe that the effect of S increases the velocity profile i.e. momentum boundary layer thickness decreases.

Figure 4, which is a graphical representation of the temperature profile for different values of suction parameter (S) versus η. We infer from this figure that temperature of the fluid decreases with increasing values of S.

Figure 5 shows the effect of heat source or sink parameter λ on the temperature profile. It is evident that the temperature decreases for increasing strength of the heat sink and due to increase of heat source strength the temperature increases. Therefore, the thickness of thermal boundary decreases with increasing the heat sink parameter, but it is increasing with the heat source parameter.

Figure 6 is plotted for the skin friction coefficient $f''(0)$ for different values of suction parameter (S) versus magnetic parameter (M). We observe that for a fixed M, $f''(0)$ increase with S. Further, skin friction coefficient $f''(0)$ strongly depending on M, is found to increase with M also it is observed that $f''(0)$ increase rapidly in the small interval $(1 \leq M \leq 2.2)$.

Figure 7, which is a representation of the local dimensionless coefficient of heat transfer $-\theta'(0)$, knows as the Nusselt number for the different values of Prandtl number (Pr) versus heat source or sink parameter λ. We observe from this figure that the rate of heat transfer increase with the value of Pr. Further, for the small value of Pr, when $\lambda < 0$ heat absorption occurs and when $\lambda > 0$ heat transfer increases.

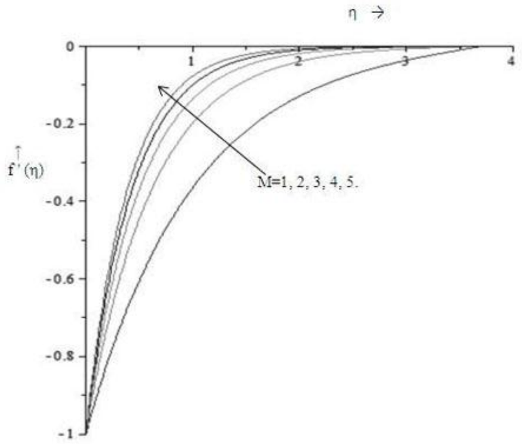

Figure1. Velocity distribution for various values of M, when S=1.0, Pr=0.71 and λ=0.25.

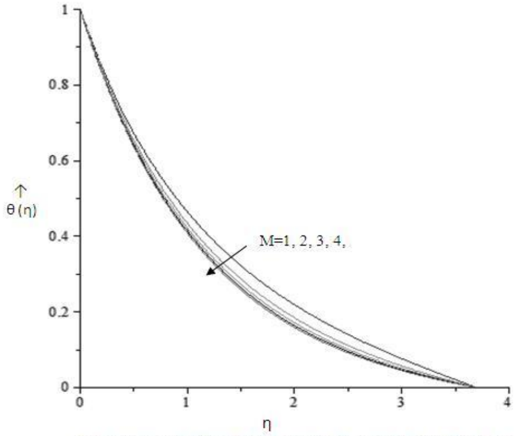

Figure2. Temperature distribution for various values of M, when S=1.0, Pr=0.71 and λ=0.5.

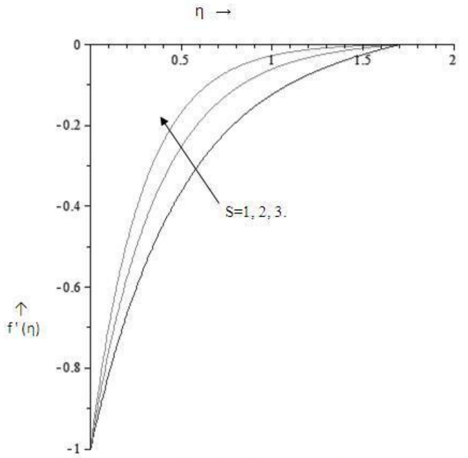

Figure3. Velocity distribution for various values of S, when M=3.0, Pr=0.71 and λ=0.5.

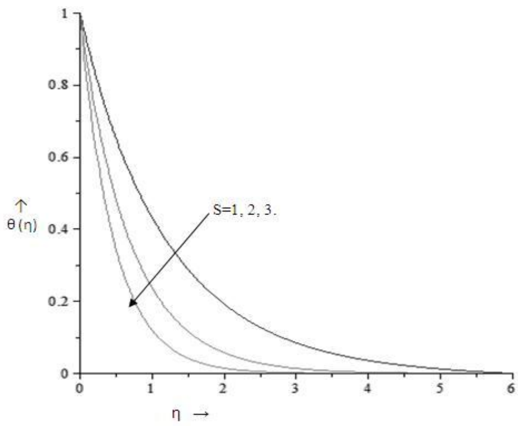

Figure4. Temperature distribution for various values of S, when M=3.0, Pr=0.71 and λ=0.5.

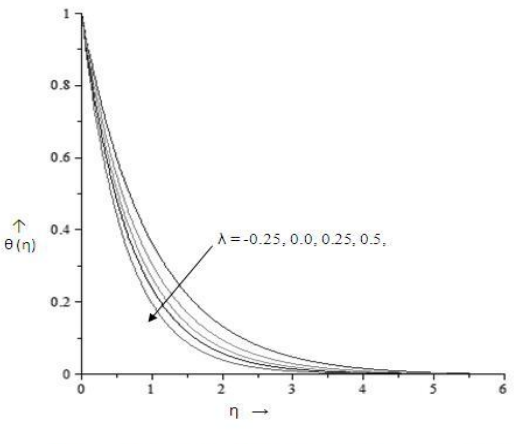

Figure5. Temperature distribution for various values of λ, when M=3.0, Pr=0.71 and S=2.0.

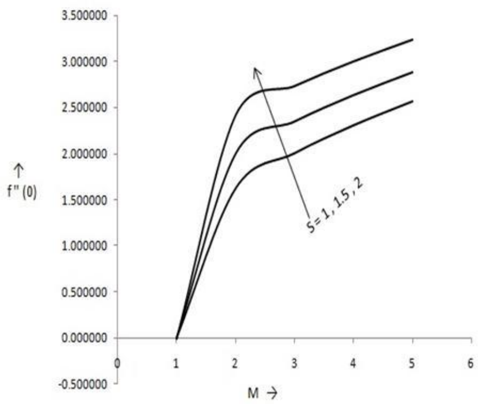

Figure6. Skin friction for various values of S, when λ=0.5, Pr=0.71

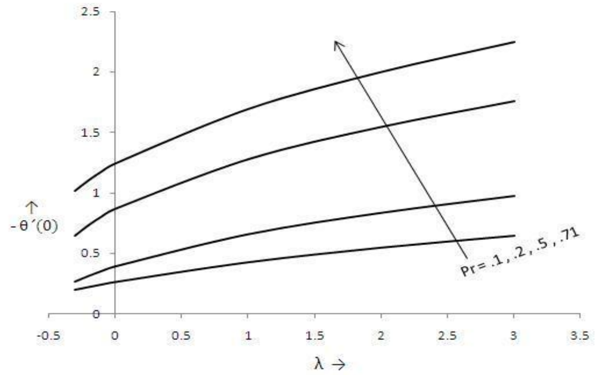

Figure7. Nusselt Number for various values of Pr. When M = 3 and S= 2

CONCLUSION

A similarity solution for the MHD boundary layer flow and heat transfer past a shrinking with wall mass suction is studied. The governing boundary layer equations were solved numerically using the Runge-Kutta-Fehlberg method using Maple software. We infer from this study that the momentum boundary layer thickness as well as temperature at a point decreases with the increasing value of magnetic parameter M and mass suction parameter S. the skin friction coefficient increases with increasing value of mass suction parameter S and magnetic parameter M. It was found that the rate of heat transfer increase with the value of Pr and for the small value of Pr, when $\lambda < 0$ heat absorption occurs and when $\lambda > 0$ heat transfer increases.

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