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EOQ Model with Finite Planning Horizon for Deteriorating Items of Exponential Demand under Shortages

R.P.Tripathi, *D.Singh and *Surbhi Aneja

Department of mathematics, Graphic Era University, Dehradun (UK) India

**Department of mathematics, SGRR (P. G) College, Dehradun*

Abstract

Inventory control is one of the major area of operations research for determining the optimal quantity and total cost. This paper establishes an EOQ model for finite planning horizon with shortages for exponentially induced demand and addresses the situation where perishable items considered. Perishable commodities are the items that can only be used in a certain period such as green vegetable, chemicals, foodstuffs etc. Mathematical formulation is established to find optimal cycle time and total cost. Numerical example is provided to illustrate the result. A sensitivity analysis is given to validate the applicability of the model. Mathematica 7.0 is used for finding numerical results.

Keywords: Exponential demand; deterioration; shortages; inventory; holding cost

2010 Mathematics Subject Classification: 90B05

1. INTRODUCTION

Inventory control is the most important part of operations research. The production of a manufacturing company depends upon the several factors in the inventory system. These factors may be labor, managerial, raw material supplied, different costs, production facilities, machine repair etc. An exponential decay can provide a good approximation for fixed lifetime perishable items. The greater quantity produced the more items perish. In this study we consider the demand to be exponential time dependent. Teng *et al*. [24] pointed out an EOQ model under trade credit with increasing demand. Hsieh *et al*. [14] extended the model for upstream and downstream trade credit to an increasing function of time. Khanra *et al*. [16] established an EOQ model for a deteriorating item with variable demand when delay in payments is permissible. Silver & meal [21] generalized the EOQ model for the case of a variable demand. Donaldson [10] established a model for linearly time dependent demand. Goyal & Chang [12] dealt with an ordinary-transfer inventory model to determine the retailer's optimal order quantity and the number of transfer per order from the ware house to the displayed area. Baker & Urban [3] considered an EOQ model for a power-form inventory level dependent demand pattern. Bar-Lev *et al*. [5] considered inventory level dependent demand type EOQ model with random yield. Urban [27] presented an inventory model considering stock-level induced demand. Min *et al*. [19] studied an EOQ model for deteriorating items for stock-level dependent demand. Wang *et al*.[28] established an inventory model under credit dependent demand. Soni [22] established "the model from two aspects (i) the demand rate is multivariate function of price and level of inventory (ii) delay in payment is permissible". Avinadav [2] formulated and analyzed two models for determining the optimal pricing under quantity for items whose demand function is known.

Lou & Wang [17] formulated and discussed an EPQ model for a manufacturer with defective items when its supplier offers an up-stream trade credit.

In case of demand is more than the production, the customer have to wait for his/her requirement. Jamal *et al*. [15] extended an EOQ model to allow for shortages. Tripathi *et al*. [25] presented an inventory model with linearly time dependent demand rate and shortages under trade credits. Abad [1] proposed a pricing problem for an item with variable rate of deterioration rate under shortages. Dye [11] extended Abad [1] model by adding both the back order cost and lost sales into the total profit. Chakrabortty *et al*. [8] considered "a manufacturing inventory model with shortages where carrying cost, shortage cost, set up cost and demand quantity are considered as fuzzy numbers".

At present many countries are suffering inflation. Hence, the effect of inflation cannot be neglected in the formulation of inventory models. Tripathi *et al.* [28] established an EOQ model for deteriorating item with linearly time dependent demand rate under inflation. Buzacott [7] provided the optimal inventory replenishment policy of deteriorating items under inflationary condition. Some related research work done by Wee & Law [29], Basu *et al*. [4] ,Chung [9], Hou [13], Taleizadeh & Nemalollahi [23], Wee *et al*. [30], Bhumia *et al*. [6], Yazdi & Honarvar [31],Rabami *et al*. [20], Luong & Karim [18] and others.

2. ASSUMPTIONS AND NOTATIONS

Assumptions:

1. The demand rate is exponential time dependent.
2. Shortages are permitted.
3. The deterioration rate θ is constant and $0 < \theta < 1$.
5. The lead time is negligible.
6. The finite planning horizon of length H divided into m equal parts.

Notations:

$Q(t)$	:	Inventory level at time t
$D(t) = \alpha e^{-\beta t}$	:	Demand rate, $\alpha > 0,\ 0 < \beta < 1$
α	:	Scale parameter
θ	:	Deterioration rate, $0 < \theta < 1$
r , & i	:	Discount & Inflation rate respectively
$R = r - i$	:	Net discount rate
T	:	Cycle time
m	:	Number of replenishments
T_j	:	The total time that is elapsed up to and including the j^{th} cycle
t_j	:	The time for inventory level in the j^{th} replenishment cycle becomes to zero
$T_j - t_j$	:	Time of shortages $(j = 1, 2, ..., m)$
I_m & I_b	:	Maximum inventory & shortages quantity level respectively
Q	:	Order quantity in one cycle
A	:	Ordering cost
c , c_1 & c_2	:	Cost, Holding & shortage cost/ unit item/ unit time respectively
t_1	:	Time for positive inventory
T^*	:	Optimal cycle time

3. MATHEMATICAL MODEL

The model being formulated here is for the finite planning horizon, which is partitioned into m equal divisions each of size $T = H/m$. So the replenishment points for the model will be $T_j = jT$. The inventory level during $[0, t_1]$ is decreased due to demand and deterioration for positive inventory. Also the level of inventory decreases due to the same demand $\alpha e^{-\beta t}$ in [t_1,T]. Differential equations governing the states of the inventory level *Q(t)* equations during [*0, T*] are:

$$\frac{dQ(t)}{dt} + \theta Q(t) = -\alpha e^{-\beta t} \quad 0 \le t \le t_1 \tag{1}$$

$$\frac{dQ(t)}{dt} = -\alpha e^{-\beta t} \quad t_1 \le t \le T \tag{2}$$

with initial condition $Q(t_1) = 0$

Solving (1) & (2), we get

$$Q(t) = \frac{\alpha}{\theta - \beta}\left\{e^{(\theta-\beta)t_1 - \theta t} - e^{-\beta t}\right\} \text{ or}$$

$$Q(t) = \frac{\alpha}{\theta - \beta}\left\{e^{(\theta-\beta)kH/m - \theta H/m} - e^{-\beta H/m}\right\} \text{ , } 0 < t < t_1 \tag{3}$$

$$Q(t) = \frac{\alpha}{\beta}(e^{-\beta t} - e^{-\beta t_1}) = \frac{\alpha}{\beta}(e^{-\beta H/m} - e^{-\beta kH/m}) \quad t_1 < t < T \tag{4}$$

$$I_m = \frac{\alpha}{\theta - \beta}\left\{e^{(\theta-\beta)kH/m} - 1\right\} \tag{5}$$

and

$$I_b = \frac{\alpha}{\beta}(e^{-\beta H/m} - e^{-\beta kH/m}) \tag{6}$$

Ordering cost $\quad C_r = A$ (7)

Holding cost is

$$C_h = c_1\int_0^{t_1} Q(t)e^{-Rt}dt = \frac{c_1\alpha}{(\theta-\beta)}\left[\frac{(\theta-\beta)}{(\beta+R)(\theta+R)}e^{-(\beta+R)kH/m} + \frac{1}{(\theta+R)}e^{(\theta-\beta)kH/m} - \frac{1}{(\beta+R)}\right] \tag{8}$$

Present value of shortage cost is

$$C_s = -c_2\int_{t_1}^{T_1} \frac{\alpha}{\beta}(e^{-\beta t} - e^{-\beta t_1})e^{-Rt}dt = -\frac{c_2\alpha}{\beta}\left[\frac{-\beta}{(\beta+R)R}e^{-(\beta+R)kH/m} - \frac{1}{(\beta+R)}e^{-(\beta+R)H/m} + \frac{1}{R}e^{-(\beta k+R)H/m}\right] \tag{9}$$

Present value of replenishment cost is

$$C_p = cI_m + ce^{-RT}\int_0^{T-t_1} \alpha e^{-\beta t}dt = c\alpha\left[\frac{1}{(\theta-\beta)}\left\{e^{(\theta-\beta)kH/m}-1\right\}-\frac{1}{\beta}e^{-RH/m}\left\{e^{-\beta(1-k)H/m}-1\right\}\right] \quad (10)$$

Present value of the total cost during H is

$$TC(m,k)=\sum_{r=0}^{m-1}TRCe^{-RjT}-Ae^{-RH}=TRC\left(\frac{1-e^{-RH}}{1-e^{-RH/m}}\right)-Ae^{-RH}=\left(C_r+C_h+C_s+C_p\right)\left(\frac{1-e^{-RH}}{1-e^{-RH/m}}\right)-Ae^{-RH} \quad (11)$$

Where $TRC = C_r + C_h + C_s + C_p$

On simplification the above expression becomes

$$TC(m,k)=\left(\frac{1-e^{-RH}}{1-e^{-RH/m}}\right)\left[\begin{array}{l} A+\frac{c_1\alpha}{(\theta-\beta)}\left\{\frac{(\theta-\beta)}{(\beta+R)(\theta+R)}e^{-(\beta+R)kH/m}+\frac{1}{(\theta+R)}e^{(\theta-\beta)kH/m}-\frac{1}{(\beta+R)}\right\} \\ -\frac{c_2\alpha}{\beta}\left\{\frac{-\beta}{(\beta+R)R}e^{-(\beta+R)kH/m}-\frac{1}{(\beta+R)}e^{-(\beta+R)H/m}+\frac{1}{R}e^{-(\beta k+R)H/m}\right\} \\ +c\alpha\left\{\frac{1}{(\theta-\beta)}\left(e^{(\theta-\beta)kH/m}-1\right)-\frac{1}{\beta}e^{-RH/m}\left(e^{-\beta(1-k)H/m}-1\right)\right\} \end{array}\right]-Ae^{-RH} \quad (12)$$

Now differentiating (12) with respect to k, we get

$$\frac{dTC(m,k)}{dk}=\left(\frac{1-e^{-RH}}{1-e^{-RH/m}}\right)\frac{\alpha H}{m}\left[\begin{array}{l}\left(\frac{c_1}{(\theta+R)}-\frac{c_2}{R}\right)e^{-(\beta+R)kH/m}+\left(\frac{c_1}{(\theta+R)}+c\right)e^{(\theta-\beta)kH/m} \\ +\frac{c_2}{R}e^{-(\beta k+R)H/m}-ce^{(\beta k-\beta-R)H/m}\end{array}\right] \quad (13)$$

The extreme value of k is obtained by solving $\frac{dTC(m,k)}{dk}=0,$ we get

$$\left(\frac{c_1}{(\theta+R)}-\frac{c_2}{R}\right)e^{-(\beta+R)kH/m}+\left(\frac{c_1}{(\theta+R)}+c\right)e^{(\theta-\beta)kH/m}+\frac{c_2}{R}e^{-(\beta k+R)H/m}-ce^{(\beta k-\beta-R)H/m}=0 \quad (14)$$

Solving (14), for a given value of m, we get optimal value of $k = k^*$. The optimal value $\left(TC^*\right)$ is determined by substituting the value of k^* and m in (12), provided they satisfy the sufficient conditions for minimizing $TC(m,k)$ i.e. $\frac{d^2TC(m,k)}{dk^2}>0$ at $m=m^*$ and $k=k^*$.

$$\frac{d^2TC(m,k)}{dk^2}=\left(\frac{1-e^{-RH}}{1-e^{-RH/m}}\right)\frac{\alpha H^2}{m^2}\left[\begin{array}{l}-\left(\frac{c_1}{(\theta+R)}-\frac{c_2}{R}\right)(\beta+R)e^{-(\beta+R)kH/m}+\left(\frac{c_1}{(\theta+R)}+c\right)(\theta-\beta)e^{(\theta-\beta)kH/m} \\ -\frac{c_2\beta}{R}e^{-(\beta k+R)H/m}-c\beta e^{(\beta k-\beta-R)H/m}\end{array}\right]>0 \quad (15)$$

It is clear from (15), $\frac{d^2TC(m,k)}{dk^2} > 0$. Thus TC* is minimum at $k = k^*$ & $m = m^*$.

4. NUMERICAL EXAMPLE

Example1. A = 100, α = 100, c = 50, β = 0.05, θ = 0.2, H = 10, c_1 = 15, m = 7, c_2 = 300, R = 0.02 in appropriate units. The corresponding optimal values are k^* = 0.633 year and Q^* = 94.48 and $TC(m^*, k^*)$ = $ 77742.

5. SENSITIVITY ANALYSIS

In order to investigate the nature of the model, all input parameters are changed one by one. It is possible to check the sensitivity of the model by varying a single input while keeping other input parameters fixed.

Table 1: Variation of optimal values with m

m	k^*	T^*	Q^*	$TC(m^*, k^*)$
2	0.806	5	525.87	107694
3	0.783	3.333	326.50	89218
4	0.750	2.500	225.91	81921
5	0.712	2	164.64	78903
6	0.673	1.667	123.791	77766
7	0.633	1.429	94.48	77742
8	0.592	1.250	72.34	78453
9	0.551	1.111	56.38	76844
10	0.509	1	41.24	81085
11	0.468	0.909	30.05	82686
12	0.426	0.833	20.57	84561
13	0.384	0.769	12.56	86553
14	0.343	0.714	5.83	88515

Table 2:

Effects of the various parameters on optimal replenishment policy, taking one parameter at a time

Parameter	Change	k^*	Q^*	$TC\left(m^*, k^*\right)$
θ	0.10	0.395	-160.51	118218.0
	0.16	0.572	43.41	85567.0
	0.20	0.633	94.48	77742.9
	0.30	0.708	152.84	71361.6
	0.36	0.728	169.43	70751.1
R	0.010	0.618	90.61	83703.4
	0.016	0.627	92.93	80095.3
	0.020	0.633	94.48	77742.9
	0.030	0.646	97.83	72261.8
	0.036	0.654	99.90	69048.5
α	50	0.633	47.24	39750.0
	80	0.633	75.58	62305.0
	100	0.633	94.48	77742.9
	120	0.633	113.38	93180.0
	150	0.633	141.72	116336.0
β	0.025	0.629	144.79	78671.1
	0.040	0.631	116.02	78163.0
	0.050	0.633	94.48	77742.9
	0.060	0.634	68.75	77445.8
	0.075	0.636	22.10	76939.4
A	50	0.633	94.48	77464.3
	80	0.633	94.48	77631.5
	100	0.633	94.48	77742.9
	120	0.633	94.48	77854.3
	150	0.633	94.48	78021.4
c	25	0.638	95.77	53656.8
	40	0.635	94.99	68112.4
	50	0.633	94.48	77742.9
	60	0.631	93.96	87368.1
	75	0.628	93.19	101796.0
c_1	7.5	0.799	137.56	60031.0
	12	0.698	111.28	69706.9
	15	0.633	94.48	77742.9
	18	0.568	77.76	87220.3
	22.5	0.474	53.74	103600.0

c_2	150 240 300 360 450	0.324 0.557 0.633 0.689 0.747	15.79 73.41 94.48 108.95 124.00	90827.1 81304.9 77742.9 75320.4 72748.4

From the above discussion the following inferences can be made:

i. If Q increases, K, Q and TC increases.
ii. If R increases, increase in K & Q, but decrease in TC.
iii. Increase of α results, decrease in Q and increase in TC.
iv. If β increases increase in K, decrease in Q and TC.
v. The increase of A leads increase in TC.
vi. If c and c_1 increase caused decrease in K and Q and increase in TC.
vii. Increase of c_2 results increase in K and Q but decrease in TC.
viii. TC is convex with the increase of m. Also K and Q will decrease with increase of m.

Graphs

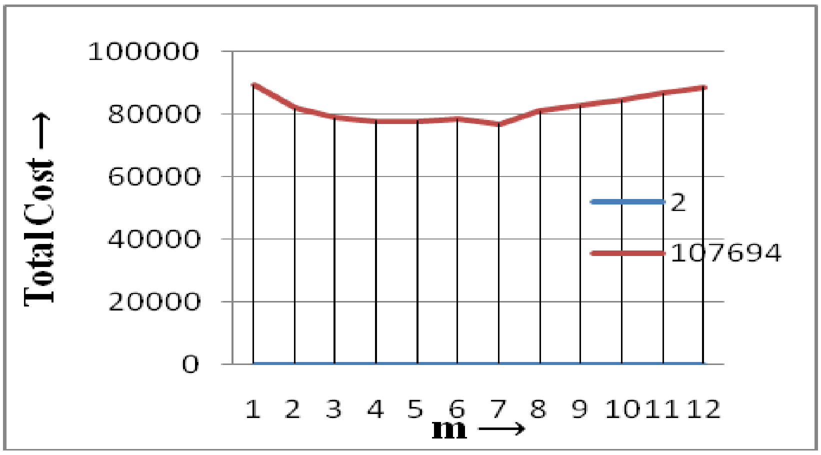

Fig. 7.1: Variation in TC w.r.t 'm'

6. CONCLUSION AND FUTURE REMARK

In actual practice the demand and deterioration both are in fluctuating state. In this paper we have presented an inventory model for deteriorating items under inflation and time discounting over finite planning horizon. Numerical examples are provided with the different situations. We have seen that the increase of number of replenishment caused the convex variation in total cost, which proved that the total cost function is convex with respect to number of replenishment. This model is useful in each type of business transaction like daily used products. The variations are significant with the total cost.

The extension of the proposed work is constant demand to probabilistic, stochastic demand and deterioration is to weibull distribution deterioration.

REFERENCES

[1] Abad, P.L. Optimal price and order size for a retailer under partial backordering. Com.& Oper. Res., 28, 53-65(2001).

[2] Avinadav, T A., Herbon,A. & Spiegel, U. Optimal ordering and pricing policy for demand functions that are separable into price and inventory age. Int. J. of Prod. Eco, 155, 406- 417(2014).

[3] Baker,R.C., T.L. Urban,T.L. A deterministic inventory system with an inventory level dependent demand rate. J. of the Oper. Res. Soc., 39, 823-831(1988).

[4] Basu,N., Sinha,S.,Kalyani,N.,Bengal,W. An inflationary inventory model with time dependent demand with weibull distribution deterioration and partial backlogging under permissible delay in payments. Con. and Cyb., 36(1), 203-217(2007).

[5] Bar-Lev,S.K.,Parlar and Perry,D. On the EOQ model with inventory-level dependent demand rate and random yield. Oper. Res. Lett., 16, 167-176(1994).

[6] Bhumia,A., Jaggi,C.K.,Sharma,A., Sharma, R. A two ware house inventory model for deteriorating item under permissible delay in payments with partial backlogging. *App.Math. & Comp.*, 232, 1125-1137(2014).

[7] Buzacott,J. Economic order quantity with inflation. *Oper. Res. Quar.*, 26(3), 553-558(1975).

[8] Chakrabortty,S.M., Dala,P.K. and Nayak .Intutionistic fuzzy optimization technique for pareto optimal situation of manufacturing inventory models with shortages. *Eur. J. of Oper. Res.*, 238, 381-387(2013).

[9] Chung,C.T. An EOQ model with deteriorating item under inflation when supplier credit linked to order quantity. Int. J. of Prod. Eco. 88(3), 307-316(2004)..

[10] Donaldson,W.A. Inventory replenishment policy for a linear trend in demand analytical solution. Oper.Res.Quart., 128, 663-670(1977).

[11] Dye,C.Y. Joint pricing and ordering policy for a deteriorating inventory wit partial backlogging. *Omega*, 35, 184-189(2007).

[12] Goyal, S.K., Chang, C.T. Optimal ordering and transfer policy for an

inventory with stock dependent demand. Euro.J. of Oper. Res. 196, 177-185 (2009).

[13] Hou,K.L. An inventory model for deteriorating item with stock-dependent consumption rate and shortages under inflation and time discounting. Euro. J. of Oper. Res. 168(2), (2006).

[14] Hsieh,T.P., Chang,H.T.,Dye,C.Y.,Wang,M.W. Optimal lot size under trade credit financing when demand and deterioration are fluctuating with time. Int. J. of Inf. and Manag. Sci., 20, 191-204(2009).

[15] Jamal, A,M.M.,Sarkar, B.R.,Wang. S. An ordering policy for deteriorating items with allowable shortage and permissible delay in payments. J. of Oper. Res. Soc., 48, 826-833(1997).

[16] Khanra, S.,Ghosh, S.K., Chaudhari, K.S.. An EOQ model for a deteriorating item with time dependent quadratic demand under permissible delay in payment. App. Math. And comp., 218, 1-9 (2011).

[17] Lou,K.R.,Wang,L. Optimal lot-sizing policy for a manufacturer with defective item in a supply chain with up-stream and down-stream trade credits. Comp. and Ind. Eng, 66, 1125-1130(2013).

[18] Luong,,H.T., Karim,R.. An integrated production inventory model of deteriorating items subject to random machine breakdown with stochastic repair time. *Int. J. of Ind. Eng. Comp.* 8 , 217 – 236 (2017).

[19] Min,J., Zhou, Y.W. , Zhao,J. An inventory model for deteriorating items under stock-dependent demand and two level trade credit. App. Math.Mod., 34, 3273-3285(2010).

[20] M.Rabbani, M., Manavizadeh,N., Balali,S. A stochastic model for indirect condition monitoring using proportional covariate model. Int. J. of Eng. Trans. A Basics, 21(1), (2008).

[21] Silver,E.A.,Meal,H.C.A simple modification of the EOQ model for the case of a varying demand rate. Prod. & Inv. Manag., 10(4), 52-65 (1969).

[22] Soni,H.N. Optimal Replenishment Policies for non-instantaneous deteriorating items with price and stock-sensitive demand under permissible delay in payments. Int. J. of Prod. Eco., 146, 259-268 (2013).

[23] Taleizadeh,A.A.,Nematollahi,M. An inventory control problem for deteriorating items with back-ordering and financial consideration**.** App. Math. Mod., 38(1), 93-109 (2014).

[24] Teng,J.T., Min, J. Pan, Q . Economic order quantity model with trade credit financing for non-decreasing demand. *Omega*, 40, 318-325 (2012).

[25] Tripathi, R.P. Economic order quantity for deteriorating item with non-decreasing demand and shortages under inflation and time discounting. *Int. J. of Eng,* 28(9), (2015).

[26] Tripathi, R.P..Singh, D., Mishra, T. Economic order quantity with linearly

time-dependent demand rate and shortages. J. of Math. and Stat., 11(1), 21-28 (2015).

[27] Urban,T.L. An extension of inventory model incorporating financing agreements with both suppliers and customers. App. Math. Mod., 36, 6323-6330 (2012).

[28] Wang,W.C., Teng, J.T. ,Lou, K.R . Seller's optimal credit period and cycle time in a supply chain for deteriorating item with maximum lifetime. Eur.J. of Oper. Res., 232, 315-321(2014).

[29] Wee, H.M., Lavo. S.T. Economic production lot size for deteriorating items taking account of the time value of money. Comp. & Oper. Res., 26(4), 545-558(1999).

[30] Wee,H.M., Huang, Y.D., Wang,W.T., Chang. Y.L. An EOQ model with partial backorders considering two backordering costs. *App. Math. and Comp.*, 232, 898- 907(2014).

[31] Yazdi,A.A., Honarvar, M. A two stage stochastic programming model of the price decision problem in the dual channel closed loop supply chains. *Ind. J. of Eng.Transaction B : App.*, 28(5), 738-746 (2015).