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Rough Soft Sets: A novel Approach

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Abstract

In this paper lower soft set , upper soft set are introduced and then the notion of Rough soft set is given. Here few interesting properties of these sets are established and some results are discussed in this contest.

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1. INTRODUCTION:

The problem of imperfect knowledge has been tackled for a long time by philosophers, logicians and mathematicians. There are many theories like Fuzzy set theory, Rough set theory and Soft set theory to tackle the problems involving uncertainty. In order to study the control problems of complicate systems and dealing with fuzzy information, American cyberneticist *L. A. Zadeh* introduced fuzzy set theory in his classical paper [8] of 1965. A polish applied mathematician and computer scientist *Zdzislaw Pawlak* introduced rough set theory in his classical paper [5] of 1982. Rough set theory is a new mathematical approach to imperfect knowledge. This theory presents still another attempt to deal with the uncertainty or vagueness. The rough set theory has attracted the attention of many researchers and practitioners who contributed essentially to its development and application.

In 1999, *Molodtsov* [4] introduced the soft set theory as a general mathematical tool for dealing with uncertainty or vagueness. Soft set theory is still a better approach to deal with problems involving uncertainty. *Molodtsov* recognized the importance of the role of parameters and introduced the theory of Soft sets. He has shown several applications of this theory in many fields like economics, engineering, medical sciences, etc.

Later, this theory became a very good source of research for many mathematicians and computer scientists of recent years because of its wide range of applicability. The concepts of Rough Soft sets and Soft Rough sets were also emerged. He has shown several applications of this theory in many fields like economics, engineering, medical sciences, etc. Later, this theory became a very good source of research for many mathematicians and computer scientists of recent years because of its wide range of applicability. The concepts of Rough Soft sets and Soft Rough sets were also emerged.

In the present work, the lower and upper soft sets are introduced in a different approach. The notion of Rough soft sets is also introduced and few results are investigated in this context. Let U be an initial universe set and E_U (or simply E) be a collection of all possible parameters with respect to U, where parameters are the characteristics or properties of objects in U. Let $P(U)$ be the collection of all subsets of U and let ϕ stand for the empty set.

Firstly some basic concepts of Soft sets and Rough sets are presented in the following consecutive sections.

2. SOFT SETS:

In this section, some basic definitions of soft sets that are needed in further study of this paper are presented.

2.1 Definition: A pair (F, A) is called a *soft set* over U, if $A \subset E$ and $F : A \to P(U)$. We write F_A for (F, A).

2.2 Definition: Let F_A and G_B be soft sets over a common universe set U and $A, B \subset E$. Then we say that

(a) F_A is a *soft subset* of G_B, denoted by $F_A \subset G_B$, if (i) $A \subset B$ and (ii) $F(e) \subset G(e)$ $\forall\, e \in A$.

(b) F_A equals G_B, denoted by $F_A = G_B$, if $F_A \subset G_B$ and $G_B \subset F_A$.

2.3 Definition: A soft set F_A over U is called a *null soft set*, denoted by Φ, if $e \in A,\ F(e) = \phi$.

2.4 Definition: A soft set F_A over U is called an *absolute soft set*, denoted by A, if $e \in A,\ F(e) = U$.

2.5 Definition: The *union* of two soft sets F_A and G_B over a common universe U is the soft set H_C, where $C = A \bigcup B,$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e) & if \quad e \in A - B \\ G(e) & if \quad e \in B - A \\ F(e) \bigcup G(e) & if \quad e \in A \bigcap B \end{cases}$$

We write $F_A \bigcup G_B = H_C$.

2.6 Definition: The *intersection* of two soft sets F_A and G_B over a common universe U is the soft set H_C, where $C = A \bigcap B,$ and for all $e \in C$, $H(e) = F(e) \bigcap G(e)$. We write $F_A \bigcap G_B = H_C$.

2.7 Definition: For a soft set F_A over U, the relative *complement* of F_A is denoted by F_A^c and is defined by $F_A^c = F_A^1$, where $F^1 : A \to P(U)$ is a mapping given by $F^1(e) = U - F(e)$ for all $e \in A$.

2.8 Definition: The *difference* of two soft sets F_A and G_B, denoted by $F_a \setminus G_b$ and it is defined by $F_a / G_b \cong F_a \cap G^c{}_B$.

3. ROUGH SETS

In this section, some basic definitions of Rough sets that are necessary for further study of this work are presented.

3.1 Definition: A relation R on a non-empty set S is said to be an *equivalence relation* on S if

(a) xRx for all $x \in S$ (*reflexivity*)

(b) $xRy \Leftrightarrow yRx$ (*symmetry*)

(c) xRy and $yRz \Rightarrow xRz$ (*transitivity*)

We denote the equivalence class of an element $x \in S$ with respect to the equivalence relation R by the symbol $R[x]$ and $R[x] = \{y \in S : yRx\}$.

3.2 Definition: Let $X \subseteq U$. Let R be an equivalence relation on U. Then we define the following.

(a) The *lower approximation* of X with respect to R is the set of all objects, which can be for certain classified as X using R. That is the set

$$R_*(X) = \{x : R[x] \subseteq X\}.$$

(b) The *upper approximation* of X with respect to R is the set of all objects, which can be possibly classified as X using R. That is the set

$$R^*(X) = \{x : R[x] \cap X \neq \phi\}.$$

(c) The *boundary region* of X with respect to R is the set of all objects, which can be classified neither as X nor as not$-X$ using R. That is the set $\mathcal{B}_R(X) = R^*(X) - R_*(X)$.

It is clear that $R_*(X) \subseteq X \subseteq R^*(X)$.

3.3 Definition: A set $X \subseteq U$ is said to be a *Rough set* with respect to an equivalence

relation R on U, if the boundary region .

$$\mathcal{B}_R(X) = R^*(X) - R_*(X) \text{ is non-empty.}$$

4. ROUGH SOFT SETS

Finally in this section, the lower soft set ,upper soft set and Rough soft sets are introduced in a different approach and few interesting results are established in this context.

4.1 Definition: Let E and U be the set of parameters and the universe set respectively. Let R be an equivalence relation on U if F_E is a soft set then we define two soft sets $\overrightarrow{F}: E \to P(U)$ and $\overleftarrow{F}: E \to P(U)$ as follows.

$$\overrightarrow{F}(e) = \overset{*}{R}(F(e))$$

$$\overleftarrow{F}(e) = \underset{*}{R}(F(e)) \text{ for every } e \in E.$$

We call the soft sets $\underset{E}{\overrightarrow{F}}$ and $\underset{E}{\overleftarrow{F}}$, the *upper soft set* and the *lower soft set* respectively.

4.2 Definition: We say that a soft set F_E, a Rough soft set if the difference $\underset{E}{\overrightarrow{F}} \setminus \underset{E}{\overleftarrow{F}}$ is a non-null soft set.

4.3 Definition: By a *Crisp soft set,* We mean a soft set F_E such that $\overrightarrow{F}_E \cong \overleftarrow{F}_E$.

4.4 Proposition: If F_E is a soft set over the universe set U then $\overleftarrow{F}_E \subseteq F_E \subseteq \overrightarrow{F}_E$

Proof: Let F_E be a soft set over the universe set U and R be an equivalence relation on U.

Let $x \in R_*(F(e)) \Rightarrow R[x] \subseteq F(e)$ for $e \in E$

$\Rightarrow R[x] \cap F(e) = R[x] \neq \phi$

$\Rightarrow x \in R^*(F(e))$

Hence $R_*(F(e)) \subseteq F(e)$ for every $e \in E$ $\to$(1)

If $x \in R_*(F(e))$ then $R[x] \subseteq F(e)$ so $x \in F(e)$

Hence $R_*(F(e)) \subseteq F(e)$ for every $e \in E$ $\to$(2)

Let $x \in F(e)$, Then $x \in R(x) \cap F(e)$

$\Rightarrow R[x] \cap F(e) \neq \phi$

$\Rightarrow x \in R^{*}(F(e))$

$\Rightarrow F(e) \subseteq R^{*}(F(e))$ for every $e \in E$ $\rightarrow (3)$

By (1), (2) ,and (3) we have $R(F(e)) \subseteq F(e) \subseteq R^{*}(F(e))$

for every $e \in E$ $\rightarrow (4)$

From (4), it follows that $\overleftarrow{F_E} \subseteq F_E \subseteq \overrightarrow{F_E}$

4.5 Proposition: If Φ_E is the null soft set and X_E is the absolute soft set, defined by $\Phi(e) = \phi$ and $X(e) = U$ for every $e \in E$, then

(a) $\overleftarrow{\Phi}_E \cong \overrightarrow{\Phi}_E \cong \Phi_E$

(b) $\overleftarrow{X_E} \cong \overrightarrow{X_E} \cong X_E$

4.6 Proposition: If F_E and G_E are any two soft sets such that $F_E \tilde{\subseteq} G_E$ then

(a) $\overleftarrow{F_E} \tilde{\subseteq} \overleftarrow{G_E}$

(b) $\overrightarrow{F_E} \tilde{\subseteq} \overrightarrow{G_E}$

4.7 Proposition: If F_E and G_E then

(a) $\overrightarrow{F_E \cup G_E} \cong \overrightarrow{F_E} \cup \overrightarrow{G_E}$

(b) $\overrightarrow{F_E \cap G_E} \tilde{\subseteq} \overrightarrow{F_E} \cap \overrightarrow{G_E}$

(c) $\overleftarrow{F_E \cap G_E} \cong \overleftarrow{F_E} \cap \overleftarrow{G_E}$

(d) $\overleftarrow{F_E} \cup \overleftarrow{G_E} \tilde{\subseteq} \overleftarrow{F_E \cup G_E}$

4.8 Proposition: If F_E is any soft set then

(a) $\overrightarrow{\overrightarrow{F_E}} \cong \overrightarrow{F_E}$

(b) $\overleftarrow{\overleftarrow{F_E}} \cong \overleftarrow{F_E}$

(c) $\overrightarrow{\overrightarrow{F_E}} \cong \overrightarrow{\overleftarrow{F_E}} \cong \overrightarrow{F_E}$

(d) $\overleftarrow{\overleftarrow{F_E}} \cong \overleftarrow{\overrightarrow{F_E}} \cong \overleftarrow{F_E}$

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