International Journal of Computational and Applied Mathematics.
ISSN 1819-4966 Volume 12, Number 2 (2017), pp. 525-536

http://www.ripublication.com

Fractional Order Generalized EPQ model

Asim Kumar Das[1*],Tapan Kumar Roy[2].

Department of Applied Mathematics, Indian Institute of Engineering Science and Technology, Shibpur, Howrah, West-Bengal, India, 711103.

Abstract

In the present article, we introduce a generalized economic production quantity (EPQ) model by using the concept of fractional calculus. In last few decades we see that fractional calculus plays an important role in the modeling of real problems in scientific and engineering fields to its more generalized form. Our objective in this paper is to developed fractional order EPQ model where classical EPQ model may be considered as the particular case of fractional order generalized EPQ model.

Keywords—Fractional differentiation, Fractional Integration, Fractional Differential Equation, Set up Cost, Holding Cost, Economic Production Quantity.

I. INTRODUCTION

The origin of fractional calculus goes back to Newton and Leibniz in the seventieth century. It is widely and efficiently used to describe many physical phenomena arising in engineering, physics, economy, and science. Recent investigations have shown that many physical systems can be represented more accurately through fractional derivative formulation. Fractional calculus, therefore find numerous applications in the field of visco-elasticity, electrical circuits, electro analytical chemistry, fractional multipoles, neuron modelling encompassing different branches of physics, chemistry , control theory and many fields of science and engineering[5-6],[8].

The application of Fractional Calculus and Fractional Differential equation has applied on some EOQ models [2-4] so far. Still many more areas of operation research models are untouched by the potential application of fractional calculus. Our objective in this paper is to develop the classical economic production quantity (EPQ) based inventory model to a generalized EPQ based inventory model emphasis on some certain assumption by using the potential application of Fractional Calculus.
Here we have applied the concept of derivative/integrals with an emphasis on Caputo and Riemann-Liouville fractional derivatives and have some interesting result and ideas that demonstrate the generalized EPQ model. Fractional derivatives and fractional integrals have interesting mathematical properties that may be utilized to developed our motivation.
In section II, we represent a basic conception on Fractional derivatives and Fractional integrals. In section III, we represent the basic concept of Classical EPQ model. In section IV, we introduce our main work which emphasizes as generalized EPQ model. Finally, In section V, we present the conclusion of our work.

II. FRACTIONAL DERIVATIVES AND INTEGRALS

A short description about fractional derivative or fractional integral can be described as follows, First, we consider a linear non homogeneous nth order ordinary differential equation ,

$$D^n y = f(x), \quad b \le x \le c \tag{2.1}$$

Then {1, x, $x^{2,}$ x^3, ...,x^{n-1}} is a fundamental set the corresponding homogeneous equation D^n y=0. f(x) is any continuous function in [b,c], then for any a$\in$(b,c),

$$y(x) = \int_a^x \frac{(x-t)^{n-1}}{(n-1)!} f(t)dt \tag{2.2}$$

is the unique solution of the equation (2.1.1) with the initial data $y^{(k)}$(a)=0,

for $0 \le k \le n-1$. Or equivalently, y(x)= ${}_aD_x^{-n}$ f(x)= $\frac{1}{\Gamma(n)}\int_a^x (x-t)^{n-1}$ f(t)dt (2.3)

Replacing n by α ,where Re(α)>0 in the above formula (2.3),we obtain the Riemann-Liouville definition of fractional integral that was reported by Liouville in 1832 and by Riemann in 1876 as

$$ {}_aD_x^{-\alpha} f(x) = {}_aJ_x^{\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_a^x (x-t)^{\alpha-1} f(t)\,dt \qquad (2.4) $$

Where ${}_aD_x^{-\alpha} f(x) = {}_aJ_x^{\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_a^x (x-t)^{\alpha-1} f(t)\,dt$ is the Riemann-Liouville integral operator. When a=0 ,(2.4) is the Riemann definition of integral and if a= -∞, (2.4) represents Liouville definition. Integral of this type were found to arise in theory of linear ordinary differential equations where they are known as Eulier transform of first kind.

If a=0 and x>0 ,then the Laplace transform solution the initial value problem

$$ D^n y(x)=f(x), \qquad x>0,\ y^{(k)}(0)=0,\ 0 \le k \le n-1 \qquad (2.5) $$

is $\bar{y}(s) = s^{-n}\,\bar{f}(s)$

(2.6)

Where $\bar{y}(s)$ and $\bar{f}(s)$ are respectively the Laplace transform of the function y(x) and f(x).

The inverse Laplace transform gives the solution of the initial value problem (2.5) as

y(x)= ${}_0D_x^{-n} f(x)$

Again from (2.6) we have y(x)= $L^{-1}\{\bar{y}(s)\}$

$$ = L^{-1}\{s^{-n}\,\bar{f}(s)\} $$

Thus we have ${}_0D_x^{-n} f(x) = L^{-1}\{s^{-n}\,\bar{f}(s)\}$ (2.7)

$$ \text{i.e } L^{-1}\{s^{-n}\,\bar{f}(s)\} = {}_0D_x^{-n} f(x) = \frac{1}{\Gamma(n)} \int_0^x (x-t)^{n-1} f(t)\,dt \qquad (2.8) $$

$$ \therefore y(x)= {}_0D_x^{-n} f(x) = L^{-1}\{s^{-n}\,\bar{f}(s)\} = \frac{1}{\Gamma(n)} \int_0^x (x-t)^{n-1} f(t)\,dt $$

This is the Riemann-Liouville integral formula for an integer n. Replacing n by real α gives the Riemann-Liouville fractional integral (2.1.3) with a=0.

III : BASIC CONCEPT OF CLASSICAL EPQ MODEL

During the first scheduling period T, production starts just after t=0 and continues up to $t = t_1$. Let the stock reaches a level I_{max}. Production is now stopped. The inventory reaches zero level at $t = T$ .and no shortages are permitted.

Notations & Assumption:

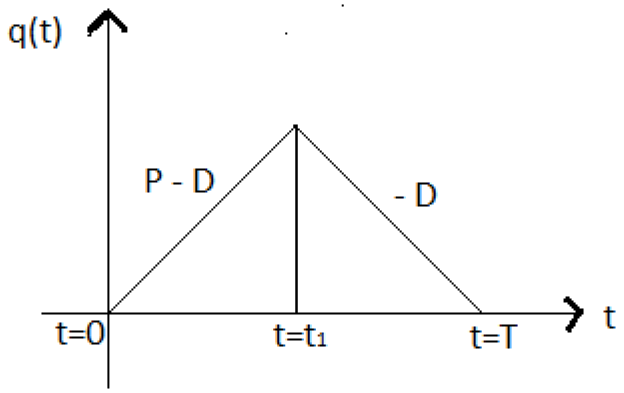
C_1 = constant carrying cost or holding cost per unit quantity per unit time,

C_3 = fixed ordering cost or set up cost per order (or production run),

P = production rate per cycle,

D = uniform demand rate per cycle.

The situation of the inventory is illustrated in figure.



If q(t) represents the inventory level at time $t \in [0, T]$, so the differential equation for the instantaneous inventory q(t) at any time t over $[0, T]$ is

$$\frac{dq(t)}{dt} = P - D \quad \text{for} \quad 0 \leq t \leq t_1 \tag{3.1}$$

$$= -D \quad \text{for} \quad t_1 \leq t \leq T \tag{3.2}$$

with initial condition q(0) = 0 (3.3)

and boundary condition q(T) = 0. (3.4)

Let $q(t_1) = I_{max}$ (3.5)

Solving (3.1) with initial condition (3.3), we get

$q(t) = (P-D)t \quad$ for $\quad 0 \leq t \leq t_1$ (3.6)

Solving (3.2) with initial condition (3.3), we get

$q(t) = -Dt + DT \quad$ for $\quad t_1 \leq t \leq T$ (3.7)

Using (3.5) in (3.5) and (3.6), we get

$$I_{max} = (P-D)t_1 \tag{3.8}$$

$$= D(T-t_1) \tag{3.9}$$

$$\therefore \quad t_1 = \frac{I_{max}}{P-D} \quad , \quad T - t_1 = \frac{I_{max}}{D}$$

and adding , we get $T = I_{max}\left(\frac{1}{P-D} + \frac{1}{D}\right)$

$$\therefore I_{max} = D\left(1 - \frac{D}{P}\right)T \tag{3.10}$$

$\therefore$ Holding Cost(HC) $= C_1\left[\int_0^{t_1} q(t)dt + \int_{t_1}^{T} q(t)dt\right]$

$$= C_1\left[(P-D)\frac{{t_1}^2}{2} + D\frac{(T-t_1)^2}{2}\right]$$

$$= \frac{C_1}{2}\left(1 - \frac{D}{P}\right)DT^2 \quad \text{[using (3.8), (3.9) and (3.10)]} \tag{3.11}$$

Total cost = Set up cost + Holding cost

$$= C_3 + \frac{C_1}{2}\left(1 - \frac{D}{P}\right)DT^2 \tag{3.12}$$

Total Average cost i.e. TAC(T) $= \frac{1}{T}\left[C_3 + \frac{C_1}{2}\left(1 - \frac{D}{P}\right)DT^2\right]$

$$= \frac{C_3}{T} + \frac{C_1}{2}\left(1 - \frac{D}{P}\right)DT \tag{3.13}$$

So the classical EPQ model is

Min $TAC(T) = \frac{C_3}{T} + \frac{C_1}{2}\left(1 - \frac{D}{P}\right)DT$

Such that $T > 0$. (3.14)

Solving (3.14), we can show that TAC(T) will be minimum for

$$T^* = \sqrt{\frac{2C_3}{C_1 D\left(1-\frac{D}{P}\right)}} \quad \& \quad TAC^*(T^*) = \sqrt{2C_1C_3D\left(1-\frac{D}{P}\right)} \tag{3.15}$$

IV. Generalized EPQ Model

$$\frac{d^\alpha q}{dt^\alpha} = P - D, \qquad \text{for } 0<t<t_1 \tag{4.1}$$

$$= -D \qquad \text{for } t_1<t<t_1+t_2=T \tag{4.2}$$

With initial condition q(0)=0, & boundary condition q(T)=0 (4.3)

And q(t_1)=$I_{\max}$ (4.4)

Taking Laplace transform of(4.1), we have

$$s^\alpha \bar{q}(s) - s^{\alpha-0-1}q(0) = \frac{(P-D)}{s}$$

$$\text{Or, } s^\alpha \bar{q}(s) = \frac{(P-D)}{s}$$

$$\text{Or, } \bar{q}(s) = \frac{(P-D)}{s^{\alpha+1}}$$

$$q(t) = L^{-1}\{\bar{q}(s)\} = (P-D)\frac{t^\alpha}{\Gamma(\alpha+1)}$$

$$\therefore q(t_1) = (P-D)\frac{t_1^\alpha}{\Gamma(\alpha+1)} = Q$$

Again we have by taking Laplace transform of (4.2)

$$s^\alpha \bar{q}(s) - s^{\alpha-0-1}q(t_1) = \frac{D}{s}$$

$$s^\alpha \bar{q}(s) = s^{\alpha-1}Q - \frac{D}{s}$$

$$s^\alpha \bar{q}(s) = \frac{Q}{s} - \frac{D}{s^{\alpha+1}}$$

$$\therefore q(t) = L^{-1}\{\bar{q}(s)\} = Q - D\frac{t^\alpha}{\Gamma(\alpha+1)} \tag{4.5}$$

$$Now T = t_1 + t_2$$

Again q(T)= 0

$$\Rightarrow \frac{DT^\alpha}{\Gamma(\alpha+1)} = Q$$

$$D\frac{(t_1+t_2)^\alpha}{\Gamma(\alpha+1)}=(P-D)\frac{t_1^\alpha}{\Gamma(\alpha+1)}$$

$\therefore$ for 0<t<t_1, $q(t)=(P-D)\dfrac{t^\alpha}{\Gamma(\alpha+1)}$ (4.6)

And for t_1<t<T, q(t)= $\dfrac{D}{\Gamma(\alpha+1)}[T^\alpha - t^\alpha]$ (4.7)

Again for t=t_1, $(P-D)\dfrac{t_1^\alpha}{\Gamma(\alpha+1)}=\dfrac{D}{\Gamma(\alpha+1)}[T^\alpha - t_1{}^\alpha]$ (4.8)

$$\Rightarrow P\frac{t_1^\alpha}{\Gamma(\alpha+1)}=\frac{D}{\Gamma(\alpha+1)}T^\alpha$$

$$\Rightarrow Pt_1^\alpha = DT^\alpha \quad (4.9)$$

Now for α=1 and β=1, we have the holding cost

$$HC_{1,1}(T)= C_1\int_0^T q(t)dt$$

$$=C_1\int_0^{t_1}(P-D)tdt + C_1\int_{t_1}^{T} D(T-t)dt$$

$$=\frac{C_1}{2}[(P-D)t_1^2 + D(T-t_1)^2]$$

$$=\frac{C_1}{2}[(P-D)t_1^2 + Dt_2{}^2]$$

$$=\frac{C_1}{2}[Dt_1t_2 + Dt_2{}^2] \qquad \text{[On using (4.8) for α=1]}$$

$$=\frac{C_1}{2}DTt_2$$

$$=\frac{C_1}{2}DT(\frac{P-D}{P})T$$

$$=\frac{C_1}{2}D(1-\frac{D}{P})T^2 \quad (4.10)$$

For any α and β=1,

$$HC_{\alpha,1}(T)=C_1\int_0^T q(t)dt$$

$$=C_1\int_0^{t_1}\frac{(P-D)}{\Gamma(\alpha+1)}t^\alpha dt + C_1\int_{t_1}^{T}\frac{D}{\Gamma(\alpha+1)}(T^\alpha - t^\alpha)dt$$

$$= \quad C_1 \frac{(P-D)}{\Gamma(\alpha+1)} \frac{t_1^{\alpha+1}}{(\alpha+1)} + C_1 \frac{D}{\Gamma(\alpha+1)} \left[T^{\alpha} t - \frac{t^{\alpha+1}}{\alpha+1} \right]_{t_1}^{T}$$

$$= \quad \frac{C_1}{\Gamma(\alpha+2)} (P-D) t_1^{\alpha+1} + C_1 \frac{D}{\Gamma(\alpha+1)} \left[T^{\alpha+1} - \frac{T^{\alpha+1}}{\alpha+1} - T^{\alpha} t + \frac{t_1^{\alpha+1}}{\alpha+1} \right]$$

$$= \frac{C_1}{\Gamma(\alpha+2)} (P-D) t_1^{\alpha+1} + \frac{C_1 D}{\Gamma(\alpha+1)} \left[\frac{\alpha T^{\alpha+1}}{\alpha+1} - t_1 (T^{\alpha} - \frac{t_1^{\alpha}}{\alpha+1}) \right]$$

$$= \frac{C_1}{\Gamma(\alpha+2)} (P-D+D) t_1^{\alpha+1} - \frac{C_1 D}{\Gamma(\alpha+1)} T^{\alpha} t_1 + \frac{C_1 D}{\Gamma(\alpha+2)} \alpha T^{\alpha+1}$$

$$= \frac{C_1 P}{\Gamma(\alpha+2)} t_1^{\alpha+1} - \frac{C_1 D}{\Gamma(\alpha+1)} T^{\alpha} t_1 + \frac{C_1 D}{\Gamma(\alpha+2)} \alpha T^{\alpha+1}$$

$$= \frac{C_1 P}{\Gamma(\alpha+2)} t_1^{\alpha+1} - \frac{C_1 D}{\Gamma(\alpha+1)} T^{\alpha} t_1 + \frac{C_1 D}{\Gamma(\alpha+2)} \alpha T^{\alpha+1}$$

$$= \frac{C_1 D}{\Gamma(\alpha+2)} T^{\alpha} t_1 - \frac{C_1 D}{\Gamma(\alpha+1)} T^{\alpha} t_1 + \frac{C_1 D}{\Gamma(\alpha+2)} \alpha T^{\alpha+1}$$

$$= \frac{C_1 D}{\Gamma(\alpha+2)} T^{\alpha} t_1 [1-(\alpha+1)] + \frac{C_1 D}{\Gamma(\alpha+2)} \alpha T^{\alpha+1}$$

$$= \quad - \frac{C_1 D}{\Gamma(\alpha+2)} \alpha T^{\alpha+1} \left(\frac{D}{P} \right)^{\frac{1}{\alpha}} + \frac{C_1 D}{\Gamma(\alpha+2)} \alpha T^{\alpha+1}$$

$$= \frac{C_1 \alpha}{\Gamma(\alpha+2)} D \left\{ 1 - \left(\frac{D}{P} \right)^{\frac{1}{\alpha}} \right\} T^{\alpha+1} \tag{4.11}$$

Now for any α and β $HC_{\alpha,\beta}(T) = C_1 D^{-\beta} q(t)$, Where q(t) is given by (4.6)&(4.7).

Now $D^{-\beta} q(t) = \frac{1}{\Gamma(\beta)} \int_0^T (t-x)^{\beta-1} q(x) dx$

$$= \frac{1}{\Gamma(\beta)} \left[\int_0^{t_1} (t-x)^{\beta-1} q(x) dx + \int_{t_1}^{T} (t-x)^{\beta-1} q(x) dx \right]$$

$$= \frac{1}{\Gamma(\beta)} \left[\int_0^{t_1} (t-x)^{\beta-1} \frac{(P-D)x^{\alpha}}{\Gamma(\alpha+1)} dx + \int_{t_1}^{T} (t-x)^{\beta-1} \frac{D}{\Gamma(\alpha+1)} (T^{\alpha} - x^{\alpha}) dx \right]$$

$$= \frac{1}{\Gamma(\beta)} [I_1 + I_2]$$

Where

$$I_1 \;=\; \int_0^{t_1} (t-x)^{\beta-1} \frac{(P-D)x^{\alpha}}{\Gamma(\alpha+1)} dx$$

$$=\frac{(P-D)}{\Gamma(\alpha+1)}\int_0^{t_1}(t-x)^{\beta-1}x^{\alpha}dx$$

And $$I_2=\int_{t_1}^{T}(t-x)^{\beta-1}\frac{D}{\Gamma(\alpha+1)}(T^{\alpha}-x^{\alpha})dx$$

$$=\frac{D}{\Gamma(\alpha+1)}\int_{t_1}^{T}(t-x)^{\beta-1}(T^{\alpha}-x^{\alpha})dx$$

$$\therefore HC_{\alpha,\beta}(T)=\frac{C_1}{\Gamma(\beta)}\left[I_1+I_2\right] \tag{4.12}$$

4.1 Generalized Total Average Cost:

Total cost = Set up cost + Holding cost

Total Average Cost (TAC)= $\frac{1}{T}$ [Total Cost(TC)]

For α=1 and β=1, Average Cost $TAC_{1,1}(T)=\frac{1}{T}[HC_{1,1}(T)+C_3]$

$$=\frac{C_1}{2}D(1-\frac{D}{P})T+\frac{C_3}{T} \tag{4.1.1}$$

So, For α=1 and β=1, the model is

$$\text{Min}\quad TAC_{1,1}(T)=\frac{C_3}{T}+\frac{C_1}{2}\left(1-\frac{D}{P}\right)DT \tag{4.1.2}$$

Such that $T>0$

Model (4.1.2) is our classical EPQ model as in (3.14).

Again for any α and β=1,

Average Cost $TAC_{\alpha,1}(T)=\frac{1}{T}[HC_{\alpha,1}(T)+C_3]$

$$=\frac{C_1\alpha}{\Gamma(\alpha+2)}D\left\{1-\left(\frac{D}{P}\right)^{\frac{1}{\alpha}}\right\}T^{\alpha}+\frac{C_3}{T}$$

Thus for any α and β=1,the model becomes,

$$MinTAC_{\alpha,1}(T)=\frac{C_1\alpha}{\Gamma(\alpha+2)}D\left\{1-\left(\frac{D}{P}\right)^{\frac{1}{\alpha}}\right\}T^{\alpha}+\frac{C_3}{T}\qquad \text{such that } T>0$$

$$=AT^{\alpha}+\frac{C_3}{T},\quad \text{Where } A=\frac{C_1\alpha}{\Gamma(\alpha+2)}D\left\{1-\left(\frac{D}{P}\right)^{\frac{1}{\alpha}}\right\} \tag{4.1.3}$$

To minimize $TAC_{\alpha,1}(T)$, we apply geometric programming method.

Here the degree of difficulty (DD) in G.P.P (4.1.3) =2-1-1=0

$$\text{Max } d(w)=\left(\frac{A}{w_1}\right)^{w_1}\left(\frac{C_3}{w_2}\right)^{w_2}$$

Subject to, $w_1+w_2=1$ (normalized condition) (4.1.4)

$\alpha w_1 - w_2=0$ (orthogonal condition) (4.1.5)

$w_1, w_2 \geq 0$.

Then solving for w_1 and w_2 from the above equation (4.2.14) and (4.2.15), we get

$$w_1=\frac{1}{\alpha+1} \text{ and } w_2=\frac{\alpha}{\alpha+1} \quad (4.1.6)$$

Again from the primal-dual relations gives, $AT^{\alpha}=w_1 d(w)$ and $\frac{C_3}{T}= w_2 d(w)$,

Then we get ,

$$\frac{A}{C_3}T^{\alpha+1}=\frac{w_1}{w_2}=\frac{1}{\alpha} \quad \Rightarrow T^{\alpha+1}=\frac{C_3}{A\alpha}\Rightarrow T=\left(\frac{C_3}{A\alpha}\right)^{\frac{1}{\alpha+1}} \quad (4.1.7)$$

$$\therefore \text{ we get, Max } d(w)=\left(\frac{A}{\frac{1}{\alpha+1}}\right)^{\frac{1}{\alpha+1}}\left(\frac{C_3}{\frac{\alpha}{\alpha+1}}\right)^{\frac{\alpha}{\alpha+1}}$$

$$= A^{\frac{1}{\alpha+1}}C_3^{\frac{\alpha}{\alpha+1}}\alpha^{-\frac{\alpha}{\alpha+1}}(\alpha+1)$$

$$\therefore MinTAC_{\alpha,1}(T) = A^{\frac{1}{\alpha+1}}C_3^{\frac{\alpha}{\alpha+1}}\alpha^{-\frac{\alpha}{\alpha+1}}(\alpha+1) \quad (4.1.8)$$

$$\text{Where } A=\frac{C_1\alpha}{\Gamma(\alpha+2)}D\left\{1-\left(\frac{D}{P}\right)^{\frac{1}{\alpha}}\right\}$$

Again for any α and β,

$$TAC_{\alpha,\beta}(T) = \frac{C_1}{\Gamma(\beta)T}\left[I_1 + I_2\right] + \frac{C_3}{T}$$

$$= \frac{C_1}{\Gamma(\beta)T}\left[\frac{(P-D)}{\Gamma(\alpha+1)}\int_0^{t_1}(t-x)^{\beta-1}x^{\alpha}dx + \frac{D}{\Gamma(\alpha+1)}\int_{t_1}^{T}(t-x)^{\beta-1}(T^{\alpha}-x^{\alpha})dx\right] + \frac{C_3}{T}$$

Again for any α and β, the model becomes,

$$MinTAC_{\alpha,\beta}(T) = \frac{C_1}{\Gamma(\beta)T}\left[\frac{(P-D)}{\Gamma(\alpha+1)}\int_0^{t_1}(t-x)^{\beta-1}x^{\alpha}dx\right.$$

$$\left. + \frac{D}{\Gamma(\alpha+1)}\int_{t_1}^{T}(t-x)^{\beta-1}(T^{\alpha}-x^{\alpha})dx\right] + \frac{C_3}{T}$$

Such that T >0 (4.1.9)

The model given in (4.1.9) is highly complicated form cannot be optimized analytically by any ordinary optimization method. But it can be optimized numerically by taking some numerical values (fractional) of α and β. It is seen that the model given by (4.1.2) is the particular case of (4.1.9) when α=1 & β=1. Again (4.1.3) is the particular case of (4.1.9) when β=1.

V. CONCLUSION

In the present paper we see that classical EPQ model may be generalized to a fractional order EPQ model where demand is constant. It is being observed that classical holding cost and classical total average cost are the particular case of generalized holding cost and generalized total average cost and hence classical EPQ model may be treated as a particular case of generalized EPQ model. We have seen that generalized EPQ model is not easy to be optimized analytically by any ordinary optimization method. In future the analytical method of optimization of this generalized EPQ model may be considerable and hence fractional calculus may be utilized to develop any other classical EOQ and EPQ model.

REFERENCES

[1] Chun –Tao. C, ‘‘An EOQ model with deteriorating items under inflation when supplier credits linked to order quantity’’ International Journal of Production Economics, 2004; Volume 88, Issue 3, 18; Pages 307-316.

[2] Das.A.K, Roy.T.K,”Role of Fractional Calculus To The Generalized Inventory Model “, ISSN-2229-371X, Volume 5, No-2, February 2014.JGRCS.

[3] Das.A.K, Roy.T.K,"FRACTIONAL ORDER EOQ MODEL WITH LINEAR TREAND OF TIME DEPENDENT DEMAND",I.J.Intelligent systems and Appllications, February 2015,03,44-45.

[4] Das.A.K, Roy.T.K,"FRACTIONAL ORDER EOQ MODEL WHEN DEMAND IS STOCK DEPENDENT","International Journal of Mathematical Archive 8(2)-2017 1-14"IISSN 2229-5046.

[5] Das.Santanu. "Functional fractional calculus for system identification and controls", ISBN 978-3-540-72702-6 Springer Berlin Heidelberg New York.

[6] Debnath.L (2003), Recent Application of Fractional Calculus to Science and Engineering, IJMS 2003; 54, 3413-3442.

[7] Donaldson, W.A., "Inventory replenishment policy for a linear trend in demand - an analytical solution", Operational Research Quarterly, 28 (1977) 663-670.

[8] Hilfer.R, Application of fractional calculus in Physics, world scientific, Singapore, 2000, Zbl0998.26002.

[9] Kicks.P,and Donaldson, W.A., "Irregular demand: assessing a rough and ready lot size formula", Journal of Operational Research Society, 31 (1980) 725-732.

[10] Kleinz.M and Osler.T.J, A child garden of Fractional Derivatives, The college Mathematics Journal, March 2000, Volume 31, Number 2, pp.82-88.

[11] Miller.K.S and Ross.B, An Introduction to the Fractional Calculus and Fractional Differential Equations, Copyright © 1993 by John Wiley & Sons, "A Wiley-Introduction Publication", Index, ISBN 0-471-58884-9 (acid free)

[12] Oldham.K.B, Spanier.J, The Fractional Calculus, Copyright © 1974 by Academic Press INC.(LONDON) LTD.

[13] Podlubny.I, The Laplace transform method for linear differential equations of the fractional order, Slovak Academy of Science Institute of Experimental Physics, June 1994.

[14] Podlubny.I, Geometric and Physical interpretation of Fractional Integral and Fractional Differentiation , Volume 5,Number 4(2002), An international Journal of Theory and Application, ISSN 1311-0454.

[15] Ritchie. E., "Practical inventory replenishment policies for a linear trend in demand followed by a period of steady demand", Journal of Operational Research Society, 31 (1980) 605-613.