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An Introduction to p-Soft τ-Algebras and their p-Soft ω-Subalgebras

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Abstract

In this paper, generalizing the notions of soft group (ring, module), soft subgroup and normal subgroup (subring and left/right ideal, sub module), we introduce the notions of p-soft τ-algebra, p-soft ω-subalgebra of a p-soft τ-algebra and study some (lattice) algebraic properties of p-soft ω-subalgebras of a p-soft τ-algebra.

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1. Introduction

The traditional view in science, especially in mathematics, is to avoid uncertainty at all levels at any cost. Thus "being uncertain" is regarded as "being unscientific". But unfortunately in real life most of the information that we have to deal with is mostly uncertain.

One of the paradigm shifts in science and mathematics in this century is to accept uncertainty as part of science and the desire to be able to deal with it, as there is very little left out in the practical real world for scientific and mathematical processing without this acceptance!

One of the earliest successful attempts in this directions is the so called Fuzzy Set Theory, introduced by Zadeh[34].

While there is enormous freedom to set the membership value for an element in a fuzzy subset, it is equally a difficult problem to set the same. In spite of this enormous

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difficulty research in Fuzzy Set Theory proved to be quite successful and several types of uncertainties were addressed both beautifully and usefully.

On the other hand, any collection of objects or any sub collection of this collection have various levels of description/understanding via adjectives/parameters. Perhaps observing this descriptive/parametric nature of collections of objects and their sub collections in the cognitive process, Molodtsov[22] defined his soft set to be a pair (F, A), where A is possibly the set of all parameters/descriptors/adjectives which describe a collection of objects U and for each parameter/descriptor/adjective in A, $F(a)$ is a sub collection in the collection of objectives U which qualify/satisfy a.

Since certain sub collections of U can be determined by a subset of A and/or a subset of A can determine a sub collection of U, a soft set (G, B) over the same collection of objects U, can be regarded as a soft subset of (F, A) if, B is a subset of A and for each b in B, Gb is a subset of Fb and the theory goes on.....

Ever since the soft sets came into existence, some mathematicians started imposing and studying algebraic structures on soft sets. Since any soft set involves two component sets, namely, a universal set and a parameter set, interestingly, *some researchers algebrized universal set and others algebrized parameter set.*

For *parameter set algebrised* research papers, one can refer to Wen[32] and Yin-Liao[33] for soft groups; Atagun-Sezgin[7] for soft substructures of rings, fields and modules.

Now in Murthy-Maheswari[24], algebrising the parameter set in a soft set, f-soft algebras of type τ or f-soft τ-algebras and f-soft subalgebras of type ω or f-soft ω-subalgebras of an f-soft τ-algebra, where ω is a subtype of τ, are introduced and some of their (lattice) algebraic properties are studied.

For *universal set algebrised* research papers, one can refer to Aktas-Cagman[2] for soft groups; Feng et al.[13] for soft semi rings; Acer et al.[1] for soft rings; Sun et al.[31] for soft modules; Sezgin-Atagun[28] for soft groups and normalistic soft groups; Sezgin et al.[29] for soft near-rings and idealistic soft near-rings; Changphas-Thongkam[11] for soft algebras in a general viewpoint; Jun[15] for soft BCK/BCI-algebras; Jun et al.[16] for soft set theory applied to ideals in d-algebras; Jun et al.[17] for soft p-ideals of soft BCI-algebras; Zhu[35] for soft BL-algebras and soft logic system BL; Burak Kurt[9] for soft algebraic structures; Kazanci et al.[19] for soft BCH-algebras; Jun-Sun[18] for soft sets in BE-algebras; Alshehri et al.[6] for soft K-algebras and Khameneh-Kilicman[20] for soft σ-algebras etc..

Now in this paper, algebrising the universal set in a soft set, p-soft algebras of type τ or p-soft τ-algebras and p-soft subalgebras of type ω or p-soft ω-subalgebras of a p-soft τ-algebra, where ω is a subtype of τ, are introduced and some of their (lattice) algebraic properties are studied.

For some notions and results of universal algebra used in this paper, one can refer to any of the books Burris-Sankappanavar[10], Cohn[12], Gratzer[14] and for all the lattice theoretic notions and results, one can refer to any of the books Szasz[27] and Birkhoff[8]. However, for some notions like subtype ω of a type τ, ω-subalgebra etc., for results related to these notions and for examples and counter examples we closely follow Murthy-Maheswari[24].

Remarks about the notation 1.1. (1) Since τ-algebras, soft sets and p-soft τ-algebras are all ordered pairs and dealt in this same paper, we had a *notational drought running into run-out of symbols and typing difficulties in the collective representation of all these notions. Consequently, we use a new notation for soft sets. However, in this case, in the concerned section, while recalling the standard notion, first we gave the so far standard definition in its common-already-journal appeared (CAJA) notation and then gave a corresponding new notation used in this paper which most suited, for a collective representation of all these concepts together without notational conflicts*, with remark after this definition whenever needed.

(2) *It may be useful to know that in general the notations of the type* (A, α_A), *where* A *is a set and* α_A *is some structure on* A, *can actually lead to such technical difficulties as* $A = B$ *implies* $\alpha_A = \alpha_B$*-assuming that the relation* $A\ R\ \alpha_A$ *is a function, which in most cases is so because it is a replacement relation* (!), *implying that one set* A *will permit one and only one such structure, namely,* α_A, *which in most cases is* **not** *expected so.*

Having observed (2), however we prefer to use the notation (A, α_A) because at any given instance we deal with one and only one underlying set A and one and only one such structure α_A.

2. Preliminaries

Now in what follows, we recall some elementary notions in Soft Set Theory making the document more self contained.

2.1. Soft Sets

In this section, the basic notions in Soft Set Theory such as soft (sub) set, empty (whole) soft set, soft union (intersection) of soft sets, OR (AND) soft set etc. are recalled.

Definition 2.1.1. [22] Let U be a universal set, $P(U)$ be the power set of U and E be a set of parameters. A pair (F, E) is called a *soft set* over U iff $F : E \to P(U)$ is a mapping defined by, for each $e \in E$, $F(e)$ is a subset of U. In other words, a soft set over U is a parametrized family of subsets of U.

Remark 2.1.2. In order to facilitate the ordered pair notation for a soft set mentioned in 2.1.1 above along with that of an ω-algebra mentioned in 2.3.1(h) of [24] previously, as we deal with *both* of them simultaneously in this same paper in Section 3, we deviate from the above notation for a soft set and adapt the following notation for convenience as follows:

Let U be a universal set. A typical *soft set* over U is an ordered pair $\mathsf{E} = (\sigma_{\mathsf{E}}, E)$, where E is a set of parameters, called the underlying set for E, $P(U)$ is the power set of U and $\sigma_{\mathsf{E}} : E \to P(U)$ is a map, called the underlying set valued map for E. Some times σ_{E} is also called the soft structure on E.

Notice that (1) a soft set is determined by *all* three namely, the universal set U, the parameter set E and the map $\sigma_E : E \to P(U)$ and *not* by any one or two of them (2) two different soft sets can very well have exactly the same underlying set or parameter set (3) as mentioned earlier in 1.1(2), here onwards our soft sets look like $\mathsf{E} = (\sigma_E, E)$ and *not* as $\mathsf{E} = (\sigma_{\mathsf{E}}, E)$ or (F, E) as defined in 2.1.2 or 2.1.1 above.

Definitions and Statements 2.1.3.

(a) [4] The *empty soft set* over U is a soft set with the empty parameter set, denoted by $\Phi = (\sigma_\phi, \phi)$. Clearly, it is unique.

(b) [4] A soft set E over U is said to be a *null soft set* over U, denoted by Φ_E, iff $\sigma_E e = \phi$, the empty set, for all $e \in E$.

(c) [3] A soft set E over U is said to be a *whole soft set* over U, denoted by U_E, iff $\sigma_E e = U$ for all $e \in E$.

(d) [26] For any pair of soft sets A and B over U, A is a *soft subset* of B, denoted by $\mathsf{A} \subseteq \mathsf{B}$, iff (i) $A \subseteq B$ (ii) $\sigma_A a \subseteq \sigma_B a$ for all $a \in A$.

The following are easy to see:

(1) Always the empty soft set Φ over U is a soft subset of every soft set A over U

(2) $\mathsf{A} = \mathsf{B}$ iff $\mathsf{A} \subseteq \mathsf{B}$ and $\mathsf{B} \subseteq \mathsf{A}$ iff $A = B$ and $\sigma_A = \sigma_B$.

For any family of soft subsets $(\mathsf{A}_i)_{i \in I}$ of E over U,

(e) [13] the *soft union* of $(\mathsf{A}_i)_{i \in I}$, denoted by $\cup_{i \in I} \mathsf{A}_i$, is defined by the soft set A, where

  (i) $A = \cup_{i \in I} A_i$

  (ii) $\sigma_A a = \cup_{i \in I_a} \sigma_{A_i} a$, where $I_a = \{i \in I / a \in A_i\}$, for all $a \in A$

(f) the *soft intersection* of $(\mathsf{A}_i)_{i \in I}$, denoted by $\cap_{i \in I} \mathsf{A}_i$, is defined by the soft set A, where

  (i) $A = \cap_{i \in I} A_i$

  (ii) $\sigma_A a = \cap_{i \in I} \sigma_{A_i} a$ for all $a \in A$

(g) [13] the *OR-soft set* of $(\mathsf{A}_i)_{i \in I}$, denoted by $\curlyvee_{i \in I} \mathsf{A}_i$, is defined by the soft set A, where

  (i) $A = \Pi_{i \in I} A_i$

  (ii) $\sigma_A \boldsymbol{a} = \cup_{i \in I} \sigma_{A_i} a_i$ for all $\boldsymbol{a} = (a_i)_{i \in I} \in A$.

(h) [13] the *AND-soft set* of $(\mathsf{A}_i)_{i\in I}$, denoted by $\curlywedge_{i\in I}\mathsf{A}_i$, is defined by the soft set A, where

(i) $A = \Pi_{i\in I}A_i$

(ii) $\sigma_A \boldsymbol{a} = \cap_{i\in I}\sigma_{A_i}a_i$ for all $\boldsymbol{a} = (a_i)_{i\in I} \in A$.

Lemma 2.1.4. For any families of soft subsets $(\mathsf{A}_i)_{i\in I}$ and $(\mathsf{B}_i)_{i\in I}$ of a soft set E over U such that $\mathsf{A}_i \subseteq \mathsf{B}_i$ for all $i \in I$, the following are true:

(a) $\cup_{i\in I}\mathsf{A}_i \subseteq \cup_{i\in I}\mathsf{B}_i$

(b) $\cap_{i\in I}\mathsf{A}_i \subseteq \cap_{i\in I}\mathsf{B}_i$

(c) $\curlyvee_{i\in I}\mathsf{A}_i \subseteq \curlyvee_{i\in I}\mathsf{B}_i$

(d) $\curlywedge_{i\in I}\mathsf{A}_i \subseteq \curlywedge_{i\in I}\mathsf{B}_i$.

Proof. All are straight forward and follows from 2.1.3 (d). ■

3. Soft Algebras

In 2007, Aktas-Cagman[2] introduced the notion of soft group. According to them, a soft group over a group G is a pair (F, A) such that $F(x)$ is a subgroup of G for all $x \in A$.

In 2008, Feng et al.[13] introduced the notions of soft semiring. According to them, a non-null soft set (η, A) over a semiring S is called a soft semiring over S, if $\eta(x)$ is a subsemiring of S for all $x \in$ Supp(η, A), where Supp$(\eta, A) = \{x \in A/\eta(x) \neq \phi\}$. Let us recall that (η, A) is a null soft set over U iff for all $x \in A$, $\eta x = \phi$, the empty subset of U.

In 2008, Sun et al.[31] introduced the notion of soft modules. According to them, a soft module over a left R-module M is a pair (F, A) such that $F(x)$ is a submodule of M for all $x \in A$.

In 2010, Acar-Koyuncu-Tanay[1] introduced the notion of soft ring. According to them, a non-null soft set (F, A) over a commutative ring R is said to be a soft ring over R, if $F(x)$ is a subring of R for all $x \in A$.

In 2011, Sezgin-Atagun[28] introduced the notion of normalistic soft group. According to them, a non-null soft set (F, A) over a group G is said to be a normalistic soft group over G, if $F(x)$ is a normal subgroup of G for all $x \in$ Supp(F, A).

3.1. p-Soft τ-Algebras and p-Soft ω-Subalgebras

In this section, generalizing the above notions of soft substructures of soft algebraic structures, the notions of p-soft algebra of type τ or p-soft τ-algebra and p-soft ω-subalgebra of a p-soft τ-algebra are introduced.

Definition and Statements 3.1.1.

(a) Let τ be a type, ω be a subtype of τ and $\mathcal{U} = (U, F_U)$ be a τ-algebra whose underlying set is the set U. A soft set $\mathsf{E} = (\sigma_E, E)$ over U is a *p-soft ω-algebra* over $\mathcal{U}$ iff for each $e \in E$, $\sigma_E e$ is an ω-subalgebra of $\mathcal{U}$.

Note: (1) we are *not* making any notational distinction between a p-soft algebra and its underlying soft set (2) if $E = \phi$, the empty set, then the soft set $\mathsf{E} = (\sigma_E, E)$ $= (\sigma_\phi, \phi)$ over U is *trivially* a p-soft ω-algebra over $\mathcal{U}$ because there is *no* $e \in E$ $= \phi$ such that $\sigma_E e = \sigma_\phi e$ is *not* an ω-subalgebra of $\mathcal{U}$ or the condition for p-soft ω-algebra is trivially satisfied.

The above p-soft ω-algebra (σ_ϕ, ϕ) over $\mathcal{U}$ is called the *empty p-soft ω-algebra* over $\mathcal{U}$.

(b) Let τ be a type, ω be a subtype of τ and $\mathcal{U}$ be a τ-algebra. For any pair of p-soft algebras A and B of types ω and τ resp. over $\mathcal{U}$, A is a p-*soft ω-subalgebra* of B iff (i) $A \subseteq B$ (ii) $\sigma_A a$ is an ω-subalgebra of $\sigma_B a$ for all $a \in A$.

Note: Whenever ω is a subtype of τ and E is a p-soft τ-algebra over $\mathcal{U}$, E_ω = $(\sigma_{E_\omega}, E_\omega)$, where $E_\omega = E$ and $\sigma_{E_\omega} e = (\sigma_E e)_\omega$, the ω-restriction of $\sigma_E e$ (cf. Note after 2.3.1(i) of [24]), for all $e \in E$, is called the *ω-restriction* of E and is a p-soft ω-algebra over $\mathcal{U}$, a p-soft ω-subalgebra of E and E is a p-soft τ-subalgebra of E.

(c) Let τ be a type, ω be a subtype of τ and $\mathcal{U}$ be a τ-algebra. For any pair of p-soft ω-subalgebras A and B of a p-soft τ-algebra E over $\mathcal{U}$, $\mathsf{A} = \mathsf{B}$ iff $A = B$ and σ_A = σ_B.

Throughout this section τ is a fixed type, ω is an arbitrary but fixed subtype of τ, $\mathcal{U}$ is a fixed (crisp) τ-algebra, the sanserif letters, A, B and E with their suffixes are p-soft (sub) algebras and any such C represents (σ_C, C).

3.2. Algebra of p-Soft ω-Subalgebras

In this section some (lattice) algebraic properties of the collection of all p-soft ω-subalgebras of a p-soft τ-algebra are studied.

Definition 3.2.1. Let τ be a type, ω be a subtype of τ and $\mathcal{U}$ be a τ-algebra. For any family of p-soft ω-subalgebras $(\mathsf{A}_i)_{i \in I}$ of a p-soft τ-algebra E over $\mathcal{U}$, the p-soft ω-subalgebra A of E, *if it exists*, with the properties (a) A_i is a p-soft ω-subalgebra of A for all $i \in I$ (b) whenever B is a p-soft ω-subalgebra of E such that A_i is a p-soft ω-subalgebra of B for all $i \in I$, A is a p-soft ω-subalgebra of B, is called the *union* of p-soft ω-subalgebras $(\mathsf{A}_i)_{i \in I}$ and is denoted by $\cup_{i \in I} \mathsf{A}_i$.

Proposition 3.2.2. For any family of p-soft ω-subalgebras $(\mathsf{A}_i)_{i\in I}$ of a p-soft τ-algebra E over $\mathcal{U}$, the soft union (σ_A, A) = A of the underlying soft sets of $(\mathsf{A}_i)_{i\in I}$, defined by $A = \cup_{i\in I} A_i$ and for all $a \in A$, $\sigma_A a = \cup_{i\in I_a} \sigma_{A_i} a$, where $I_a = \{i \in I / a \in A_i\}$, is a p-soft ω-subalgebra of E such that A is the union of p-soft ω-subalgebras of $(\mathsf{A}_i)_{i\in I}$ or $\mathsf{A} = \cup_{i\in I}\mathsf{A}_i$ whenever

(1) $(A_i)_{i\in I}$ are pairwise disjoint (or)

(2) $(\mathsf{A}_i)_{i\in I}$ is a chain of p-soft ω-subalgebras of E and ω is finitary.

Proof. (1) It follows from 2.1.3(e), 3.1.1 (a) and (b) and 3.2.1.

(2) It follows from 2.1.3(e), 3.1.1 (a) and (b), 2.3.3 of [24] and 3.2.1.

The following Example shows that the above Proposition is *not* true if $(A_i)_{i\in I}$ are *not* pairwise disjoint (or) $(\mathsf{A}_i)_{i\in I}$ is *not* a chain of p-soft ω-subalgebras of E but ω is finitary.

Example 3.2.3. Let τ, ω, $\mathcal{X}$, $\mathcal{Y}_1$ and $\mathcal{Y}_2$ be as in 2.3.4 of [24]. Let $\mathsf{E} = (\{(e_1, \mathcal{X}), (e_2, \mathcal{X})\}, \{e_1, e_2\})$ be the p-soft τ-algebra over $\mathcal{X}$, $\mathsf{A}_1 = (\{(e_1, \mathcal{Y}_1)\}, \{e_1\})$ and $\mathsf{A}_2 = (\{(e_1, \mathcal{Y}_2), (e_2, \mathcal{X})\}, \{e_1, e_2\})$.

Then $(\mathsf{A}_i)_{i=1,2}$ are p-soft ω-subalgebras of E but the soft union of their underlying soft sets does *not* define a union of p-soft ω-subalgebras.

In the above Example ω is finitary and $(\mathsf{A}_i)_{i=1,2}$ is *not* a chain of p-soft ω-subalgebras of E. So, the same Example also shows that the above Proposition is *not* true if only ω is finitary but $(\mathsf{A}_i)_{i\in I}$ is *not* a chain of p-soft ω-subalgebras of E.

The following Example shows that the above Proposition is *not* true if, only $(\mathsf{A}_i)_{i\in I}$ is a chain of p-soft ω-subalgebras of E but ω is *not* finitary.

Example 3.2.4. Let τ, ω, $\mathcal{X}$ and $(\mathcal{Y}_i)_{i\in\mathbb{N}}$ be as in 2.3.5 of [24]. Let $\mathsf{E} = (\sigma_E, E)$, where $E = \mathbb{N}$ and σ_E be the *constant map* defined by, for all $e \in E$, $\sigma_E e = \mathcal{X}$, be the p-soft τ-algebra over $\mathcal{X}$ and $\mathsf{A}_i = (\sigma_{A_i}, A_i)$, where $A_i = \{n/n = 1, 2, \ldots, i\}$ and σ_{A_i} be the *constant map* defined by, for all $a \in A_i$, $\sigma_{A_i} a = \mathcal{Y}_i$, for all $i \in \mathbb{N}$.

Then $(\mathsf{A}_i)_{i\in\mathbb{N}}$ is a chain of p-soft ω-subalgebras of E such that the soft union of the underlying soft sets of the p-soft ω-subalgebras $(\mathsf{A}_i)_{i\in\mathbb{N}}$ does *not* define a p-soft ω-subalgebra of E.

Definition 3.2.5. Let τ be a type, ω be a subtype of τ and $\mathcal{U}$ be a τ-algebra. For any family of p-soft ω-subalgebras $(\mathsf{A}_i)_{i\in I}$ of a p-soft τ-algebra E over $\mathcal{U}$, the p-soft ω-subalgebra A of E, *if it exists*, with the properties (a) A is a p-soft ω-subalgebra of A_i for all $i \in I$ (b) whenever B is a p-soft ω-subalgebra of E such that B is a p-soft ω-subalgebra of A_i for all $i \in I$, B is a p-soft ω-subalgebra of A, is called the *intersection* of p-soft ω-subalgebras $(\mathsf{A}_i)_{i\in I}$ and is denoted by $\cap_{i\in I}\mathsf{A}_i$.

Proposition 3.2.6. For any subtype ω of a type τ such that $d(\omega)_0 \neq \phi$ and for any p-soft τ-algebra E over $\mathcal{U}$, the following are true:

(1) For any family of p-soft ω-subalgebras $(\mathsf{A}_i)_{i\in I}$ of E, the soft intersection (σ_A, A) = A of the underlying soft sets of $(\mathsf{A}_i)_{i\in I}$, defined by $A = \cap_{i\in I} A_i$ and for all $a \in A$, $\sigma_A a = \cap_{i\in I}\sigma_{A_i} a$, is a p-soft ω-subalgebra of E such that A is the intersection of p-soft ω-subalgebras of $(\mathsf{A}_i)_{i\in I}$ or $\mathsf{A} = \cap_{i\in I}\mathsf{A}_i$

(2) For any soft subset A of the underlying soft set of E, the intersection of all p-soft ω-subalgebras of E whose underlying soft sets contain the soft set A is the unique smallest p-soft ω-subalgebra of E containing A.

Proof. (1) follows from 2.1.3(f), 3.1.1 (a) and (b), 2.3.7(1) of [24] and 3.2.5.

(2) follows from (1).

Definition 3.2.7. For any subtype ω of a type τ such that $d(\omega)_0 \neq \phi$, for any p-soft τ-algebra E over $\mathcal{U}$ and for any soft subset A of the underlying soft set of E, the unique smallest p-soft ω-subalgebra of E defined as in (2) above is called the *p-soft ω-subalgebra generated* by A of E and is denoted by $(\mathsf{A})_\mathsf{E}$. In other words, $\cap_{\mathsf{A}\subseteq\mathsf{B},\mathsf{B}\text{ is a p-soft }\omega-\text{subalgebra of }\mathsf{E}}\mathsf{B} = (\mathsf{A})_\mathsf{E}$.

Lemma 3.2.8. For any subtype ω of a type τ such that $d(\omega)_0 \neq \phi$ and for any p-soft τ-algebra E over $\mathcal{U}$, $(\mathsf{A})_\mathsf{E} = \mathsf{C}$ iff $C = A$ and for all $a \in A$, $\sigma_C a = (\sigma_A a)_\mathcal{U}$.

Proof. It follows from 3.1.1(c), 2.3.8 of [24] and the fact that B, where $B = A$ and for each $a \in A$, $\sigma_B a = (\sigma_A a)_\mathcal{U}$, is also an p-soft ω-subalgebra of E. ■

The following Example shows that the above Proposition is *not* true if $d(\omega)_0 = \phi$.

Example 3.2.9. Let $\tau, \omega, \mathcal{X}, \mathcal{Y}_1$ and $\mathcal{Y}_2$ be as in 2.3.9 of [24]. Let E=$(\{(e_1, \mathcal{X}), (e_2, \mathcal{X})\}, \{e_1, e_2\})$ be the p-soft τ-algebra over $\mathcal{X}$, $\mathsf{A}_1 = (\{(e_1, \mathcal{X}), (e_2, \mathcal{Y}_1)\}, \{e_1, e_2\})$ and $\mathsf{A}_2 = (\{(e_2, \mathcal{Y}_2)\}, \{e_2\})$.

Then $(\mathsf{A}_i)_{i=1,2}$ are p-soft ω-subalgebras of E but the soft intersection of their underlying soft sets does *not* define a intersection of p-soft ω-subalgebras.

Corollary 3.2.10. For any subtype ω of a type τ such that $d(\omega)_0 \neq \phi$ and for any p-soft τ-algebra E over $\mathcal{U}$, the set $\mathscr{S}^p_\omega(\mathsf{E})$ of all p-soft ω-subalgebras of E is a complete lattice with

(1) the largest element $1_{\mathscr{S}^p_\omega(\mathsf{E})} = \mathsf{E}_\omega$, the ω-restriction of E

(2) the least element $0_{\mathscr{S}^p_\omega(\mathsf{E})} = \mathsf{E}_0$, where $E_0 = \phi$, the empty set, and $\sigma_{E_0} = \sigma_\phi$, the empty map (cf. (2) of Note after 3.1.1(a)), and

for any family $(\mathsf{A}_i)_{i\in I}$ of p-soft ω-subalgebras of E,

(3) $\wedge_{i\in I}\mathsf{A}_i = \cap_{i\in I}\mathsf{A}_i$ and with

(4) $\vee_{i\in I}\mathsf{A}_i = \cap\{\mathsf{B} \in \mathscr{S}^p_\omega(\mathsf{E})/\mathsf{A}_i \cap \mathsf{B} = \mathsf{A}_i$ for all A_i in $(\mathsf{A}_i)_{i\in I}\}$.

Proof. It follows from Notes after 3.1.1 (a) and (b), 3.2.6(1) and 2.2.1(a) of [24]. ■

Proposition 3.2.11. For any families of p-soft ω and τ-algebras $(\mathsf{A}_i)_{i\in I}$ and $(\mathsf{B}_i)_{i\in I}$ resp. over $\mathcal{U}$ such that A_i is a p-soft ω-subalgebra of B_i for all $i \in I$, $\cup_{i\in I}\mathsf{A}_i$ is a p-soft ω-subalgebra of $\cup_{i\in I}\mathsf{B}_i$ whenever

(1) $(B_i)_{i\in I}$ are pairwise disjoint (or)

(2) $(\mathsf{A}_i)_{i\in I}$ and $(\mathsf{B}_i)_{i\in I}$ are chains of p-soft ω and τ-algebras resp. over $\mathcal{U}$ and τ is finitary.

Proof. (1) It follows from 3.2.2(1) and 3.1.1(b).

(2) It follows from 3.2.2(2), 2.1.4(a) and 3.1.1(b). ■

The following Example shows that the above Proposition is *not* true if $(B_i)_{i\in I}$ are *not* pairwise disjoint.

Example 3.2.12. Let τ, ω, $\mathcal{X}$, $\mathcal{Y}_1$ and $\mathcal{Y}_2$ be as in 2.3.4 of [24]. Let B_1 = $(\{(e_1, \mathcal{Y}_1)\}$, $\{e_1\})$ and B_2 = $(\{(e_1, \mathcal{Y}_2), (e_2, \mathcal{X})\}, \{e_1, e_2\})$. Then $(\mathsf{B}_i)_{i=1,2}$ are p-soft τ-algebras over $\mathcal{X}$. Let A_1 = B_1 and A_2 = $(\{(e_2, \mathcal{X})\}, \{e_2\})$.

Then $(\mathsf{A}_i)_{i=1,2}$ are p-soft ω-algebras over $\mathcal{X}$; A_1 is a p-soft ω-subalgebra of B_1; A_2 is a p-soft ω-subalgebra of B_2 and since $(A_i)_{i=1,2}$ are disjoint, by 3.2.2(1), $\mathsf{A}_1 \cup \mathsf{A}_2$ = A is a p-soft ω-algebra over $\mathcal{X}$ but the soft union of the underlying soft sets of the p-soft τ-algebras $(\mathsf{B}_i)_{i=1,2}$ over $\mathcal{X}$ does *not* define a p-soft τ-algebra over $\mathcal{X}$.

The following Example shows that the above Proposition is *not* true if, only $(\mathsf{A}_i)_{i\in I}$ is a chain of p-soft ω-algebras over $\mathcal{U}$ and τ is finitary but $(\mathsf{B}_i)_{i\in I}$ is *not* a chain of p-soft τ-algebras over $\mathcal{U}$.

Example 3.2.13. Let τ, ω, $\mathcal{X}$, $\mathcal{Y}_1$ and $\mathcal{Y}_2$ be as in 2.3.4 of [24]. Let B_1 = $(\{(e_1, \mathcal{X})$, $(e_2, \mathcal{Y}_1)\}, \{e_1, e_2\})$ and B_2 = $(\{(e_1, \mathcal{X}), (e_2, \mathcal{Y}_2)\}, \{e_1, e_2\})$. Then $(\mathsf{B}_i)_{i=1,2}$ are p-soft τ-algebras over $\mathcal{X}$ but *not* a chain. Let A_1 = $(\{(e_1, \mathcal{Y}_1)\}$, $\{e_1\})$ and A_2 = B_2.

Then $(\mathsf{A}_i)_{i=1,2}$ are p-soft ω-algebras over $\mathcal{X}$; A_1 is a p-soft ω-subalgebra of both A_2 and B_1; A_2 is a p-soft ω-subalgebra of B_2 and since $(\mathsf{A}_i)_{i=1,2}$ is a chain of p-soft ω-algebras, by 3.2.2(2), $\mathsf{A}_1 \cup \mathsf{A}_2$ = A is a p-soft ω-algebra over $\mathcal{X}$ but the soft union of the underlying soft sets of the p-soft τ-algebras $(\mathsf{B}_i)_{i=1,2}$ over $\mathcal{X}$ does *not* define a p-soft τ-algebra over $\mathcal{X}$.

The following Example shows that the above Proposition is *not* true if, only $(\mathsf{B}_i)_{i\in I}$ is a chain of p-soft τ-algebras over $\mathcal{U}$ and τ is finitary but $(\mathsf{A}_i)_{i\in I}$ is *not* a chain of p-soft ω-algebras over $\mathcal{U}$.

Example 3.2.14. Let τ, ω, $\mathcal{X}$, $\mathcal{Y}_1$ and $\mathcal{Y}_2$ be as in 2.3.4 of [24]. Let B_1 = $(\{(e_1, \mathcal{X})\}$, $\{e_1\})$ and B_2 = $(\{(e_1, \mathcal{X}), (e_2, \mathcal{X})\}, \{e_1, e_2\})$. Then $(\mathsf{B}_i)_{i=1,2}$ are p-soft τ-algebras over $\mathcal{X}$ and also a chain. Let A_1 = $(\{(e_1, \mathcal{Y}_1)\}, \{e_1\})$ and A_2 = $(\{(e_1, \mathcal{Y}_2), (e_2, \mathcal{X})\}, \{e_1, e_2\})$.

Then $(\mathsf{A}_i)_{i=1,2}$ are p-soft ω-algebras over $\mathcal{X}$; A_1 is a p-soft ω-subalgebra of B_1; A_2 is a p-soft ω-subalgebra of B_2 and since $(\mathsf{B}_i)_{i=1,2}$ is a chain of p-soft τ-algebras, by 3.2.2(2), $\mathsf{B}_1 \cup \mathsf{B}_2 = \mathsf{B}$ is a p-soft τ-algebra over $\mathcal{X}$ but the soft union of the underlying soft sets of the p-soft ω-algebras $(\mathsf{A}_i)_{i=1,2}$ over $\mathcal{X}$ does *not* define a p-soft ω-algebra over $\mathcal{X}$.

The following Example shows that the above Proposition is *not* true if, only $(\mathsf{A}_i)_{i\in I}$ and $(\mathsf{B}_i)_{i\in I}$ are chains of p-soft ω and τ-algebras resp. over $\mathcal{U}$ but τ is *not* finitary.

Example 3.2.15. Let τ, ω, $\mathcal{X}$ and $(\mathsf{A}_i)_{i\in\mathbb{N}}$ be as in 3.2.4. Then $(\mathsf{A}_i)_{i\in\mathbb{N}}$ is a chain of p-soft ω-algebras over $\mathcal{X}$. Let $\mathsf{B}_i = (\sigma_{B_i}, B_i)$, where $B_i = \{n/n = 1, 2, \ldots, i+1\}$ and σ_{B_i} be the constant map defined by, for all $b \in B_i$, $\sigma_{B_i} b = \mathcal{Z}_i$, where $\mathcal{Z}_i$ is a τ-subalgebra of $\mathcal{X}$ defined as in 2.3.14 of [24], for all $i \in \mathbb{N}$.

Then $(\mathsf{B}_i)_{i\in\mathbb{N}}$ is a chain of p-soft τ-algebras over $\mathcal{X}$; A_i is a p-soft ω-subalgebra of B_i for all $i \in \mathbb{N}$ and the soft unions of the underlying soft sets of the p-soft ω and τ-algebras $(\mathsf{A}_i)_{i\in\mathbb{N}}$ and $(\mathsf{B}_i)_{i\in\mathbb{N}}$ resp. over $\mathcal{X}$ does *not* define a p-soft ω and τ-algebras resp. over $\mathcal{X}$.

Proposition 3.2.16. For any families of p-soft ω and τ-algebras $(\mathsf{A}_i)_{i\in I}$ and $(\mathsf{B}_i)_{i\in I}$ resp. over $\mathcal{U}$ such that A_i is a p-soft ω-subalgebra of B_i for all $i \in I$, $\cap_{i\in I}\mathsf{A}_i$ is a p-soft ω-subalgebra of $\cap_{i\in I}\mathsf{B}_i$ whenever $d(\omega)_0 \neq \phi$.

Proof. It follows from 3.2.6(1), 2.1.4(b) and 3.1.1(b). ■

The following Example shows that the above Proposition is *not* true if $d(\omega)_0 = \phi$.

Example 3.2.17. Let $\tau, \omega, \mathcal{X}, \mathcal{Y}_1$ and $\mathcal{Y}_2$ be as in 2.3.16 of [24]. Let B_1=$(\{(e_1, \mathcal{X}), (e_2, \mathcal{X})\}, \{e_1, e_2\})$ and $\mathsf{B}_2 = (\{(e_2, \mathcal{X})\}, \{e_2\})$. Then $(\mathsf{B}_i)_{i=1,2}$ are p-soft τ-algebras over $\mathcal{X}$. Let $\mathsf{A}_1 = (\{(e_1, \mathcal{X}), (e_2, \mathcal{Y}_1)\}, \{e_1, e_2\})$ and $\mathsf{A}_2 = (\{(e_2, \mathcal{Y}_2)\}, \{e_2\})$.

Then $(\mathsf{A}_i)_{i=1,2}$ are p-soft ω-algebras over $\mathcal{X}$; A_1 is a p-soft ω-subalgebra of B_1; A_2 is a p-soft ω-subalgebra of B_2 and since $d(\tau)_0 \neq \phi$, by 3.2.6(1), $\mathsf{B}_1 \cap \mathsf{B}_2 = \mathsf{B}$ is a p-soft τ-algebra over $\mathcal{X}$ but the soft intersection of the underlying soft sets of the p-soft ω-algebras $(\mathsf{A}_i)_{i=1,2}$ over $\mathcal{X}$ does *not* define a p-soft ω-algebra over $\mathcal{X}$.

Definition 3.2.18. Let τ be a type, ω be a subtype of τ and $\mathcal{U}$ be a τ-algebra. Let $(\mathsf{A}_i)_{i\in I}$ be a family of p-soft ω-algebras over $\mathcal{U}$. Then the OR-soft set A over U is an OR-p-soft ω-algebra of $(\mathsf{A}_i)_{i\in I}$ over $\mathcal{U}$, denoted by $\curlyvee_{i\in I}\mathsf{A}_i$, whenever it defines a p-soft ω-algebra.

Lemma 3.2.19. For any family of p-soft ω-algebras $(\mathsf{A}_i)_{i\in I}$ over $\mathcal{U}$, the OR-soft set A, where $A = \Pi_{i\in I}A_i$ and for all $\boldsymbol{a} = (a_i)_{i\in I} \in A$, $\sigma_A \boldsymbol{a} = \cup_{i\in I}\sigma_{A_i}a_i$, is a p-soft ω-algebra over $\mathcal{U}$ whenever for all $(a_i)_{i\in I} \in A$, $(\sigma_{A_i}a_i)_{i\in I}$ is a chain of ω-subalgebras of $\mathcal{U}$ and ω is finitary.

Proof. It follows from 2.1.3(g), 3.1.1(a) and 2.3.3 of [24]. ■

The following Example shows that the above Lemma is *not* true if, only ω is finitary but

$(\sigma_{A_i} a_i)_{i\in I}$ is *not* a chain of ω-subalgebras of $\mathcal{U}$ for all $(a_i)_{i\in I} \in A$.

Example 3.2.20. Let τ, ω, $\mathcal{X}$, $\mathcal{Y}_1$ and $\mathcal{Y}_2$ be as in 2.3.4 of [24]. Let $\mathsf{A}_1 = (\{(a_1, \mathcal{Y}_1)\}, \{a_1\})$ and $\mathsf{A}_2 = (\{(a_2, \mathcal{Y}_2)\}, \{a_2\})$.

Then $(\mathsf{A}_i)_{i=1,2}$ are p-soft ω-algebras over $\mathcal{X}$ such that $\{\sigma_{A_1} a_1, \sigma_{A_2} a_2\}$ is *not* a chain of ω-subalgebras of $\mathcal{X}$ and the OR-soft set of the underlying soft sets of the p-soft ω-algebras $(\mathsf{A}_i)_{i=1,2}$ over $\mathcal{X}$ does *not* define an OR-p-soft ω-algebra over $\mathcal{X}$.

The following Example shows that the above Lemma is *not* true if, only $(\sigma_{A_i} a_i)_{i\in I}$ is a chain of ω-subalgebras of $\mathcal{U}$, for all $(a_i)_{i\in I} \in A$, but ω is *not* finitary.

Example 3.2.21. Let $(\mathsf{A}_i)_{i\in\mathbb{N}}$ be as in 3.2.4. Then $(\mathsf{A}_i)_{i\in\mathbb{N}}$ are p-soft ω-algebras over $\mathcal{X}$; σ_{A_i} is the constant map defined by $\sigma_{A_i} a = \mathcal{Y}_i$ for all $a \in A_i$; $(\mathcal{Y}_i)_{i\in\mathbb{N}}$ is a chain of ω-subalgebras of $\mathcal{X}$ and the OR-soft set of the underlying soft sets of the p-soft ω-algebras $(\mathsf{A}_i)_{i\in\mathbb{N}}$ over $\mathcal{X}$ does *not* define an OR-p-soft ω-algebra over $\mathcal{X}$.

Proposition 3.2.22. For any families of p-soft ω and τ-algebras $(\mathsf{A}_i)_{i\in I}$ and $(\mathsf{B}_i)_{i\in I}$ resp. over $\mathcal{U}$ such that A_i is a p-soft ω-subalgebra of B_i for all $i \in I$, $\curlyvee_{i\in I}\mathsf{A}_i$ is a p-soft ω-subalgebra of $\curlyvee_{i\in I}\mathsf{B}_i$ whenever for each $\boldsymbol{a} = (a_i)_{i\in I} \in \Pi_{i\in I} A_i$, $(\sigma_{A_i} a_i)_{i\in I}$ and for each $\boldsymbol{b} = (b_i)_{i\in I} \in \Pi_{i\in I} B_i$, $(\sigma_{B_i} b_i)_{i\in I}$ are chains of ω and τ-subalgebras resp. of $\mathcal{U}$ and τ is finitary.

Proof. It follows from 3.2.19, 2.1.4(c) and 3.1.1(b). ■

The following Example shows that the above Proposition is *not* true if, only $(\sigma_{B_i} b_i)_{i\in I}$ is a chain of τ-subalgebras of $\mathcal{U}$ for each $\boldsymbol{b} = (b_i)_{i\in I} \in \Pi_{i\in I} B_i$ and τ is finitary but $(\sigma_{A_i} a_i)_{i\in I}$ is *not* a chain of ω-subalgebras of $\mathcal{U}$ for each $\boldsymbol{a} = (a_i)_{i\in I} \in \Pi_{i\in I} A_i$.

Example 3.2.23. Let τ, ω, $\mathcal{X}$, $\mathcal{Y}_1$ and $\mathcal{Y}_2$ be as in 2.3.4 of [24]. Let $\mathsf{B}_1 = (\{(a_1, \mathcal{Y}_1)\}, \{a_1\})$ and $\mathsf{B}_2 = (\{(a_2, \mathcal{X})\}, \{a_2\})$. Then $(\mathsf{B}_i)_{i=1,2}$ are p-soft τ-algebras over $\mathcal{X}$ and $\{\sigma_{B_1} a_1, \sigma_{B_2} a_2\}$ is a chain of τ-subalgebras of $\mathcal{X}$. Let A_1=B_1 and A_2=$(\{(a_2, \mathcal{Y}_2)\}, \{a_2\})$.

Then $(\mathsf{A}_i)_{i=1,2}$ are p-soft ω-algebras over $\mathcal{X}$; $\{\sigma_{A_1} a_1, \sigma_{A_2} a_2\}$ is *not* a chain of ω-subalgebras of $\mathcal{X}$; A_1 is a p-soft ω-subalgebra of B_1; A_2 is a p-soft ω-subalgebra of B_2 and since $\{\sigma_{B_1} a_1, \sigma_{B_2} a_2\}$ is a chain of τ-subalgebras of $\mathcal{X}$ and τ is finitary, by 3.2.19, $\mathsf{B}_1 \curlyvee \mathsf{B}_2 = \mathsf{B}$ is a p-soft τ-algebra over $\mathcal{X}$ but the OR-soft set of the underlying soft sets of the p-soft ω-algebras $(\mathsf{A}_i)_{i=1,2}$ over $\mathcal{X}$ does *not* define a p-soft ω-algebra over $\mathcal{X}$.

The following Example shows that the above Proposition is *not* true if, only $(\sigma_{A_i} a_i)_{i\in I}$ is a chain of ω-subalgebras of $\mathcal{U}$ for each $\boldsymbol{a} = (a_i)_{i\in I} \in \Pi_{i\in I} A_i$ and τ is finitary but $(\sigma_{B_i} b_i)_{i\in I}$ is *not* a chain of τ-subalgebras of $\mathcal{U}$ for each $\boldsymbol{b} = (b_i)_{i\in I} \in \Pi_{i\in I} B_i$.

Example 3.2.24. Let τ, ω, $\mathcal{X}$, $\mathcal{Y}_1$, B_1 and B_2 be as in 3.2.12. Then $(\mathsf{B}_i)_{i=1,2}$ are p-soft τ-algebras over $\mathcal{X}$ and $\{\sigma_{B_1} e_1, \sigma_{B_2} e_1\}$ is *not* a chain of τ-subalgebras of $\mathcal{X}$. Let $\mathsf{A}_1 = \mathsf{B}_1$ and $\mathsf{A}_2 = (\{(e_2, \mathcal{Y}_1)\}, \{e_2\})$.

Then $(\mathsf{A}_i)_{i=1,2}$ are p-soft ω-algebras over $\mathcal{X}$; $\{\sigma_{A_1} e_1, \sigma_{A_2} e_2\}$ is a chain of ω-subalgebras of $\mathcal{X}$; A_1 is a p-soft ω-subalgebra of B_1; A_2 is a p-soft ω-subalgebra of B_2 and since

$\{\sigma_{A_1} e_1, \sigma_{A_2} e_2\}$ is a chain of ω-subalgebras of $\mathcal{X}$ and ω is finitary, by 3.2.19, $\mathsf{A}_1 \curlyvee \mathsf{A}_2$ = A is a p-soft ω-algebra over $\mathcal{X}$ but the OR-soft set of the underlying soft sets of the p-soft τ-algebras $(\mathsf{B}_i)_{i=1,2}$ over $\mathcal{X}$ does *not* define a p-soft τ-algebra over $\mathcal{X}$.

The following Example shows that the above Proposition is *not* true if, only $(\sigma_{A_i} a_i)_{i\in I}$ and $(\sigma_{B_i} b_i)_{i\in I}$ are chains of ω and τ-subalgebras of $\mathcal{U}$ for each $\boldsymbol{a} = (a_i)_{i\in I} \in \Pi_{i\in I} A_i$ and for each $\boldsymbol{b} = (b_i)_{i\in I} \in \Pi_{i\in I} B_i$ resp., but τ is *not* finitary.

Example 3.2.25. Let τ, ω, $\mathcal{X}$ and $(\mathcal{Y}_i)_{i\in\mathbb{N}}$ be as in 2.3.5 of [24]. Let $\mathsf{B}_i = (\sigma_{B_i}, B_i)$, where $B_i = \{n/n = 1, 2, ..., i\}$ and σ_{B_i} be the *constant map* defined by, for all $b \in B_i$, $\sigma_{B_i} b = \mathcal{Y}_i$, for all $i \in \mathbb{N}$. Then $(\mathsf{B}_i)_{i\in\mathbb{N}}$ are p-soft τ-algebras over $\mathcal{X}$ and $(\sigma_{B_i} b_i)_{i\in\mathbb{N}} = (\mathcal{Y}_i)_{i\in\mathbb{N}}$ is a chain of τ-subalgebras of $\mathcal{X}$. Let $\mathsf{A}_i = (\sigma_{A_i}, A_i)$, where $A_i = \{i\}$ and σ_{A_i} be the *constant map* defined by, for all $a \in A_i$, $\sigma_{A_i} a = \mathcal{Y}_i$, for all $i \in \mathbb{N}$.

Then $(\mathsf{A}_i)_{i\in\mathbb{N}}$ are p-soft ω-algebras over $\mathcal{X}$; $(\sigma_{A_i} a_i)_{i\in\mathbb{N}} = (\mathcal{Y}_i)_{i\in\mathbb{N}}$ is a chain of ω-subalgebras of $\mathcal{X}$; A_i is a p-soft ω-subalgebra of B_i for all $i \in \mathbb{N}$ and the OR-soft sets of the underlying soft sets of the p-soft ω and τ-algebras $(\mathsf{A}_i)_{i\in\mathbb{N}}$ and $(\mathsf{B}_i)_{i\in\mathbb{N}}$ resp. over $\mathcal{X}$ do *not* define p-soft ω and τ algebras resp. over $\mathcal{X}$ as in 3.2.21.

Definition 3.2.26. Let τ be a type, ω be a subtype of τ and $\mathcal{U}$ be a τ-algebra. Let $(\mathsf{A}_i)_{i\in I}$ be a family of p-soft ω-algebras over $\mathcal{U}$. Then the AND-soft set A over U is an AND-p-soft ω-algebra of $(\mathsf{A}_i)_{i\in I}$ over $\mathcal{U}$, denoted by $\curlywedge_{i\in I} \mathsf{A}_i$, whenever it defines a p-soft ω-algebra.

Lemma 3.2.27. For any family of p-soft ω-algebras $(\mathsf{A}_i)_{i\in I}$ over $\mathcal{U}$, the AND-soft set A, where $A = \Pi_{i\in I} A_i$ and for all $\boldsymbol{a} = (a_i)_{i\in I} \in A$, $\sigma_A \boldsymbol{a} = \cap_{i\in I} \sigma_{A_i} a_i$, is a p-soft ω-algebra over $\mathcal{U}$ whenever $d(\omega)_0 \neq \phi$.

Proof. It follows from 2.1.3(h), 3.1.1(a) and 2.3.7(1) of [24]. ■

The following Example shows that the above Lemma is *not* true if $d(\omega)_0 = \phi$.

Example 3.2.28. Let τ, ω, $\mathcal{X}$, $\mathcal{Y}_1$ and $\mathcal{Y}_2$ be as in 2.3.9 of [24]. Let $\mathsf{A}_1 = (\{(a_1, \mathcal{Y}_1)\}, \{a_1\})$ and $\mathsf{A}_2 = (\{(a_2, \mathcal{Y}_2)\}, \{a_2\})$.

Then $(\mathsf{A}_i)_{i=1,2}$ are p-soft ω-algebras over $\mathcal{X}$ but the AND-soft set of their underlying soft sets does *not* define a AND-p-soft ω-algebra over $\mathcal{X}$.

Proposition 3.2.29. For any families of p-soft ω and τ-algebras $(\mathsf{A}_i)_{i\in I}$ and $(\mathsf{B}_i)_{i\in I}$ resp. over $\mathcal{U}$ such that A_i is a p-soft ω-subalgebra of B_i for all $i \in I$, $\curlywedge_{i\in I} \mathsf{A}_i$ is a p-soft ω-subalgebra of $\curlywedge_{i\in I} \mathsf{B}_i$ whenever $d(\omega)_0 \neq \phi$.

Proof. It follows from 3.2.27, 2.1.4(d) and 3.1.1(b). ■

The following Example shows that the above Proposition is *not* true if $d(\omega)_0 = \phi$.

Example 3.2.30. Let $\tau, \omega, \mathcal{X}, \mathcal{Y}_1$ and $\mathcal{Y}_2$ be as in 2.3.16 of [24]. Let B_1=$(\{(a_1, \mathcal{X})\}, \{a_1\})$ and $\mathsf{B}_2 = (\{(a_2, \mathcal{X})\}, \{a_2\})$. Then $(\mathsf{B}_i)_{i=1,2}$ are p-soft τ-algebras over $\mathcal{X}$. Let $\mathsf{A}_1 =$

$(\{(a_1, \mathcal{Y}_1)\}, \{a_1\})$ and $\mathsf{A}_2 = (\{(a_2, \mathcal{Y}_2)\}, \{a_2\})$.

Then $(\mathsf{A}_i)_{i=1,2}$ are p-soft ω-algebras over $\mathcal{X}$; A_1 is a p-soft ω-subalgebra of B_1; A_2 is a p-soft ω-subalgebra of B_2 and since $d(\tau)_0 \neq \phi$, by 3.2.27, $\mathsf{B}_1 \curlywedge \mathsf{B}_2 = \mathsf{B}$ is a p-soft τ-algebra over $\mathcal{X}$ but the AND-soft set of the underlying p-soft ω-algebras $(\mathsf{A}_i)_{i=1,2}$ over $\mathcal{X}$ does *not* define a p-soft ω-algebra over $\mathcal{X}$ as in 3.2.28.

4. Conclusion

In this paper, generalizing the notions of soft group (ring, module), soft subgroup and normal subgroup (subring and left/right ideal, sub module), we introduced the notions of p-soft τ-algebra, p-soft ω-subalgebra of a p-soft τ-algebra and studied some standard (lattice) algebraic properties of p-soft ω-subalgebras of a p-soft τ-algebra.

In [25], we continued the study further with that of maximal p-soft ω-subalgebras and p-soft ω-homomorphisms of p-soft τ-algebras.

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