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Stability of Jungck-Noor Iteration

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Abstract

The aim of this paper is to obtain the stability results of jungck-Noor and modified generalised jungck-Ishikawa iteration procedure satisfying some contractive condition.

Keywords: Stability of iterative procedure, modified generalised jungck-Ishikawa iteration, jungck-Noor iteration, fixed point, multi-valued mapping.

1. INTRODUCTION

Let (X, d) be a metric space and $\{x_n\}$ is a sequence defined by $x_{n+1} = Tx_n, n = 0,1,2$ ...converges to a fixed point p .Then this sequence is called stable if there exist an approximate sequence $\{y_n\}$ because of rounding errors and numerical approximations of functions, such that $\lim_{n\to\infty} y_n = p$ if and only if $\lim_{n\to\infty} \in_n = 0$. Several authors established the stability results using various contraction conditions (see, for example, Berinde [1], Czerwik [2], Harder and Hicks [3,4], Osilike [8,9], Rhoades [12,13], Singh et al [14,15]). Ostrowiski [10] was the first who obtained stability results in metric space. Harder and Hicks [3, 4] obtained stability results for general iterative procedure. Singh et al [15] discussed the stability results of Jungck and Jungck –Mann iterative procedures for a pair of Jungck-Osilike type maps in metric space, which is as follows:

Theorem 1.1 [15] Let *S* and *T* be maps on an arbitrary set *Y* with values in *X* such that $TY \subseteq SY$ and SY or TY is a complete subspace of *X*. Let z be a coincidence point of *T* and *S*, that is,
$Sz = Tz = p$ (say). Let $x_0 \in Y$ and let the sequence $\{Sx_n\}$ generated by $Sx_{n+1} = Tx_n$,$n = 0, 1...$ converge to p. Let $\{Sy_n\} \subset X$ and define $\epsilon_n = d(Sy_{n+1}, Ty_n), n = 0,1 ...$ If the pair (S,T) is a
J-contraction, that is, *S* and *T* satisfy

$$d(Tx, Ty) \leq kd(Sx, Sy), 0 \leq k < 1,$$

for all $x, y \in Y$, then
(I) $d(p, Sy_{n+1}) \leq d(p, Sx_{n+1}) + k^{n+1}d(Sx_0, Sy_0) + \sum_{i=0}^{n} k^{n-i}\epsilon_i$
further,
(II) $\lim_{n\to\infty} Sy_n = p$ if and only if $\lim_{n\to\infty} \in_n = 0$.

In this paper, we discuss the stability of Jungck-Noor iteration procedure and modified Jungck-Ishikawa iteration procedure for multivalued mapping using some contraction condition. Our work is motivated by Singh et al [15].

2. PRELIMINARIES

We use following definitions in our paper.

Definition 2.1 [5] For any $x_0 \in X$, the Jungck-Picard iteration $\{Sx_n\}_{n=0}^{\infty}$ is defined by

$$Sx_{n+1} = TSx_n, n = 0,1,2 ... \quad (1)$$

This procedure becomes Picard iteration when $S = id$ where id is the identity map in X.

Definition 2.2 [15] For any $x_0 \in X$, the Jungck-Mann iteration $\{Sx_n\}_{n=0}^{\infty}$ is defined by

$$Sx_{n+1} = (1 - \alpha_n)Sx_n + \alpha_n Tx_n \quad (2)$$

where $\{\alpha_n\}_{n=0}^{\infty}$ are real sequences in [0,1) such that $\sum_{n=0}^{\infty} \alpha_n = \infty$.

Definition 2.3[7] For any $x_0 \in X$, the Jungck-Ishikawa iteration $\{Sx_n\}_{n=0}^{\infty}$ is defined by

$$Sx_{n+1} = (1-\alpha_n)Sx_n + \alpha_n Ty_n \quad (3)$$
$$Sy_n = (1-\beta_n)Sx_n + \beta_n Tx_n$$

Where n=0,1,2... and $\{\alpha_n\}_{n=0}^{\infty}, \{\beta_n\}_{n=0}^{\infty}$ satisfies $(i)\alpha_0 = 1$ $(ii) 0 \le \alpha_n, \beta_n \le 1, n \ge 0$ $(iii) \sum_{n=0}^{\infty} \alpha_n = \infty$ $(iv) \sum_{j=0}^{n} \prod_{i=j+1}^{n} \{1-\alpha_i + a\alpha_i\}$ converges.

Definition 2.4 [6] For any $x_0 \in X$, the Jungck-Noor iteration $\{Sx_n\}_{n=0}^{\infty}$ is defined by

$$Sx_{n+1} = (1-\alpha_n)Sx_n + \alpha_n Tz_n \quad (4)$$
$$Sz_n = (1-\beta_n)Sx_n + \beta_n Tt_n$$
$$St_n = (1-\gamma_n)Sx_n + \gamma_n Tx_n$$

Where n= 0,1,2... and $\{\alpha_n\}_{n=0}^{\infty}, \{\beta_n\}_{n=0}^{\infty}$ and $\{\gamma_n\}_{n=0}^{\infty}$ satisfies $(i)\alpha_0 = 1$ $(ii) 0 \le \alpha_n, \beta_n, \gamma_n \le 1$,

$n \ge 0$ $(iii) \sum_{n=0}^{\infty} \alpha_n = \infty$ $(iv) \sum_{j=0}^{n} \prod_{i=j+1}^{n} \{1-\alpha_i + a\alpha_i\}$ converges.

Here we define a new modified generalised Jungck-Ishikawa iteration procedure $\{Sx_n\}_{n=0}^{\infty}$ as

For any $x_0 \in X$,

$$Sx_{n+1} = (1-\alpha_n)Sx_n + \beta_n Tx_n + (\alpha_n - \beta_n)Tz_n \quad (5)$$
$$Sz_n = (1-\gamma_n)Sx_n + \gamma_n Tx_n$$

Where n= 0,1,2... and $\{\alpha_n\}_{n=0}^{\infty}, \{\beta_n\}_{n=0}^{\infty}$ and $\{\gamma_n\}_{n=0}^{\infty}$ satisfies

$(i)\alpha_0 = 1$

$(ii) 0 \le \alpha_n, \beta_n, (\alpha_n - \beta_n), \gamma_n \le 1,\ n \ge 0$

$(iii) \sum_{n=0}^{\infty} \alpha_n = \infty$

$(iv) \sum_{j=0}^{n} \prod_{i=j+1}^{n} \{1-\alpha_i + a\alpha_i\}$ converges.

Note: 1) If $\beta_n = 0$ we get the Jungck-Ishikawa iteration

2) If $\beta_n = \gamma_n = 0$ we get the Jungck-Mann iteration

Definition 2.5 [11] Let (X, d) be a metric space and Y be an arbitrary nonempty set.

$CL(X) = \{A; A$ is a non empty closed subset of X},

$N(X) = \{A; A$ is a non-empty subset of X},

For $x \in X$ and $B \in N(X)$ is defined as

$D(a, B) = \inf\{d(a, b): b \in B\}$

$H(A, B) = \max\{\sup D(a, B); a \in A, \sup D(A, b); b \in B\}$

H is called a generalized Hausdorff on $CL(X)$.

Definition 2.6 [14] Let (X,d) be a metric space and $Y \subseteq X$. Let $T: Y \to CL(X), S: Y \to X$ and SY or TY is a complete subspace of X. Let T and S have a common fixed point p. For any $x_0 \in Y$ there exist a sequence $\{Sx_n\}$ generated by $Sx_{n+1} \in Tx_n, n = 1,2 \ldots$ converges to p. Let $\{Sy_n\} \subseteq X$ and set $\epsilon_n = H(Sy_{n+1}, Ty_n), n = 0,1,2 \ldots$ then this iterative procedure is (S,T) stable if
$\lim_{n\to\infty} \epsilon_n = 0$ implies that $\lim_{n\to\infty} Sy_n = p$.

3. MAIN RESULT

Theorem 3.1: Let (X,d) be a metric space and $Y \subseteq X$. Let $T: Y \to CL(X), S: Y \to X$ and SY or TY is a complete subspace of X. Let T and S have a common fixed point p. For any $x_0 \in Y$ there exist a sequence defined in (definition 2.4) converges to p. Let $\{Sy_n\} \subseteq X$ such that

$$Sy_{n+1} = (1-\alpha_n)Sy_n + \alpha_n Tv_n$$

$$Sv_n = (1-\beta_n)Sy_n + \beta_n Ts_n$$
$$Ss_n = (1-\gamma_n)Sy_n + \gamma_n Ty_n$$

And set

$$\epsilon_n = H(Sy_{n+1}, (1-\alpha_n)Sy_n + \alpha_n Tv_n)$$

satisfying the condition

$$H(Tx,Ty) \le ad(Sx,Sy) + LD(Sx,Tx) \qquad (6)$$

For $a \in (0,1)$ and $L \ge 0$, then

A) $$\begin{aligned} d(p, Sy_{n+1}) \le\ & d(p, Sx_{n+1}) + \prod_{i=0}^{n}(1-\alpha_i + a\alpha_i) d(Sx_0, Sy_0) \\ & + La^2 \sum_{j=0}^{n} \alpha_j \beta_j \gamma_j \prod_{i=j+1}^{n}(1-\alpha_i + a\alpha_i)\, D(Sx_i, Tx_i) \\ & + La \sum_{j=0}^{n} \alpha_j \beta_j \prod_{i=j+1}^{n}(1-\alpha_i + a\alpha_i) D(St_i, Tt_i) \\ & + L \sum_{j=0}^{n} \alpha_j \prod_{i=j+1}^{n}(1-\alpha_i + a\alpha_i) D(Sz_i, Tz_i) \\ & + \sum_{j=0}^{n} \prod_{i=j+1}^{n}(1-\alpha_i + a\alpha_i)\, \epsilon_j \end{aligned} \qquad (7)$$

Where the product is 1 when $j = n$. Also

B) $\lim_{n\to\infty} Sy_n = p$ if and only if $\lim_{n\to\infty} \epsilon_n = 0$

Proof: By the triangle inequality, we have

$$d(p, Sy_{n+1}) \leq d(p, Sx_{n+1}) + d(Sx_{n+1}, Sy_{n+1})$$

$$\leq d(p, Sx_{n+1}) + H((1-\alpha_n)Sx_n + \alpha_n Tz_n, Sy_{n+1})$$

$$\leq d(p, Sx_{n+1}) + H[(1-\alpha_n)Sx_n + \alpha_n Tz_n, (1-\alpha_n)Sy_n + \alpha_n Tv_n] +$$

$$H[(1-\alpha_n)Sy_n + \alpha_n Tv_n, Sy_{n+1}]$$

$$\leq d(p, Sx_{n+1}) + (1-\alpha_n)d(Sx_n, Sy_n) + \alpha_n H(Tz_n, Tv_n) + \epsilon_n$$

From (6),

$$\leq d(p, Sx_{n+1}) + (1-\alpha_n)d(Sx_n, Sy_n) + \alpha_n[ad(Sz_n, Sv_n) + LD(Sz_n, Tz_n) + \epsilon_n \tag{8}$$

But,

$$d(Sz_n, Sv_n) = H[(1-\beta_n)Sx_n + \beta_n Tt_n, (1-\beta_n)Sy_n + \beta_n Ts_n]$$

$$\leq (1-\beta_n)d(Sx_n, Sy_n) + \beta_n H(Tt_n, Ts_n)]$$

$$\leq (1-\beta_n)d(Sx_n, Sy_n) + \beta_n[ad(St_n, Ss_n) + LD(St_n, Tt_n)]$$

$$\leq (1-\beta_n)d(Sx_n, Sy_n) + \beta_n ad(St_n, Ss_n) + \beta_n LD(St_n, Tt_n)] \tag{9}$$

Again,

$$d(St_n, Ss_n) = H[(1-\gamma_n)Sx_n + \gamma_n Tx_n, (1-\gamma_n)Sy_n + \gamma_n Ty_n]$$

$$\leq (1-\gamma_n)d(Sx_n, Sy_n) + \gamma_n H(Tx_n, Ty_n)]$$

$$\leq (1-\gamma_n)d(Sx_n, Sy_n) + \gamma_n[ad(Sx_n, Sy_n) + LD(Sx_n, Tx_n)]$$

$$\leq (1-\gamma_n)d(Sx_n, Sy_n) + \gamma_n ad(Sx_n, Sy_n) + \gamma_n LD(Sx_n, Tx_n)]$$

$$\leq d(Sx_n, Sy_n) + \gamma_n LD(Sx_n, Tx_n)$$

Now equation (9) becomes,

$$\leq (1-\beta_n)d(Sx_n, Sy_n) + \beta_n a[d(Sx_n, Sy_n) + \gamma_n LD(Sx_n, Tx_n)] + \beta_n LD(St_n, Tt_n)$$

$\leq d(Sx_n, Sy_n) - \beta_n d(Sx_n, Sy_n) + a\beta_n d(Sx_n, Sy_n) + \beta_n \gamma_n aLD(Sx_n, Tx_n) + \beta_n LD(St_n, Tt_n)$

$\leq d(Sx_n, Sy_n) + \beta_n \gamma_n aLD(Sx_n, Tx_n) + \beta_n LD(St_n, Tt_n)$

Equation (8) becomes,

$d(p, Sy_{n+1}) \leq d(p, Sx_{n+1}) + (1-\alpha_n)d(Sx_n, Sy_n) + \alpha_n a[d(Sx_n, Sy_n) + \beta_n \gamma_n aLD(Sx_n, Tx_n) + \beta_n LD(St_n, Tt_n) + \alpha_n LD(Sz_n, Tz_n) + \epsilon_n$

$\leq d(p, Sx_{n+1}) + (1-\alpha_n + a\alpha_n)d(Sx_n, Sy_n) + a^2\alpha_n\beta_n\gamma_n LD(Sx_n, Tx_n) + a\alpha_n\beta_n LD(St_n, Tt_n) + \alpha_n LD(Sz_n, Tz_n) + \epsilon_n$

Also,

$d(Sx_n, Sy_n) = H[(1-\alpha_{n-1})Sx_{n-1} + \alpha_{n-1}Tz_{n-1}, Sy_n)]$

$\leq H[(1-\alpha_{n-1})Sx_{n-1} + \alpha_{n-1}Tz_{n-1}, (1-\alpha_{n-1})Sy_{n-1} + \alpha_{n-1}Tv_{n-1}] + H[(1-\alpha_{n-1})Sy_{n-1} + \alpha_{n-1}Tv_{n-1}, Sy_n]$

$\leq H[(1-\alpha_{n-1})Sx_{n-1} + \alpha_{n-1}Tz_{n-1}, (1-\alpha_{n-1})Sy_{n-1} + \alpha_{n-1}Tv_{n-1}] + \epsilon_{n-1}$

$\leq (1-\alpha_{n-1})d(Sx_{n-1}, Sy_{n-1}) + \alpha_{n-1}H(Tz_{n-1}, Tv_{n-1}) + \epsilon_{n-1}$

$\leq (1-\alpha_{n-1})d(Sx_{n-1}, Sy_{n-1}) + \alpha_{n-1}[ad(Sz_{n-1}, Sv_{n-1}) + LD(Sz_{n-1}, Tz_{n-1})] + \epsilon_{n-1}$

$\leq (1-\alpha_{n-1})d(Sx_{n-1}, Sy_{n-1}) + \alpha_{n-1}ad(Sz_{n-1}, Sv_{n-1}) + \alpha_{n-1}LD(Sz_{n-1}, Tz_{n-1}) + \epsilon_{n-1}$

Now,

$d(Sz_{n-1}, Sv_{n-1}) = H[(1-\beta_{n-1})Sx_{n-1} + \beta_{n-1}Tt_{n-1}, (1-\beta_{n-1})Sy_{n-1} + \beta_{n-1}Ts_{n-1}]$

$\leq (1-\beta_{n-1})d(Sx_{n-1}, Sy_{n-1}) + \beta_{n-1}H(Tt_{n-1}, Ts_{n-1})]$

$\leq (1-\beta_{n-1})d(Sx_{n-1}, Sy_{n-1}) + \beta_{n-1}[ad(St_{n-1}, Ss_{n-1}) + LD(St_{n-1}, Tt_{n-1})]$

$$\leq (1-\beta_{n-1})d(Sx_{n-1}, Sy_{n-1}) + \beta_{n-1}ad(St_{n-1}, Ss_{n-1}) + \beta_{n-1}LD(St_{n-1}, Tt_{n-1})] \qquad (10)$$

Again,

$$d(St_{n-1}, Ss_{n-1}) = H[(1-\gamma_{n-1})Sx_{n-1} + \gamma_{n-1}Tx_{n-1}, (1-\gamma_{n-1})Sy_{n-1} + \gamma_{n-1}Ty_{n-1}]$$

$$\leq (1-\gamma_{n-1})d(Sx_{n-1}, Sy_{n-1}) + \gamma_{n-1}H(Tx_{n-1}, Ty_{n-1})]$$

$$\leq (1-\gamma_{n-1})d(Sx_{n-1}, Sy_{n-1}) + \gamma_{n-1}[ad(Sx_{n-1}, Sy_{n-1}) + LD(Sx_{n-1}, Tx_{n-1})]$$

$$\leq (1-\gamma_{n-1})d(Sx_{n-1}, Sy_{n-1}) + \gamma_{n-1}ad(Sx_{n-1}, Sy_{n-1}) + \gamma_{n-1}LD(Sx_{n-1}, Tx_{n-1})]$$

$$\leq d(Sx_{n-1}, Sy_{n-1}) + \gamma_{n-1}LD(Sx_{n-1}, Tx_{n-1})$$

Now equation (10) becomes,

$$d(Sz_{n-1}, Sv_{n-1}) \leq (1-\beta_{n-1})d(Sx_{n-1}, Sy_{n-1})$$
$$+\beta_{n-1}a[d(Sx_{n-1}, Sy_{n-1}) + \gamma_{n-1}LD(Sx_{n-1}, Tx_{n-1})]$$
$$+\beta_{n-1}LD(St_{n-1}, Tt_{n-1})$$
$$\leq d(Sx_{n-1}, Sy_{n-1}) - \beta_{n-1}d(Sx_{n-1}, Sy_{n-1}) + a\beta_{n-1}d(Sx_{n-1}, Sy_{n-1})$$
$$+\beta_{n-1}\gamma_{n-1}aLD(Sx_{n-1}, Tx_{n-1}) + \beta_{n-1}LD(St_{n-1}, Tt_{n-1})$$
$$\leq d(Sx_{n-1}, Sy_{n-1}) + \beta_{n-1}\gamma_{n-1}aLD(Sx_{n-1}, Tx_{n-1}) + \beta_{n-1}LD(St_{n-1}, Tt_{n-1})$$

So,

$$d(Sx_n, Sy_n) \leq (1-\alpha_{n-1})d(Sx_{n-1}, Sy_{n-1})$$
$$+\alpha_{n-1}a[d(Sx_{n-1}, Sy_{n-1}) + \beta_{n-1}\gamma_{n-1}aLD(Sx_{n-1}, Tx_{n-1})$$
$$+\beta_{n-1}LD(St_{n-1}, Tt_{n-1})] + \alpha_{n-1}LD(Sz_{n-1}, Tz_{n-1}) + \epsilon_{n-1}$$

Thus,

$$d(p, Sy_{n+1}) \leq d(p, Sx_{n+1}) +$$
$$(1-\alpha_n + a\alpha_n)[(1-\alpha_{n-1})d(Sx_{n-1}, Sy_{n-1}) + a\alpha_{n-1}d(Sx_{n-1}, Sy_{n-1})$$
$$+a^2\alpha_{n-1}\beta_{n-1}\gamma_{n-1}LD(Sx_{n-1}, Tx_{n-1}) + a\alpha_{n-1}\beta_{n-1}LD(St_{n-1}, Tt_{n-1})$$
$$+\alpha_{n-1}LD(Sz_{n-1}, Tz_{n-1}) + \epsilon_{n-1}] + a^2\alpha_n\beta_n\gamma_nLD(Sx_n, Tx_n)$$
$$+a\alpha_n\beta_nLD(St_n, Tt_n) + \alpha_nLD(Sz_n, Tz_n) + \epsilon_n$$
$$\leq d(p, Sx_{n+1}) + (1-\alpha_n + a\alpha_n)(1-\alpha_{n-1} + a\alpha_{n-1})d(Sx_{n-1}, Sy_{n-1})$$
$$+(1-\alpha_n + a\alpha_n)(a^2\alpha_{n-1}\beta_{n-1}\gamma_{n-1}L)D(Sx_{n-1}, Tx_{n-1})$$

$$+(1-\alpha_n+a\alpha_n)(a\alpha_{n-1}\beta_{n-1}L)D(St_{n-1},Tt_{n-1})$$

$$+(1-\alpha_n+a\alpha_n)\alpha_{n-1}LD(Sz_{n-1},Tz_{n-1})+(1-\alpha_n+a\alpha_n)\in_{n-1}$$

$$+a^2\alpha_n\beta_n\gamma_nLD(Sx_n,Tx_n)+a\alpha_n\beta_nLD(St_n,Tt_n)+$$
$\alpha_nLD(Sz_n,Tz_n)+\in_n$

Repeating (n-1) times, yield

$$\leq d(p,Sx_{n+1})+\prod_{i=0}^{n}(1-\alpha_i+a\alpha_i)d(Sx_{0,}Sy_0)$$
$$+La^2\sum_{j=0}^{n}\alpha_j\beta_j\gamma_j\prod_{i=j+1}^{n}(1-\alpha_i+a\alpha_i)\,D(Sx_{i,}Tx_i)$$
$$+La\sum_{j=0}^{n}\alpha_j\beta_j\prod_{i=j+1}^{n}(1-\alpha_i+a\alpha_i)D(St_{i,}Tt_i)$$
$$+L\sum_{j=0}^{n}\alpha_j\prod_{i=j+1}^{n}(1-\alpha_i+a\alpha_i)D(Sz_{i,}Tz_i)$$
$$+\sum_{j=0}^{n}\prod_{i=j+1}^{n}(1-\alpha_i+a\alpha_i)\in_j$$

Part (A) proved.
To prove (B),

Suppose $\lim_{n\to\infty}Sy_n=p$ then,

$$\in_n=H(Sy_{n+1},(1-\alpha_n)Sy_n+\alpha_nTv_n)$$
$$\leq d(Sy_{n+1},p)+H(p,(1-\alpha_n)Sy_n+\alpha_nTv_n)$$
$$\leq d(Sy_{n+1},p)+(1-\alpha_n)d(p,Sy_n)+\alpha_nH(Tp,Tv_n)$$
$$\leq d(Sy_{n+1},p)+(1-\alpha_n)d(p,Sy_n)+\alpha_n[ad(Sp,Sv_n)+LD(Sp,Tp)]$$
$$\to 0\ as\ n\to\infty.$$

Suppose

$\lim_{n\to\infty}\in_n=0$. Let A denotes the lower triangular matrix with entries

$$a_{nj}=\alpha_j\prod_{i=j+1}^{n}(1-\alpha_i+a\alpha_i)$$

Then A is multiplicative so that,

$$\lim_{n\to\infty}L\sum_{j=0}^{n}\alpha_j\prod_{i=j+1}^{n}(1-\alpha_i+a\alpha_i)D(Sz_{i,}Tz_i)=0$$

$$\lim_{n\to\infty}La\sum_{j=0}^{n}\alpha_j\beta_j\prod_{i=j+1}^{n}(1-\alpha_i+a\alpha_i)D(St_{i,}Tt_i)=0$$

$$\lim_{n\to\infty}La^2\sum_{j=0}^{n}\alpha_j\beta_j\gamma_j\prod_{i=j+1}^{n}(1-\alpha_i+a\alpha_i)D(Sx_{i,}Tx_i)=0$$

Let B be the lower triangular matrix with entries

$$b_{nj} = \prod_{i=j+1}^{n}(1-\alpha_i + a\alpha_i)$$

Condition (iv) implies that $\lim_{n\to\infty}\prod_{i=j+1}^{n}(1-\alpha_i + a\alpha_i) = 0$

This completes the proof.

Corollary 3.1: Let (X,d) be a metric space and $Y \subseteq X$. Let $T,S\colon Y \to X$ and $SY\ or\ TY$ is a complete subspace of X. Let T and S have a common fixed point p. For any $x_0 \in Y$ there exist a sequence defined in (definition 2.4) converges to p. Let $\{Sy_n\} \subseteq X$ and set $\in_n = d(Sy_{n+1}, (1-\alpha_n)Sy_n + \alpha_n Tv_n)$ satisfying the condition

$$d(Tx,Ty) \le ad(Sx,Sy) + Ld(Sx,Tx)$$

For $a \in (0,1)$ and $L \ge 0$, then

A) $$\begin{aligned} d(p,Sy_{n+1}) \le\ & d(p,Sx_{n+1}) + \prod_{i=0}^{n}(1-\alpha_i + a\alpha_i)d\big(Sx_{0,}Sy_0\big) \\ & + La^2\sum_{j=0}^{n}\alpha_j\beta_j\gamma_j\prod_{i=j+1}^{n}(1-\alpha_i + a\alpha_i)\, d\big(Sx_{i,}Tx_i\big) \\ & + La\sum_{j=0}^{n}\alpha_j\beta_j\prod_{i=j+1}^{n}(1-\alpha_i + a\alpha_i)d\big(St_{i,}Tt_i\big) \\ & + L\sum_{j=0}^{n}\alpha_j\prod_{i=j+1}^{n}(1-\alpha_i + a\alpha_i)d\big(Sz_{i,}Tz_i\big) \\ & + \sum_{j=0}^{n}\prod_{i=j+1}^{n}(1-\alpha_i + a\alpha_i) \in_j \end{aligned}$$

Where the product is 1 when $j = n$. Also
B) $\lim_{n\to\infty} Sy_n = p$ if and only if $\lim_{n\to\infty} \in_n = 0$.

Proof: If $T$and S are single-valued mappings then we replace $\in_n = H(Sy_{n+1}, (1-\alpha_n)Sy_n + \alpha_n Tv_n)$ by $\in_n = d(Sy_{n+1}, (1-\alpha_n)Sy_n + \alpha_n Tv_n)$,$H(Tx,Ty)$ by $d(Tx,Ty)$ and $D(Sx,Ty)$ by $d(Sx,Ty)$.

Theorem 3.2: Let (X,d) be a metric space and $Y \subseteq X$. Let $T\colon Y \to CL(X), S\colon Y \to X$ and $SY\ or\ TY$ is a complete subspace of X. Let T and S have a common fixed point p. For any $x_0 \in Y$ there exist a sequence defined in (5) converges to p. Let $\{Sy_n\} \subseteq X$ and set

$\in_n = H(Sy_{n+1}, (1-\alpha_n)Sy_n + \alpha_n Tv_n)$ satisfying the condition

$$H(Tx,Ty) \le ad(Sx,Sy) + LD(Sx,Tx)$$

For $a \in (0,1)$ and $L \ge 0$, then

A) $$\begin{aligned} d(p,Sy_{n+1}) \le\ & d(p,Sx_{n+1}) + \prod_{i=0}^{n}(1-\alpha_i + a\alpha_i)d\big(Sx_{0,}Sy_0\big) \\ & + L\sum_{j=0}^{n}(\beta_j + \big(\alpha_j - \beta_j\big)a\gamma_j)\prod_{i=j+1}^{n}(1-\alpha_i + a\alpha_i)\, D\big(Sx_{i,}Tx_i\big) \end{aligned}$$

$$+L\sum_{j=0}^{n}(\alpha_j-\beta_j)\prod_{i=j+1}^{n}(1-\alpha_i+a\alpha_i)D(Sz_{i,}Tz_i)$$
$$+\sum_{j=0}^{n}\prod_{i=j+1}^{n}(1-\alpha_i+a\alpha_i)\in_j$$

where the product is 1 when $j=n$. Also

B) $\lim_{n\to\infty} Sy_n = p$ if and only if $\lim_{n\to\infty} \in_n = 0$.

Proof: The proof of this theorem is similar as of theorem 1.1

REFERENCES

[1] Berinde V., Iterative Approximation of Fixed Points, Springer Verlag, Lectures Notes in Mathematics, 2007.

[2] Czerwik S., Dlutek K. and Singh S. L., Roundoff stability of iteration procedures for set-valued operators in b-metric spaces, J. Natur. Phys. Sci., Vol. 15, No. 1-2, 2001, pp. 1–8.

[3] Harder A. M. and Hicks T. L., A stable iteration procedure for nonexpansive mappings, Math. Japonica. Vol. 33, 1988, pp. 687–692.

[4] Harder A. M. and Hicks T L., Stability results for fixed point iteration procedures, Math. Japonica, Vol. 33, 1988, pp. 693-706.

[5] Jungck G., Commuting mappings and fixed points, The American Mathematical Monthly, vol. 83, no. 4, 1976, pp. 261–263.

[6] Olatinwo M. O., A generalization of some convergence results using a Jungck-Noor three-step iteration process in arbitrary Banach space, Fasciculi Mathematici, no. 40, 2008, pp. 37–43.

[7] Olatinwo M. O. and Imoru C. O., Some convergence results for the Jungck-Mann and Jungck-Ishikawa processes in the class of generalized Zamfirescu operators, Acta Math. Univ. Comenianae, Vol. LXXVII, No. 2, 2008, pp. 299-304.

[8] Osilike M. O., Stability results for fixed point iteration procedures, J. Nigerian Math. Soc., Vol.14/15, 1995/96, pp. 17–29.

[9] Osilike M. O., A stable iteration procedure for quasi-contractive maps, Indian J. Pure Appl. Math., Vol. 27, No. 1, 1996, pp. 25–34.

[10] Ostrowski M., The round – off stability of iterations, Z. Angew. Math. Mech., Vol. 47, No. 1, 1967, pp. 77-81.

[11] Popa V, Fixed point theorems for implicit contractive mappings, Stud. Cercet. S¸tiint¸., Ser. Mat.,Univ. Bacˇau, vol. 7, 1997, pp. 127–133.

[12] Rhoades B. E., Fixed point theorems and stability results for fixed point iteration procedures, Indian J. Pure Appl. Math., Vol. 21, No. 1, 1990, pp. 1–9.

[13] Rhoades B. E., Fixed point theorems and stability results for fixed point iteration procedures. II, Indian J. Pure Appl. Math., Vol. 24, No. 11, 1993, pp. 691–703.

[14] Singh S. L., and Prasad Bhagwati, Some coincidence theorems and stability of iterative procedures, Comput. Math. Appl. Vol. 55, No. 11, 2008, pp. 2512-2520.

[15] Singh S. L., Bhatnagar Charu and Mishra S. N., Stability of Jungck-type iterative procedures, Int. J. Math. Sci., No. 19, 2005, pp. 3035-3043.