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Effect of Peclet Number on the Dispersion of a Solute in a Blood flowing in Non-Uniform Tube

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Abstract:

The effect of peclet number on blood flow in a tube of varying cross section is analyzed mathematically, treating blood as a homogeneous Newtonian fluid. The nonlinear differential equation are solved by using finite difference scheme and the effect of peclet number on concentration in different axial position are calculated.

Keywords: Dispersion, Peclet Number, concentration, Reynolds number.

1. INTRODUCTION:

The principal task of blood is to provide oxygen and nourishment to the tissues and the organs and to collect waste substances. The study of mass transfer and diffusion phenomena inside the arterial lumen and through the vascular wall in Blood vessels occupies an important position.

The concentration in the fluid is obtained by two phenomena- diffusion and convection. Convection is the process of the solute being carried downstream because of the flow, while diffusion is a molecular mechanism Batchelor [14].A number of models on Solute dynamics have been developed. Taylor [17] studied the dispersion due to convection alone, and outlined an elegant mathematical approximation for the case where molecular diffusion plays an important role. Aris [16] studied the problem in tubes of arbitrary cross-section. Lighthill [15] obtained an elegant solution to the dispersion of a solute in a laminar tube. Gill and Sankarasubramanian [13] obtained the exact solution to the unsteady convective diffusion equation for immiscible displacement in fully developed laminar flow in tubes. Hunt [11] obtained an analytical solution. Recently some mathematical models coupling 3D flow and solute dynamics have been developed. They are defined in a finite arterial segment of arbitrary shape, where inflow solute distribution is provided. Rappitsch et al.[10] studied numerically modelling of shear-dependent mass transfer in large arteries. They describe blood flow by the unsteady 3D incompressible Navier-Strokes equations for Newtonian fluids; solute transport is modelled by the advection-diffusion equation. They, obtained solution by using finite element method. A numerical simulation is carried out for pulsatile mass transport in a 3D arterial bend to demonstrate the influence of the arterial flow patterns in wall permeability characteristics and transmural mass transfer. In the content of atherogenesis, mass transport refers to the movement of atherogenic molecules from flowing blood into the artery wall, or vice versa. Prosi et al. [6] consider absorption and exchange through the vascular tissues . All these models provide the local concentration pattern and useful to understand the relationship between the local flow patterns, the nourishing of arterial tissues and possible pathologies derived when such a process is altered. Quarteroni et al.[7] studied mathematically and numerically, modeling of solute Dynamics in blood flow and Arterial walls. They considered the Navier-Strokes equation for an incompressible fluid, describing the blood velocity and pressure fields, are coupled with an advection, diffusion equation for the solute concentration.

It is known that geometrical effects, such as curvature, will strongly affect the flow pattern and consequently the concentration of gases and substances dissolved in the blood, Moore et al. [9] . It is worth to investigate , and to what extent, the geometry and haemodynamic factors are responsible for anomalous accumulation and altered absorption of substances on the arterial wall, leading to atherosclerotic lesions and degenerative process, Stangeby et al. [8] . Pontrelli et al. [5] studied mass transport and diffusion processes of a substance dissolved in the blood and shown the characteristics of the long wave propagation of solute in a curved vessel.

A comparative study on Mathematical analysis for unsteady dispersion of solutes in blood stream studied by D.S Sankar et.al [11] treating blood as Herschel-Bulkley fluid. Nurul Aini et.al [2] investigated dispersion of solute in blood through circular channel plates with the effect of chemical reaction, treating blood as casson fluid model. Jagadeesha et al. [3] studied solute transfer in a power law fluid flow through permeable tube.

Pontrelli et al. [4] studied a mathematical model for mass transport and diffusion phenomena in the arterial lumen and obtained analytical and numerical solutions for the characteristics of the long wave propagation and the role played by the geometry on the solute distribution and demonstrate the strong influence of curvature in artery. However, all these analyses have been developed for rigid tubes of uniform cross-section. The study of flow through tubes of non-uniform cross-section has attracted many research workers. Thus, we have examined the problem numerically for low Reynold's number flow of a viscous incompressible fluid in a circular tube of non-uniform cross-section to analyze the effect of Peclet number on concentration.

2. MATHEMATICAL FORMULATION:

The blood flow in the artery is assumed to be a homogeneous Newtonian fluid, which flows in a tube of slowly varying cross-section. The cylindrical coordinate system (R,θ,X) is chosen, in which X- axis coincides with the axis of the tube. The flow of the blood is assumed to be steady and axi-symmetric. Let U and W be the velocity components in the R and X direction respectively. The governing equation for such flows in non-dimensional form are

$$U\frac{\partial U}{\partial R}+W\frac{\partial U}{\partial X}=-\frac{1}{\rho}\frac{\partial P}{\partial R}+\nu\left[\frac{\partial^2 U}{\partial R^2}+\frac{1}{R}\frac{\partial U}{\partial R}-\frac{U}{R^2}+\frac{\partial^2 U}{\partial X^2}\right] \quad (1)$$

$$U\frac{\partial W}{\partial R}+W\frac{\partial W}{\partial X}=-\frac{1}{\rho}\frac{\partial P}{\partial X}+\nu\left[\frac{\partial^2 W}{\partial R^2}+\frac{1}{R}\frac{\partial W}{\partial R}+\frac{\partial^2 W}{\partial X^2}\right] \quad (2)$$

$$\frac{\partial U}{\partial R}+\frac{U}{R}+\frac{\partial W}{\partial X}=0 \quad (3)$$

The specific transport equation for such flow is given by

$$U\frac{\partial C}{\partial R}+W\frac{\partial C}{\partial X}=D\left[\frac{\partial^2 C}{\partial R^2}+\frac{1}{R}\frac{\partial C}{\partial R}\right] \quad (4)$$

Different substances are dissolved in blood, transported through the stream and possibly exchanged through the arterial wall. For simplicity, the presences of one solute only is considered and its concentration denoted by *C*. let assume that, the axial diffusion is small compared to radial diffusion and the tube length to diameter is so large that there are no end effects. Where ρ , ν and D are respectively the density, coefficient of kinematic viscosity and coefficient of diffusivity are taken constant.

The corresponding boundary conditions are

For fluids:

U = 0 , W=0 on R = S(X) (5)

For solute:

$C=C_1$= input concentration at X = 0 (6)

$$\frac{\partial C}{\partial R} = 0 \text{ on } R = 0 \quad (7)$$

$$\frac{\partial C}{\partial R} = 0 \text{ on } R = S(X) \quad (8)$$

Where boundary condition (6) assumes a uniform and constant concentration of solute at the entrance of the tube. The boundary conditions (7) and (8) assures an axisymmetry condition of the concentration profile.

The appropriate non-dimensional variables are

$$u = \frac{U}{U_o}, w = \frac{W}{U_o}, c = \frac{C}{C_o}, x = \frac{DX}{a_o^2 U_o}, \; r = \frac{R}{a_o S^2(x)}$$

Where U_o, C_o are the representative velocity and concentration respectively, and a_o is the radius of the tube at X = 0.

Where $S(x) = S\left(\frac{a_o^2 U_o x}{D}\right)$,Pe= Peclet number $= \frac{a_o U_o}{D}$, Re= Reynolds number $= \frac{a_o U_o}{\nu}$

The governing equation for such flows in non-dimensional form are

For fluid:

$$u\frac{\partial u}{\partial r} + \frac{w}{pe}\frac{\partial u}{\partial x} = -\frac{\partial p}{\partial r} + \frac{S(x)}{Re}\left\{\frac{1}{S^2(x)}\left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2}\right] + \frac{\partial^2 u}{\partial x^2}\right\} \quad (9)$$

$$u\frac{\partial w}{\partial r}+\frac{w}{pe}\frac{\partial w}{\partial x}=-\frac{1}{pe}\frac{\partial p}{\partial x}+\frac{S(x)}{Re}\left\{\frac{1}{S^2(x)}\left[\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right]+\frac{1}{pe^2}\frac{\partial^2 w}{\partial x^2}\right\} \qquad (10)$$

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{S(x)}{pe}\frac{\partial w}{\partial x}=0 \qquad (11)$$

For Solute:

$$\frac{1}{S(x)}\frac{u}{pe}\frac{\partial c}{\partial r}+w\frac{\partial c}{\partial x}=\frac{1}{S^2(x)}\left[\frac{\partial^2 c}{\partial r^2}+\frac{1}{r}\frac{\partial c}{\partial r}\right] \qquad (12)$$

The boundary conditions are

u = 0 , w=0 on r =1 (13)

3. SOLUTIONS:

To obtain the concentration, the axial and radial velocities for the flow through a tube are obtained from equations (9--11) with appropriate boundary conditions by Manton's work [12]. Hence, without giving a descriptive method of solution, we directly write the simplified solution of velocity radial axial velocities as follows:

$$u=4\varepsilon\frac{\left(r^3-r\right)}{peS^2(x)} \qquad (14)$$

$$w=\frac{1}{S^2(x)}\left[4\left(1-r^2\right)+\frac{Sc}{S^2(x)}\left(\frac{8}{9}r^6-4r^4+4r^2-\frac{8}{9}\right)\right] \qquad (15)$$

Where Sc=Pe/Re is the Schmidt number, ε is small parameter, which incorporates the slow variation in cross-section.

Boundary condition to be imposed on the transport equation in dimensional are

c = 1 at x =0 (16)

$$\frac{\partial c}{\partial r}=0 \text{ at } r=0 \qquad (17)$$

$$\frac{\partial c}{\partial r}=0 \text{ at } r=1. \qquad (18)$$

The expressions (14) and (15), when substituted in equation (12) and solved along with the boundary conditions (16--18) will yield the concentration of the solute. However, analytical solution of the resulting differential equation is not possible; we have opted for numerical solutions.

A finite difference scheme that can be used to numerically approximate a given differential equation. To obtain a discretization for a differential equation, it is possible to obtain a finite difference formula for every term in the differential equation and then combine these formulas in the obvious manner. Just replace each term in the differential equation with its finite difference approximation. We used Crank-Nicolson approximation.

4. RESULT AND DISCUSSION:

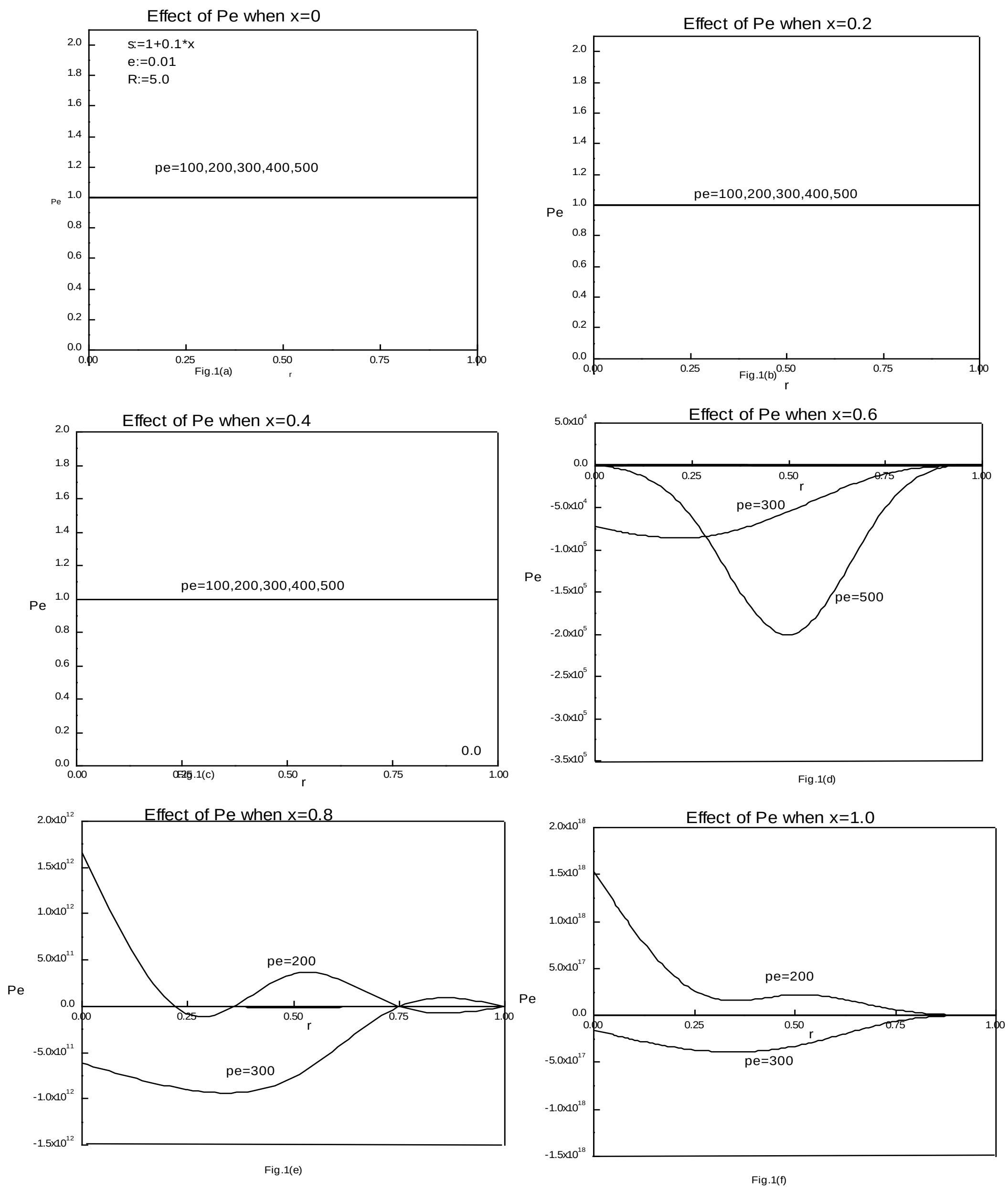
Our goal is to study the effect of Peclet number on concentration c in different axial position in a tube. The values of Peclet number Pe are taken in the range 100-500.

Here we take geometry as $S(x) = 1 + (0.1)x$

The Peclet number $\frac{a_o U_o}{D}$, is the ratio of the species transport by fluid convective motion to the species transport by molecular diffusion that is Pe, is measure of the mass transfer by convection compared to that due to diffusion. It is large, then both convective and diffusion play key roles in different parts of the flow filed. If Pe is small, then species transport is dominated by diffusion. If the convective velocity is zero, then Pe=0, mass transfer occurs only by diffusion and the concentration profile of the transport species can be obtained from a solution of the diffusion equation.

The concentration profiles in the radial direction for different values of Peclet number (Pe) at different axial positions are shown in figures 1(a)- 1(f) . it is interesting to note that for all values of Pe, the influence of Pe on concentration is negligible in the entrance region of the tube. This trend is reversed towards the discharge end. However for large values of Pe, the variation of concentration with Pe is significant. Also, it may be observed that as Pe is increases, concentration decreases at the discharge end of the tube. In each case concentration decreases with radial distance from tube axis to the wall. The concentration profiles becomes flat near the exist of the tube. This decrease in concentration is expected due to the loss of solute and it is not uniform all along the radial coordinate. Similar studies have been reported by Hunt [11] in terms of the values of Pe.

Large Pe which have been considered imply that convection is more pronounced than diffusion. Thus, the solute initially near wall and centerline move downstream as a result of the flow rather than getting diffused. The solute in the end of the tube is convicted and diffused.



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