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# Optimal Integrated Inventory Policy for Constantly Deteriorating Units with Random Input

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**Abstract:**
In the classical EOQ model, it is tactically assumed that the retailer receives the requisitioned quantity and there is no deterioration of units in inventory. However, in practice, we have observed that due to number of reasons the quantity received does not match with the quantity ordered. In this paper we analyse an integrated inventory policy for manufacturer and retailer when units received by retailer are random and deteriorate constantly. Manufacturer offers a trade credit to gain profits of a long term sustainable supply chain. A conciliation factor is suggested to share the benefits between supplier and retailer. The proposed model is validated by a numerical example and sensitivity analysis of inventory parameter is being carried out. This model is useful to entrepreneurs of small and medium-sized enterprises (SMEs) or managers involved in supply chain management for packed food, beverages, cosmetics and fashion goods.
**Keywords:** Integrated inventory, trade credit, constant deterioration, joint profit, random input.

## 1 INTRODUCTION

Globalization of the business forced the business players to device win-win strategy to survive in this competitive world. Profit of one member of supply chain must not be at the cost of another's profit. This vision inspired researchers to work on integrated models that can safe guard overall supply chain profit as well as individual profit of supply chain members. Goyal (1976) formulated a single vendor single-buyer integrated inventory model. Banerjee (1986) extended above model when vendor follows lot-for-lot production policy. Goyal and Gupta (1989) reviewed on integrated inventory and some future areas have been identified. Shah and Pandey (2009) studied deteriorating inventory for advertisement and stock dependent demand. Shah and Shukla (2010), Shah and Shukla (2011), Annadurai (2013), Shah (2015) extended integrated model for deteriorating stocks.

Due to variety of reasons, viz, machine's breakdown, natural calamity, worker's strike, electricity failure, shortage of raw materials etc., it is possible that supplier does not match with the quantity ordered from retailer. These cases can be model and analysed using be a random variable depending on the quantity ordered. Silver (1976) developed an EOQ model when the quantity received is uncertain. Kalro and Gohil (1982) extended Silver's result to allow shortages. Shah and Shah (1992-a, 1992-b) developed optimal ordering policy when units received are random in nature and subject to constant rate of deterioration for infinite/finite production rate. Yano and Lee (1995) reviewed the literature on quantitatively oriented approach for determining lot-sizes when production or procurement yields are random. They discussed the issues relating to the modelling cost yield uncertainty and performance of the system with random yield. Shah and Trivedi (1999) extended model for exponentially deteriorating inventory with random lead time. Soni et al. (2006) derived EOQ model with two level credits. Shah and Gor (2009) developed an integrated inventory model when units received are random.

Among various methods of sales promotion offering credit period for full payment is one that acts very positively for long term relationship among members of supply chain. Various researchers and practitioners have used it in different ways to analyse its merits. Goyal (1985) formulated a mathematical model when trade credit is offered by the vendor to the buyer. Jamal (2000) formulated model for optimal shipment such that profit maximize. Shinn and Hwang (2003) derived model for optimal pricing and ordering policies for retailers. Chang (2004) formulated for order linked trade credit. Hu (2011) extended to integrated inventory policy with price-and-credit-linked demand. Seifert et al. (2013) reviewed on trade credit. Sarkar et al. (2015) studied trade credit policy for items subjected to time dependent deterioration and possess fixed lifetime.

In EOQ implicitly assume that inventory is depleted by customer's demand alone. This assumption is quite valid for non-perishable or non deteriorating inventory items. However, there are numerous types of inventory whose utility does not remain constant over time. In this case, inventory is depleted not only by demand but also by deterioration. Ghare and Schrader (1963) assumed with constant demand to derive revised economic order quantity. Cohen (1977) analyzed the RPLS problem for an exponentially deteriorating product. Hariga (1995) examined the inventory lot-sizing problem with deteriorating items and continuous time-dependent demand. Aggarwal and Jaggi (1995), Rau et al. (2006), Shinn and Hwang (1997), Sarker et al. (2001), Shah (2013) have proposed and analysed inventory models with different deteriorating conditions.

In the proposed model, an integrated supplier – retailer supply chain is considered. Units kept in retailer's stock are subject to constant deterioration as well as receives uncertain input from supplier. Joint and individual profit is analysed in both isolated and non – isolated systems. A conciliation factor is used to balance the retailer's individual profit in the non – isolated scenario using trade credit offered by supplier. An algorithm is proposed to calculate optimal number of shipments, cycle time, quantity and thus maximum joint profit. Co-operative scenario of supply chain not only improves joint profit substantially but also lead to long term growing and sustainable supply chain. Contributions made by different authors that helped and inspired to write this paper are mentioned in table (1).

**Table 1** Contribution by different authors

| Author | Integrated model | Deterioration | Random input | Optimal shipment | Conciliation factor | Trade credit |
|---|---|---|---|---|---|---|
| Ghare and Schrader(1963) | | ✔ | | | | |
| Goyal (1976) | ✔ | | | | | |
| Silver (1976) | | | ✔ | | | |
| Cohen(1977) | | ✔ | | | | |
| Kalro and Gohil (1982) | | | ✔ | | | |
| Goyal (1985) | | | ✔ | | | ✔ |
| Banerjee (1986) | ✔ | | | | | |
| Goyal and Gupta (1989) | ✔ | | | | | |
| Shah and Shah(1992a) | | ✔ | ✔ | | | |
| Shah and Shah(1992b) | | ✔ | ✔ | | | |
| Yano and Lee (1995) | | | ✔ | | | |
| Aggarwal and Jaggi(1995) | | ✔ | | | | |
| Hariga(1995) | | ✔ | | | | |
| Shinn and Hwang(1997) | | ✔ | | | | |
| Shah and Trivedi (1999) | | ✔ | ✔ | | | |
| Jamal(2000) | | | | ✔ | | ✔ |
| Sarker et al.(2001) | | ✔ | | | | ✔ |
| Shinn and Hwang (2003) | | | | ✔ | | ✔ |
| Chang(2004) | | | | | | ✔ |
| Rau et al.(2006) | ✔ | ✔ | | | | ✔ |
| Soni et al.(2006) | ✔ | | ✔ | | | ✔ |
| Shah and Gor (2009) | ✔ | | ✔ | ✔ | ✔ | ✔ |
| Shah and Pandey (2009) | | ✔ | | | | ✔ |
| Shah and Shukla(2010) | | ✔ | | | | ✔ |
| Hu (2011) | ✔ | | | | | ✔ |
| Shah and Shukla(2011) | | ✔ | | | | ✔ |
| Seifert et al.(2013) | ✔ | | | | | ✔ |
| Shah et al.(2013) | ✔ | ✔ | | | | ✔ |
| Annadurai (2013) | ✔ | ✔ | | | | ✔ |
| Shah (2015) | ✔ | ✔ | | | | ✔ |
| Sarkar et al. (2015) | | | | | | |
| Proposed model | ✔ | ✔ | ✔ | ✔ | ✔ | ✔ |

## 2 NOTATIONS AND ASSUMPTIONS

### 2.1 Notations

*Inventory parameters for manufacturer*

| | |
|---|---|
| $A_m$ | Set up costs ($) |
| $C_m$ | Production cost / unit ($) |
| $h_m$ | Holding cost / unit / annum ($) |
| $M$ | Credit period offered by manufacturer to retailer |
| $D$ | Demand rate |
| $\pi_m$ | Manufacturer's profit when Y-units are received ($) |
| $E_m$ | Manufacturer's expected profit per unit time ($) |
| $T(Y)$ | Manufacturer replenishment time when Y-units are received (years) |

*Inventory parameters for retailer*

| | |
|---|---|
| $A_r$ | Ordering cost($) |
| $C_r$ | Purchase cost per unit($) |
| $Q$ | Retailer's order quantity per order |
| $n$ | Number of shipment |
| $h_r$ | Holding cost for retailer ($) |
| $S$ | Selling price/unit time ($) |
| $I_e$ | Interest earned/unit/annum |
| $\theta$ | Constant deterioration |
| $Q_m(t/Y)$ | Manufacturer-retailer combined inventory level when Y-units are received |
| $Q_r(t/Y)$ | Retailer's inventory level Y-units are received |
| $\pi_r$ | Retailer's profit when Y-units are received ($) |
| $E_r$ | Retailer's expected profit per unit time ($) |
| $\pi$ | Joint expected total profit per unit time ($) |
| $E(Y)$ | $mQ$ ,the mean of received random variable Y when Q units are received , m>0 |
| $E(T(Y))$ | Expected cycle time when Y-units are received = $E(Y)/D = mQ/D$ |
| $V(Y)$ | $=\sigma_0^2+\sigma_1^2 Q^2$, the variance of the received random variable |

Necessary condition for proposed model is $S > C_r > C_m$

### 2.2 Assumptions

1) In this model of supply chain a single – manufacturer and retailer under single item is in consideration.
2) The demand is deterministic and constant during the period under review.
3) Replenishment rate is instantaneous and no shortages allowed.

4) Items in inventory are subject to constant deterioration $\theta$ $(0<\theta<1)$.
5) The quantity requisitioned does not necessarily match with the quantity received but is random variable following a normal distribution $Q$ denotes the quantity requisitioned and $Y$ is the quantity received then is a random variable with mean and variance as $E(Y)=m\mathrm{Q}$ and $V(\mathrm{Y})=\sigma_0^2+\sigma_1^2 Q^2$ respectively where $b>0$ is the bias factor and $\sigma_0^2$ and $\sigma_1^2$ ($\sigma_0^2 \geq \sigma_1^2$) are non- negative constants. $Q$ is the decision variable.
6) Trade credit of $M$ units is offered by the manufacturer to attract the retailer to cooperate in the integrated strategy

## 3 MODEL FORMULATION

In this section, we formulate an integrated inventory model where quantity requisitioned does not necessarily match with the quantity received but it is a random variable it following a normal distribution for a constant deteriorating units. A conciliation factor is suggested to share the benefits.

### 3.1 Manufacturer's total profit

For the manufacturer, the total profit per unit time is composed of sales revenue, set-up cost and holding cost. These components are evaluated as following.

*3.1.1 Sales Revenue :*

The on hand inventory of the manufacturer depletes with constant demand rate then differential equation that describes the instantaneous state of inventory at any instant of time t is given by

$$\frac{\mathrm{d}Q_m(t/T)}{\mathrm{d}t}+\theta Q_m(t/Y)=-D;\quad 0\leq t\leq T(\mathrm{Y}) \tag{1}$$

Using boundary condition $Q_m(T(\mathrm{Y}))=0$ we get solution of differential equation (1) as

$$Q_m(t/Y)=\frac{D}{\theta}\left(\mathrm{e}^{\theta(T(\mathrm{Y})-t)}-1\right) \tag{2}$$

At $t=0$ we get initial quantity

$$Q_m(y)=\frac{D}{\theta}\left(\mathrm{e}^{\theta T(y)}-1\right) \tag{3}$$

Therefore The sales revenue during $\left[0,T(\mathrm{Y})\right]$ is given by

$$SR_m=(C_r-C_m)Q_m(y) \tag{4}$$

*3.1.2 Setup cost:*

$$\text{Constant set up cost is } SC_m=A_m \tag{5}$$

*3.1.3 Holding cost:*

The manufacturer's inventory level when Y-units are received, in the integrated two-echelon inventory model is the difference between the manufacturer-retailer combined average inventory level and the retailer's average inventory level. Thus, manufacturer's holding cost during $[0, T(\mathrm{Y})]$ is given by

$$HC_m = h_{\mathrm{m}}\left(\int_0^{T(\mathrm{Y})} Q_m\left(t \,/\, y\right)\mathrm{d}t - \mathrm{n}\int_0^{\frac{\mathrm{T(Y)}}{\mathrm{n}}} Q_r\left(t \,/\, y\right)\mathrm{d}t\right) = \frac{h_m D}{\theta^2}\left(n - 1 + \mathrm{e}^{\theta T(y)} - n\mathrm{e}^{\frac{\theta T(Y)}{n}}\right) \quad (6)$$

ConseConsequently, manufacturer net profit is sales revenue minus the total relevant cost, which can be expressed as following

$$\pi_m\left(n, y\right) = SR_m - SC_m - HC_m \quad (7)$$

Therefore Manufacturer expected profit for $Q$ - units is

$$E_{\mathrm{m}}\left(Q, n\right) = \frac{E\left(\pi_{\mathrm{m}}\left(\mathrm{Y}\right)\right)}{E\left(T\left(Y\right)\right)} = \frac{D^2}{\theta m Q}\left(C_r - C_m\right)\left(\mathrm{e}^{\frac{\theta D}{mQ}} - 1\right) - \frac{A_m \mathrm{D}}{\mathrm{mQ}} - \frac{D^2 h_m}{\theta^2 \mathrm{mQ}}\left(n - 1 + \mathrm{e}^{\frac{\theta D}{mQ}} - n\mathrm{e}^{\frac{\theta D}{mQn}}\right) \quad (8)$$

## 3.1 Retailer’s total profit :

For the manufacturer, the total profit per unit time is composed of sales revenue, setup cost and holding cost. These components are evaluated as following.

### *3.2.1 Sales Revenue :*

The on hand inventory of the manufacturer depletes with constant demand rate then differential equation that describes the instantaneous state of inventory at any instant of time t is given by

$$\frac{\mathrm{d}Q_r\left(\mathrm{t/Y}\right)}{\mathrm{d}t} + \theta Q_r\left(t \,/\, Y\right) = -D; 0 \le t \le \frac{T\left(Y\right)}{n} \quad (9)$$

Using boundary condition, $Q_r\left(\frac{T\left(Y\right)}{n}\right) = 0$, we get solution of differential eq.(9) as

$$Q_{\mathrm{r}}\left(t \,/\, T\right) = \frac{D}{\theta}\left(\mathrm{e}^{\theta\left(\frac{T(\mathrm{Y})}{n} - t\right)} - 1\right) \quad (10)$$

At $t = 0$ we get initial quantity

$$Q_{\mathrm{r}}\left(Y\right) = \frac{D}{\theta}\left(\mathrm{e}^{\frac{\theta T(Y)}{n}} - 1\right) \quad (11)$$

Therefore the sales revenue during $\left[0, \frac{T\left(\mathrm{Y}\right)}{n}\right]$ is given by

$$SR_r = \left(\mathrm{S} - C_r\right)Q_r\left(\mathrm{Y}\right) \quad (12)$$

### *3.2.2 Ordering cost*:

Constant ordering cost is $OC_r = nA_r$ (13)

*3.2.3. Holding cost:*

The retailer's inventory level when Y-units are received, in the interval $\left[0, \frac{T(\mathrm{Y})}{n}\right]$ is given by

$$HC_r = h_{\mathrm{r}}\left(\int_0^{\frac{\mathrm{T(Y)}}{\mathrm{n}}} Q_r(t/y)\mathrm{d}t\right) = \frac{h_{\mathrm{r}}D}{\theta}\left(-T(Y) + \frac{\mathrm{e}^{\theta T(\mathrm{Y})}}{\theta} - \frac{1}{\theta}\right) \tag{14}$$

Consequently, manufacturer net profit is sales revenue minus the total relevant cost, which can be expressed as following

$$\pi_r(n, y) = SR_r - OC_r - HC_r \tag{15}$$

Therefore Retailer expected profit for $Q$ - units is

$$E_r(Q,n) = \frac{E(\pi_r(y))}{E(T(y))} = \frac{D^2}{\theta mQ}(\mathrm{S} - C_r)\left(\mathrm{e}^{\frac{\theta D}{mQn}} - 1\right) - \frac{nA_r\mathrm{D}}{\mathrm{mQ}} - \frac{h_{\mathrm{r}}D^2}{\theta\mathrm{mQ}}\left(-\frac{D}{mQ} + \frac{\mathrm{e}^{\frac{\theta D}{mQ}}}{\theta} - \frac{1}{\theta}\right) \tag{16}$$

### 3.2 Joint total profit per unit time:

The joint expected total profit is sum of $E_{\mathrm{r}}(Q,n)$ and $E_{\mathrm{m}}(Q,n)$, where $n$ optimal number of shipment $(n \in I_+)$ and $Q$ is optimal quantity, Q is continuous function.

$$\pi(\mathrm{Q}, n) = E_{\mathrm{r}}(Q,n) + E_{\mathrm{m}}(Q,n) \tag{17}$$

## 4 COMPUTATIONAL ALGORITHM

Two cases may arise: Either the manufacturer or the retailer takes an isolated decision or by conciliation they agree upon a joint policy.

*Case 1: When the manufacturer and retailer take isolated decision*

For retailer's profit $E_r$ to be maximum, for fix $n$. Set the partial derivative of $E_r$ with respect to $Q$ to be zero which yields by subject to $Q$ from $E_{\mathrm{r}}(Q,n)$, so finally we obtain optimal $Q^*(n)$

For the manufacturer to maximize $E_m$. Since $n$ is a discrete positive integer, it must satisfy

$$E_m(Q^*(n), n-1) \le E_m(Q^*(n), n) \ge E_m(Q^*(n), n+1) \tag{18}$$

Therefore, the total expected cost per unit time when the manufacturer-retailer does not agree to have integration is

$$E_{NI} = \max_{\mathrm{n}}\left\{\max_{\mathrm{n}} E_r(Q^*(n), n) + E_m\right\} \tag{19}$$

*Case 2: When manufacturer-retailer agrees to have integration*

In this case, $\pi$ is optimized jointly. The optimal value of $Q$ and $n$ must satisfy the following conditions simultaneously. $\frac{\partial \pi}{\partial Q}=0$ and $\pi(Q,n-1)\le \pi(Q,n)\ge \pi(Q,n+1)$

Hence, the total expected profit under integration, $E_I$ is given by

$$E_I = \max_{Q,n}\left(E_m, E_r\right) \tag{20}$$

## 5. NUMERICAL EXAMPLE AND OBSERVATIONS:

Consider one numerical example with following values in appropriate units

$D=400(units),\ m=0.4,\ C_m=10(\$),\ C_r=12(\$), S=19(\$),\ h_r=4\%,\ h_m=4\%,$
$n=3,\ \theta=0.01,\ A_r=300(\$),\ A_m=1000(\$)$

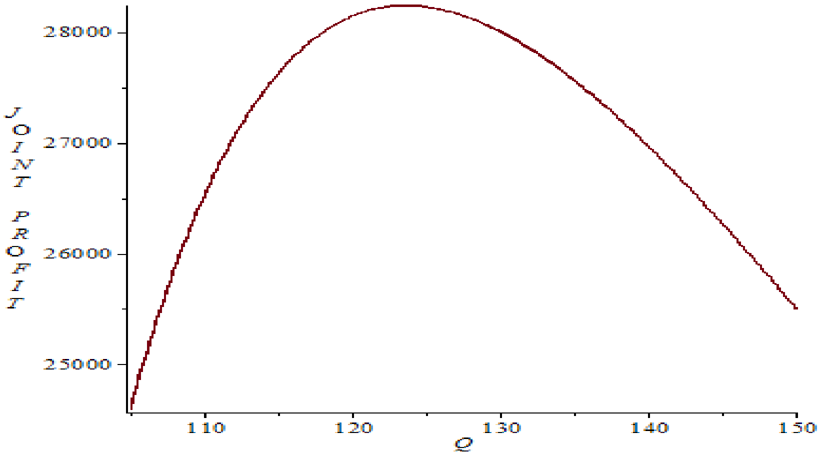


**Fig. 1** Joint Profit verses Order quantity for Integrated System

Model and algorithm is validated using numerical data. Results for both the integrated and isolated methods are discussed in table 2 further concavity of joint profit for integrated system is shown in figure 1.

**Table 2:** The optimal solution without and with integrated strategy

| Cases | Isolated Decision | Integrated Decision |
|---|---|---|
| $n$ | 3 | 3 |
| $Q$(units) | 139 | 124 |
| $E_m$(\$) | 15148.99 | 17164.73 |
| $E_r$ (\$) | 11900.96 | 11114.52 |
| $\pi$ (\$) | 27049.9 | 28279.25 |
| PIPI (%) | | 4.5 |
| M (in months) | | 1.957508 |

Clearly, $E_{\mathrm{I}}$ is greater than $E_{\mathrm{NI}}$ the integrated profit increment is (say)

$$\mathrm{I}_1 = E_I - E_{NI} \tag{21}$$

and its percentage is

$$\mathrm{PIPI} = \frac{E_I - E_{NI}}{E_{NI}} \times 100 \tag{22}$$

Let the retailer profit increment is defined by $S_b = \lambda S_1$ where $\lambda$ is the conciliation factor for benefit sharing. When $\lambda = 1$ all benefit goes to manufacturer only. For $\lambda = 0$ , all total benefit goes to retailer only. When $\lambda = 0.5$ , the total profit is equally distributed between manufacturer and retailer. If $I_e$ is the interest rate the present value of the unit cost after time M is $\mathrm{e}^{-I_e M}$ .The length of the retailer's credit period $M$ can be computed by solving the equation

$$DC_{\mathrm{r}}\left(1 - e^{-I_e M}\right) = S_b \tag{23}$$

This gives

$$M = \frac{1}{\mathrm{D}} \ln\left(\frac{DC_r}{DC_r - \lambda s_1}\right) \tag{24}$$

In Table 2 the comparative study of two cases an isolated and an integrated decision is presented. The manufacturer benefits $2015.77 while the retailer's loses $786.44. Therefore, the retailer will be reluctant to accept a joint strategy. To motivate the retailer to cooperate, the manufacturer offers a credit period of 59 days. The joint total profit is increase by 4.5%. Graphical comparison for joint profit in isolated and integrated system is shown in figure 2. Bifurcation of retailer's and manufacturer's profit is shown in figure 3. These comparisons and observations gives a clear conclusion that integrated supply chain is always lead to better profit. Loss observed by retailer can be manage by retailer by offering a credit period and then sharing the benefit. This creates a win – win situation and lead to long term sustainable supply chain.

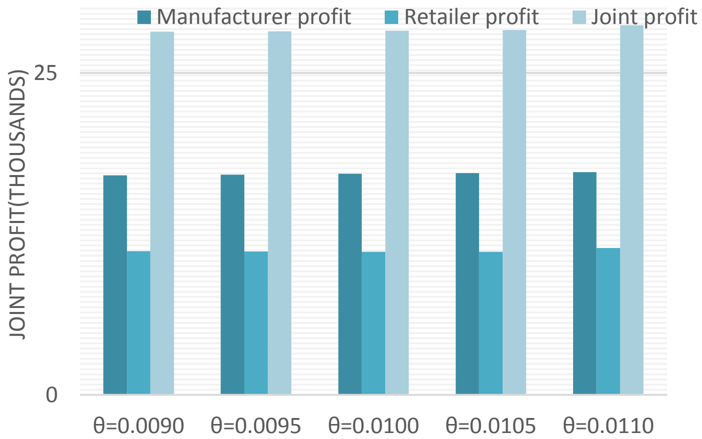


**Fig. 3** Comparison of Joint and Individual Profit for integrated system

## 6. SENSITIVITY ANALYSIS

For the above example we carried out sensitivity analysis to find the critical inventory parameters the change in manufacture, retailer and joint profit is studied by varying inventory parameters as -10%, -5%, 5% and 10%. The results are shown in Figure 4.

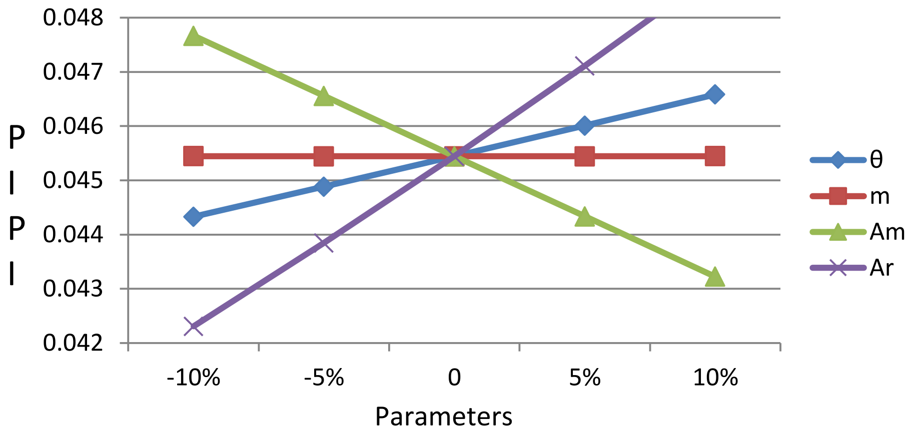


**Figure 4.** Sensitivity of parameters towards percentage of integrated profit increment (PIPI)

The sensitivity analysis of $\theta, m, A_m$ and $A_r$ are carried out in Figure 1. It has shown that $m$ no effect on percentage of integrated profit increment .It is observed that an increase in $\theta$ and $A_r$ increase percentage of integrated profit increment where $A_m$ decrease the percentage of integrated profit increment.

## 7. CONCLUSION:

In this paper an integrated model for constant deteriorating stock is proposed under random input. Model provides algorithm to calculate optimal number of shipments for joint profit maximization. Joint profit for both retailer and manufacture is calculated and analysed with individual profits. Though the integrated joint profit increases supplier's profit, the retailer's profit decreases due to random input in his inventory. Observations from numerical data and sensitivity analysis clearly show that percentage of integrated profit increase as rate of deterioration increase. A credit period is offered to retailer from supplier to share the advantage of integrated system. Observations also advocate that integrated system work very well for the supply chain deals with highly deteriorating items.

## 8. FUTURE SCOPE

The proposed model can be extended for deteriorating items with a two parameter Weibull-distribution. In addition we could consider the selling price as well as stock dependent demand. Finally we could generalize the model for shortages, quantity or price discount, preservative technology investment and inflation rates and others different economic background.

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