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Mathematical Modelling and Analysis of Cocurrent Imbibition Phenomenon in Inclined Heterogeneous Porous Medium

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Abstract

A mathematical model is developed for cocurrent imbibition phenomenon occurring in inclined heterogeneous porous medium during secondary oil recovery process. For the mathematical modelling, we consider the heterogeneous porous medium and the porosity and the permeability of the heterogeneous porous medium are as functions of variable. The mathematical formulation leads to a one dimensional nonlinear partial differential equation. Homotopy series solution is obtained for governing equation. The solution of governing equation represents saturation of injected water for cocurrent imbibition phenomenon. The graphical and numerical representations are given by Mathematica software.

Keywords: Fluid flow; Heterogeneous porous medium; Cocurrent imbibition phenomenon; Homotopy analysis method.

1. INTRODUCTION

In the current research paper, we have discussed the cocurrent imbibition phenomenon in inclined heterogeneous porous medium. Spontaneous imbibition is defined as the displacement of non-wetting phase by wetting phase in porous medium by means of capillary force. Spontaneous imbibition is most important phenomenon in oil recovery process. Spontaneous imbibition may be classified as cocurrent imbibition and countercurrent imbibition. The main difference between two mechanisms for imbibition is the direction of flow. In cocurrent imbibition, the wetting and non-wetting phases move in the same direction [1-5]. In countercurrent imbibition, the wetting and non-wetting phases move in the opposite directions [1-3, 6-9]. When a porous medium is partially filled with wetting phase, oil recovery is dominated by cocurrent imbibition phenomenon [2]. Most of all the researchers have been discussed cocurrent imbibition phenomenon in homogeneous porous medium with different aspects [1-5]. Bourblaux and Kalaydjian [1] have discussed experimental study of cocurrent and countercurrent flows in natural porous media. Pooladi-Darvish and Firoozabadi [2] have studied the similarities and differences of cocurrent and countercurrent imbibition and pointed out the consequences for practical applications. Fazeli at al. [3] have applied homotopy perturbation method for cocurrent and countercurrent imbibition in fractured porous media. Yadav and Mehta [4] have obtained series solution for cocurrent imbibition during immiscible two-phase flow through porous media. Patel and Desai [5] have discussed the cocurrent imbibition phenomenon in inclined homogeneous porous medium. Many authors have discussed different phenomenon in heterogeneous porous medium. For example, Verma [10] have discussed the behavior of fingering in a displacement process in heterogeneous porous medium with capillary pressure. Patel, Mehta and Patel [7] have discussed imbibition phenomenon in heterogeneous porous media.

During secondary oil recovery process, it is assumed that the water is injected into oil formatted inclined heterogeneous porous medium. The velocity of oil and the velocity of water are considered under gravitational effect and inclination effect in the investigated flow system. For the sake of mathematical study, we assumed that the porosity and permeability of inclined heterogeneous porous medium are as functions of variable x only. The schematic representation of finger is considered as suggested by Scheidegger and Johnson [11]. Thus only average cross-sectional area occupied by fingers is considered, neglecting the size and shape of individual finger. Hence the average cross-sectional area occupied by injected water is defined as the saturation of injected water $S_w(x,t)$ at distance x and time t.

The mathematical model is developed for cocurrent imbibition phenomenon in inclined heterogeneous porous medium. One dimensional nonlinear partial differential equation is governed by it. Homotopy series solution is obtained with appropriate boundary conditions. The solution of governing equation represents the saturation of injected water at distance x and time t for cocurrent imbibition phenomenon.

2. MATHEMATICAL MODELLING

One dimensional two-phase immiscible and incompressible flow in porous medium is governed by the generalized Darcy's law for each phase as [12-14]:

$$V_w = -\frac{k_w}{\delta_w} K\left(\frac{\partial P_w}{\partial x} + \rho_w g \sin\theta\right) \tag{1}$$

$$V_o = -\frac{k_o}{\delta_o} K\left(\frac{\partial P_o}{\partial x} + \rho_o g \sin\theta\right) \quad (2)$$

where V_w and V_o are the velocities of water and oil respectively, k_w and k_o are the relative permeabilities of water and oil respectively, δ_w and δ_o are the constant viscosities of water and oil respectively, $K = K(x)$ is the variable permeability of the inclined heterogeneous porous medium, P_w and P_o are the pressures of water and oil respectively, ρ_w and ρ_o are the constant densities of water and oil respectively, g is the acceleration due to gravity, θ is the angle of inclination with porous matrix.

The law of conservation of mass for incompressible flow gives

$$P\frac{\partial S_w}{\partial t} + \frac{\partial V_w}{\partial x} = 0 \quad (3)$$

where $P = P(x)$ is the variable porosity of heterogeneous porous medium.

Due to the surface tension and the curvature of the interfaces between the two fluids within the small pores, the pressure in the non-wetting (oil) fluid is higher than the pressure in the wetting fluid (water). The difference between these two pressures is the capillary pressure (P_c). We accept, as an empirical fact, that the capillary pressure is a function of phase saturation [14, 15]:

$$P_c(S_w) = P_o - P_w. \quad (4)$$

Consider the capillary pressure is a continuous linear function of phase saturation as [16]:

$$P_c(S_w) = -\beta S_w \quad (5)$$

where β is a constant.

Assume that the analytical relationship between relative permeability and phase saturation as [11]:

$$k_w = S_w \text{ and } k_o = 1 - \alpha S_w \quad (6)$$

where α is a constant.

The sum of velocity of injected water and velocity of oil is total velocity V_t in cocurrent imbibition phenomenon as [17]:

$$V_w + V_o = V_t. \quad (7)$$

For the investigated flow system in heterogeneous porous medium, we assume porosity and permeability as functions of variable x only [10],

$$P = P(x) = \frac{1}{a_1 - a_2 x} \quad (8)$$

$$K = K(x) = K_0(1 + bx) \tag{9}$$

where a_1, a_2, K_0 and b are positive constants. Since $P(x)$ can't exceed unity, we assume that $a_1 - a_2 x \geq 1$.

For the sake of simplicity, we consider $K \propto P$ [18],

$$K = K_c P \tag{10}$$

where K_c is a constant.

Combining (1), (2) and (7) results in

$$\frac{k_w}{\delta_w} K\left(\frac{\partial P_w}{\partial x} + \rho_w g \sin\theta\right) + \frac{k_o}{\delta_o} K\left(\frac{\partial P_o}{\partial x} + \rho_o g \sin\theta\right) = -V_t\,. \tag{11}$$

Substituting (4) into (11)

$$\left(K\frac{k_w}{\delta_w} + K\frac{k_o}{\delta_o}\right)\frac{\partial P_w}{\partial x} + K\frac{k_o}{\delta_o}\frac{\partial P_c}{\partial x} + K\left(\rho_w \frac{k_w}{\delta_w} + \rho_o \frac{k_o}{\delta_o}\right) g \sin\theta = -V_t\,. \tag{12}$$

Solving (12) for $\frac{\partial P_w}{\partial x}$

$$\frac{\partial P_w}{\partial x} = -\left(K\frac{k_w}{\delta_w} + K\frac{k_o}{\delta_o}\right)^{-1}\left(K\frac{k_o}{\delta_o}\frac{\partial P_c}{\partial x} + K\left(\rho_w \frac{k_w}{\delta_w} + \rho_o \frac{k_o}{\delta_o}\right) g \sin\theta + V_t\right). \tag{13}$$

Using (1) and (13), we get

$$V_w = -\frac{k_w}{\delta_w}\left(\frac{k_w}{\delta_w} + \frac{k_o}{\delta_o}\right)^{-1}\left(K\frac{k_o}{\delta_o}(\rho_w - \rho_o) g \sin\theta - K\frac{k_o}{\delta_o}\frac{\partial P_c}{\partial x} - V_t\right). \tag{14}$$

The pressure of water can be expressed in the form

$$P_w = \frac{P_w + P_o}{2} + \frac{P_w - P_o}{2} = \overline{P} - \frac{1}{2}P_c \tag{15}$$

where $\overline{P}$ is the mean pressure which is constant, therefore (12) implies

$$K\left(\rho_w \frac{k_w}{\delta_w} + \rho_o \frac{k_o}{\delta_o}\right) g \sin\theta + \frac{K}{2}\left(\frac{k_o}{\delta_o} - \frac{k_w}{\delta_w}\right)\frac{\partial P_c}{\partial x} = -V_t\,. \tag{16}$$

Therefore (14) reduces to

$$V_w = \frac{K}{2}\frac{k_w}{\delta_w}\frac{\partial P_c}{\partial x} - K\frac{k_w}{\delta_w}\rho_w g \sin\theta\,. \tag{17}$$

Using (17) and (3), we get

$$P\frac{\partial S_w}{\partial t} + \frac{\partial}{\partial x}\left[\frac{K}{2}\frac{k_w}{\delta_w}\frac{\partial P_c}{\partial x} - K\frac{k_w}{\delta_w}\rho_w g \sin\theta\right] = 0\,. \tag{18}$$

Since $K = K_c P, k_w = S_w$ and $P_c = -\beta S_w$, we have

$$\frac{\partial S_w}{\partial t} = \frac{\beta K_c}{2\delta_w}\left[\frac{\partial}{\partial x}\left(S_w \frac{\partial S_w}{\partial x}\right) + S_w \frac{\partial S_w}{\partial x}\frac{1}{P}\frac{\partial P}{\partial x}\right] + \frac{K_c \rho_w g \sin\theta}{\delta_w}\left[\frac{\partial S_w}{\partial x} + S_w \frac{1}{P}\frac{\partial P}{\partial x}\right]. \quad (19)$$

Since

$$\frac{1}{P}\frac{\partial P}{\partial x} = \frac{\partial(\log P)}{\partial x} = \frac{\partial}{\partial x}\left(-\log a_1 + \frac{a_2 x}{a_1}\right)(\textit{neglecting higher order terms of } x) = \frac{a_2}{a_1}.$$

Using dimensionless variables

$$X = \frac{x}{l},\ T = \frac{\beta K_c t}{2\delta_w l^2}$$

(19) becomes

$$\frac{\partial S_w}{\partial T} = \frac{\partial}{\partial X}\left[S_w \frac{\partial S_w}{\partial X}\right] + A\frac{\partial S_w}{\partial X} + BS_w \frac{\partial S_w}{\partial X} + ABS_w \quad (20)$$

where $A = \frac{2l\rho_w g \sin\theta}{\beta}$, $B = \frac{a_2 l}{a_1}$ and $S_w(x,t) = S_w(X,T)$

is the governing nonlinear partial differential equation for the cocurrent imbibition phenomenon in the inclined heterogeneous porous medium and $S_w(X,T)$ is the solution of (20) which represents the saturation of injected water at distance X and time T. The following boundary conditions are used for solving (20):

$$S_w(0,T) = 0 \text{ and } S_w(1,T) = \frac{1+T}{3}. \quad (21)$$

3. SOLUTION BY HOMOTOPY ANALYSIS METHOD

The homotopy analysis method proposed by Liao [19] in his Ph.D. thesis for solving nonlinear differential equation. This is a very powerful and effective method to solve nonlinear problems. Various nonlinear differential equations are solved by homotopy analysis method [5, 8, 9, 19-28].

According to (20), we define a nonlinear operator $\aleph$ as

$$\aleph[\phi(X,T;q)] = \phi(X,T;q)\frac{\partial^2\phi(X,T;q)}{\partial X^2} + \left\{\frac{\partial\phi(X,T;q)}{\partial X}\right\}^2 + A\frac{\partial\phi(X,T;q)}{\partial X} + B\phi(X,T;q)\frac{\partial\phi(X,T;q)}{\partial X} + AB\phi(X,T;q) - \frac{\partial\phi(X,T;q)}{\partial T}. \quad (22)$$

Our boundary conditions (21) suggest us to choose the initial guess as

$$S_{w_0}(X,T)=\frac{TX+X^2}{3} \tag{23}$$

with the auxiliary linear operator

$$L[\phi(X,T;q)]=\frac{\partial^2\phi(X,T;q)}{\partial X^2} \tag{24}$$

which satisfies the property $L[C_1X+C_2]=0$.

Let $q\in[0,1]$ denote the homotopy-parameter, $c_0\neq 0$ the convergence control parameter [29], $H(X,T)\neq 0$ an auxiliary function. Liao [19] constructed the zeroth-order deformation equation

$$(1-q)L[\phi(X,T;q)-S_{w_0}(X,T)]=c_0\,q\,H(X,T)\aleph[\phi(X,T;q)] \tag{25}$$

subject to the boundary conditions $\phi(0,T;q)=0$ and $\phi(1,T;q)=\frac{1+T}{3}$. Note that the initial guess $S_{w_0}(X,T)$ satisfies the boundary conditions. Thus when $q=0$ we have the initial guess

$$\phi(X,T;0)=S_{w_0}(X,T). \tag{26}$$

When $q=1$, the zeroth-order deformation equation (25) provided

$$\phi(X,T;1)=S_w(X,T). \tag{27}$$

Thus as the homotopy-parameter q increases from 0 to 1, $\phi(X,T;q)$ indeed varies continuously from the initial guess $S_{w_0}(X,T)$ to the exact solution $S_w(X,T)$ of the original equation (20). Assume that the auxiliary linear operator L, the initial guess $S_{w_0}(X,T)$, the convergence control parameter c_0 and the auxiliary function $H(X,T)$ are chosen properly that the Maclaurin series of $\phi(X,T;q)$ expanded with respect to q i.e.

$$\phi(X,T;q)=S_{w_0}(X,T)+\sum_{m=1}^{\infty}S_{w_m}(X,T)q^m \tag{28}$$

where

$$S_{w_m}(X,T)=\frac{1}{m!}\left.\frac{\partial^m\phi(X,T;q)}{\partial q^m}\right|_{q=0} \tag{29}$$

converges at $q=1$. Due to (27) and (28), we have the homotopy series solution

$$S_w(X,T)=S_{w_0}(X,T)+\sum_{m=1}^{\infty}S_{w_m}(X,T). \tag{30}$$

Write $\overrightarrow{S_{w_n}} = \{S_{w_0}(X,T), S_{w_1}(X,T), \ldots, S_{w_n}(X,T)\}$. Differentiating the zeroth-order deformation equation (25) m times with respect to the homotopy-parameter q and dividing them by $m!$ and then setting $q=0$, we have the high-order deformation equation

$$L\left[S_{w_m}(X,T) - \chi_m S_{w_{m-1}}(X,T)\right] = c_0\, H(X,T)\Re_m\left(\overrightarrow{S_{w_{m-1}}}\right) \quad (31)$$

subject to boundary conditions $S_{w_m}(0,T)=0$ and $S_{w_m}(1,T)=0$, $m \geq 1$ (32)

where
$$\Re_m\left(\overrightarrow{S_{w_{m-1}}}\right) = \frac{1}{(m-1)!}\frac{\partial^{m-1}\aleph[\phi(X,T;q)]}{\partial q^{m-1}}\bigg|_{q=0}, \quad m \geq 1 \quad (33)$$

and
$$\chi_m = \begin{cases} 0, & if\ m \leq 1, \\ 1, & if\ m > 1. \end{cases} \quad (34)$$

For the sake of simplicity, assume $H(X,T)=1$. Using the initial guess and the auxiliary linear operator, it is easy to solve the linear ordinary differential equations (31)-(34). The special solution of (31) is

$$S^*_{w_m}(X,T) = \chi_m S_{w_{m-1}}(X,T) + c_0\, L^{-1}\left[\Re_m\left(\overrightarrow{S_{w_{m-1}}}\right)\right] \quad (35)$$

where L^{-1} denotes the inverse operator of L. Then the solution of the high-order deformation equation (31) is

$$S_{w_m}(X,T) = S^*_{w_m}(X,T) + C_1 X + C_2 \quad (36)$$

where the coefficients C_1 and C_2 are determined by the boundary conditions (32). Hence the approximate analytical solution of nonlinear partial differential equation (20) takes the following form:

$$S_w(X,T) = \frac{TX + X^2}{3} + c_0\left(\begin{array}{l} -\frac{AX}{9} - \frac{BX}{90} - \frac{ABX}{36} - \frac{TX}{9} - \frac{ATX}{6} - \frac{BTX}{36} - \frac{ABTX}{18} \\ -\frac{T^2X}{18} - \frac{BT^2X}{54} + \frac{ATX^2}{6} + \frac{T^2X^2}{18} - \frac{X^3}{18} + \frac{AX^3}{9} + \frac{TX^3}{9} \\ +\frac{ABTX^3}{18} + \frac{BT^2X^3}{54} + \frac{X^4}{18} + \frac{ABX^4}{36} + \frac{BTX^4}{36} + \frac{BX^5}{90} \end{array}\right) + \cdots. \quad (37)$$

4. RESULTS AND DISCUSSION

The solution contains convergence control parameter c_0 which is most important parameter to obtain convergent homotopy series solution of nonlinear partial differential equation. The c_0 -curve helps us to obtain the proper value of c_0. The line segment almost parallel to the horizontal axis in the c_0 -curve is the valid interval of c_0 [5, 8, 9, 21-23, 26, 28]. The convergent homotopy series solution have discussed using c_0 -curve by different researchers as Patel and Desai [5, 8, 9, 28], Darvishi and Khani [21], Ghotbi at al. [22], Abbasbandy at al. [23], Liao [24], Fariborzi and Naghshband [26]. The BVPh 1.1, a Mathematica package [24] is used to plot the c_0 -curves. The value of following constants are assumed as $l = 1, \rho_w = 0.1, g = 9.8, \beta = 2$.

4.1 The c_0 -curves of $S_{w_{XX}}(0,0)$

The homotopy analysis solution is obtained for cocurrent imbibition phenomenon in inclined heterogeneous porous medium and its convergence depends on c_0 which is chosen from c_0 -curve. Fig. 1 - 3 show the c_0 -curve of $S_{w_{XX}}(0,0)$ for 30th order approximation for angle of inclination $\theta = 0^\circ, \theta = 5^\circ$ and $\theta = 10^\circ$ respectively. The proper value of $c_0 = -0.05$ chosen for convergent homotopy analysis solution of cocurrent imbibition phenomenon in inclined $(\theta = 0^\circ, \theta = 5^\circ, \theta = 10^\circ)$ heterogeneous porous medium.

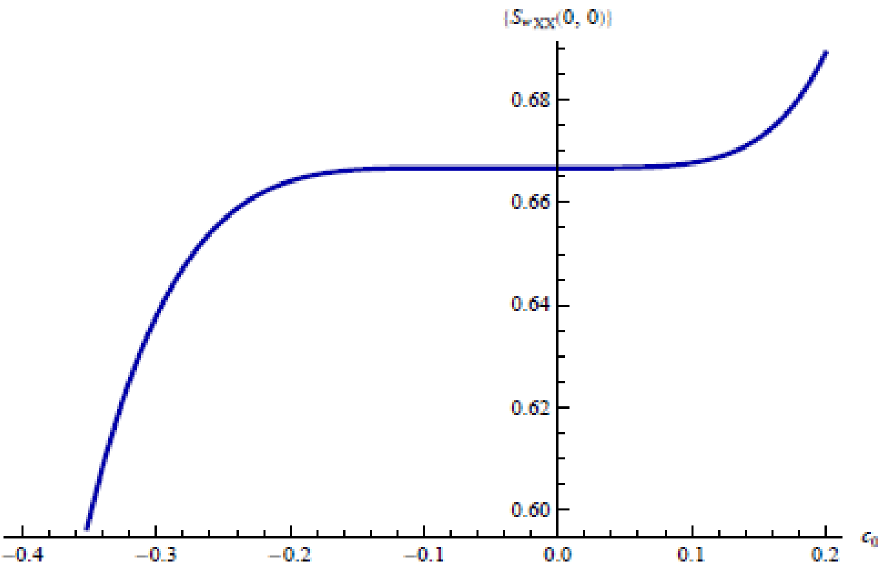

Figure 1: The c_0 -curve of $S_{w_{XX}}(0,0)$ for $\theta = 0^\circ$.

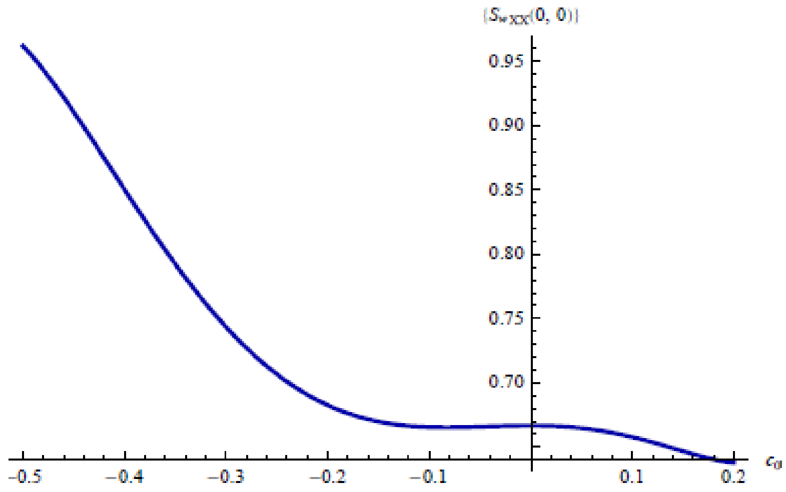

Figure 2: The c_0 -curve of $S_{w_{XX}}(0,0)$ for $\theta = 5^{\circ}$.

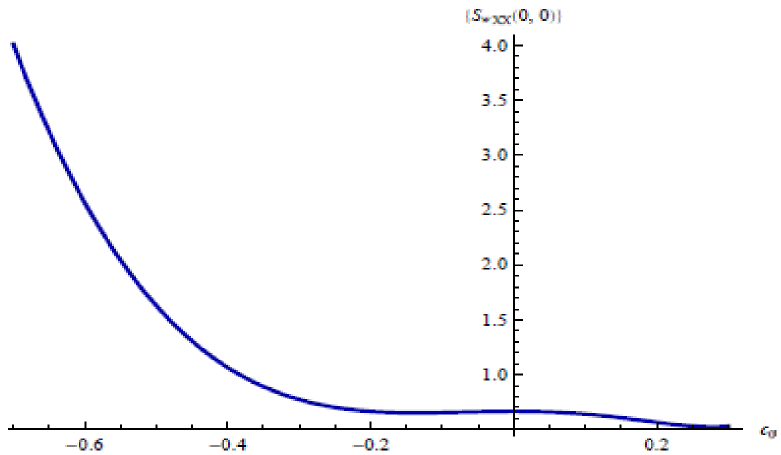

Figure 3: The c_0 -curve of $S_{w_{XX}}(0,0)$ for $\theta = 10^{\circ}$.

4.2 Numerical interpretation of solution

Table 1-3 indicate the numerical values of saturation of injected water for cocurrent imbibition phenomenon in heterogeneous porous medium with angle of inclination $\theta = 0^{\circ}, \theta = 5^{\circ}$ and $\theta = 10^{\circ}$ respectively.

Table 1: Numerical values of the saturation of injected water for $\theta = 0^\circ$.

T	$X=0.1$	$X=0.2$	$X=0.3$	$X=0.4$	$X=0.5$	$X=0.6$	$X=0.7$	$X=0.8$	$X=0.9$	$X=1$
0.1	0.0079782	0.0229830	0.0451758	0.0744627	0.1105000	0.1527179	0.2003618	0.2525440	0.3083037	0.3666667
0.2	0.0133488	0.0335191	0.0605489	0.0942292	0.1341183	0.1795746	0.2298015	0.2839035	0.3409435	0.4000000
0.3	0.0188933	0.0443428	0.0762660	0.1143454	0.1580524	0.2066856	0.2594209	0.3153679	0.3736254	0.4333333
0.4	0.0246107	0.0554496	0.0923181	0.1347989	0.1822872	0.2340357	0.2892065	0.3469273	0.4063442	0.4666667
0.5	0.0304996	0.0668349	0.1086968	0.1555773	0.2068084	0.2616102	0.3191453	0.3785725	0.4390952	0.5000000
0.6	0.0365589	0.0784942	0.1253933	0.1766686	0.2316017	0.2893949	0.3492251	0.4102946	0.4718739	0.5333333
0.7	0.0427874	0.0904228	0.1423990	0.1980608	0.2566535	0.3173762	0.3794342	0.4420854	0.5046763	0.5666667
0.8	0.0491838	0.1026161	0.1597055	0.2197422	0.2819502	0.3455407	0.4097613	0.4739368	0.5374984	0.6000000
0.9	0.0557468	0.1150696	0.1773042	0.2417012	0.3074788	0.3738759	0.4401959	0.5058417	0.5703367	0.6333333
1.0	0.0624751	0.1277785	0.1951868	0.2639264	0.3332267	0.4023695	0.4707278	0.5377931	0.6031880	0.6666667

Table 2: Numerical values of the saturation of injected water for $\theta = 5^\circ$.

T	$X=0.1$	$X=0.2$	$X=0.3$	$X=0.4$	$X=0.5$	$X=0.6$	$X=0.7$	$X=0.8$	$X=0.9$	$X=1$
0.1	0.0097402	0.0263320	0.0498361	0.0800617	0.1165793	0.1587506	0.2057738	0.2567388	0.3106864	0.3666667
0.2	0.0153699	0.0373119	0.0657641	0.1004253	0.1407763	0.1861178	0.2356193	0.2883755	0.3434644	0.4000000
0.3	0.0211738	0.0485766	0.0820285	0.1211262	0.1652728	0.2137213	0.2656271	0.3201035	0.3762768	0.4333333
0.4	0.0271506	0.0601216	0.0986205	0.1421519	0.1900538	0.2415462	0.2957843	0.3519133	0.4091187	0.4666667
0.5	0.0332993	0.0719420	0.1155312	0.1634900	0.2151052	0.2695781	0.3260784	0.3837961	0.4419858	0.5000000
0.6	0.0396183	0.0840332	0.1327518	0.1851284	0.2404128	0.2978030	0.3564976	0.4157435	0.4748738	0.5333333
0.7	0.0461065	0.0963905	0.1502736	0.2070551	0.2659630	0.3262076	0.3870306	0.4477477	0.5077790	0.5666667
0.8	0.0527625	0.1090090	0.1680879	0.2292583	0.2917427	0.3547791	0.4176667	0.4798013	0.5406978	0.6000000
0.9	0.0595849	0.1218839	0.1861862	0.2517265	0.3177388	0.3835051	0.4483957	0.5118973	0.5736268	0.6333333
1.0	0.0665724	0.1350106	0.2045600	0.2744484	0.3439388	0.4123736	0.4792079	0.5440292	0.6065632	0.6666667

Table 3: Numerical values of the saturation of injected water for $\theta = 10^\circ$.

T	$X=0.1$	$X=0.2$	$X=0.3$	$X=0.4$	$X=0.5$	$X=0.6$	$X=0.7$	$X=0.8$	$X=0.9$	$X=1$
0.1	0.0115398	0.0297336	0.0545446	0.0856902	0.1226614	0.1647585	0.2111404	0.2608814	0.3130304	0.3666667
0.2	0.0174309	0.0411593	0.0710275	0.1066482	0.1474320	0.1926297	0.2413846	0.2927896	0.3459433	0.4000000
0.3	0.0234964	0.0528669	0.0878390	0.1279308	0.1724855	0.2207190	0.2717739	0.3247755	0.3788830	0.4333333
0.4	0.0297350	0.0648515	0.1049701	0.1495254	0.1978072	0.2490118	0.3022959	0.3568301	0.4118449	0.4666667
0.5	0.0361453	0.0771083	0.1224118	0.1714196	0.2233830	0.2774940	0.3329385	0.3889451	0.4448250	0.5000000
0.6	0.0427259	0.0896324	0.1401552	0.1936011	0.2491988	0.3061521	0.3636904	0.4211126	0.4778194	0.5333333
0.7	0.0494755	0.1024189	0.1581913	0.2160580	0.2752414	0.3349730	0.3945407	0.4533251	0.5108246	0.5666667
0.8	0.0563926	0.1154630	0.1765116	0.2387786	0.3014975	0.3639443	0.4254792	0.4855757	0.5438373	0.6000000
0.9	0.0634758	0.1287596	0.1951072	0.2617513	0.3279545	0.3930539	0.4564960	0.5178580	0.5768546	0.6333333
1.0	0.0707237	0.1423040	0.2139694	0.2849648	0.3546000	0.4222902	0.4875820	0.5501657	0.6098736	0.6666667

4.3 Graphical interpretation of solution

The graph of saturation of injected water v/s distance X for fixed time $T = 0.1, 0.2, \ldots, 1$ for $\theta = 0^\circ$ is given in Fig. 4. Fig. 5 shows the graph of saturation of injected water v/s distance X for fixed time $T = 0.1, 0.2, \ldots, 1$ for $\theta = 5^\circ$. The graph of saturation of injected water v/s distance X for fixed time $T = 0.1, 0.2, \ldots, 1$ for $\theta = 10^\circ$ is shown in Fig. 6.

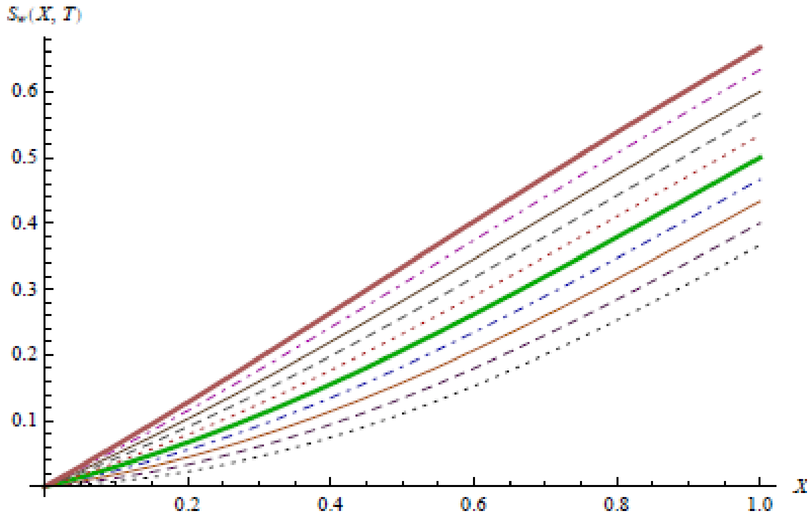

Figure 4: Saturation of water v/s distance for fixed time $T = 0.1, 0.2, \ldots, 1$ for $\theta = 0^\circ$.

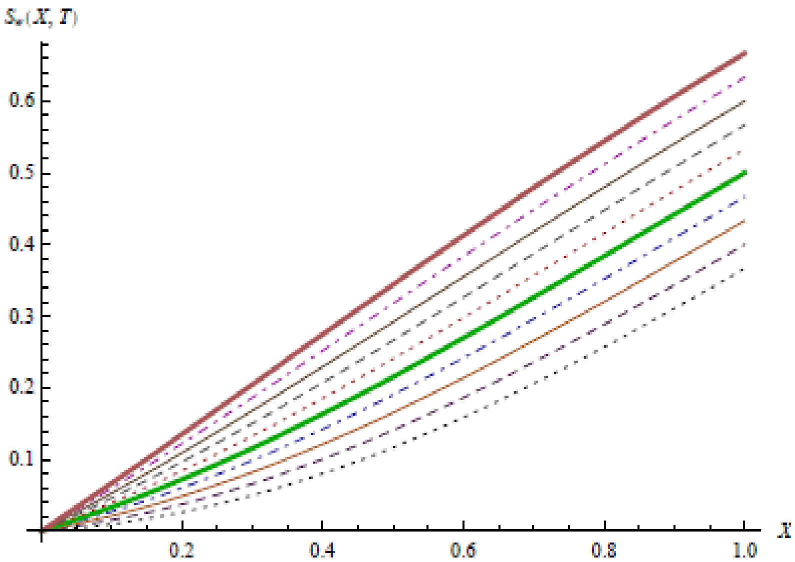

Figure 5: Saturation of water v/s distance for fixed time $T = 0.1, 0.2, \ldots, 1$ for $\theta = 5^\circ$.

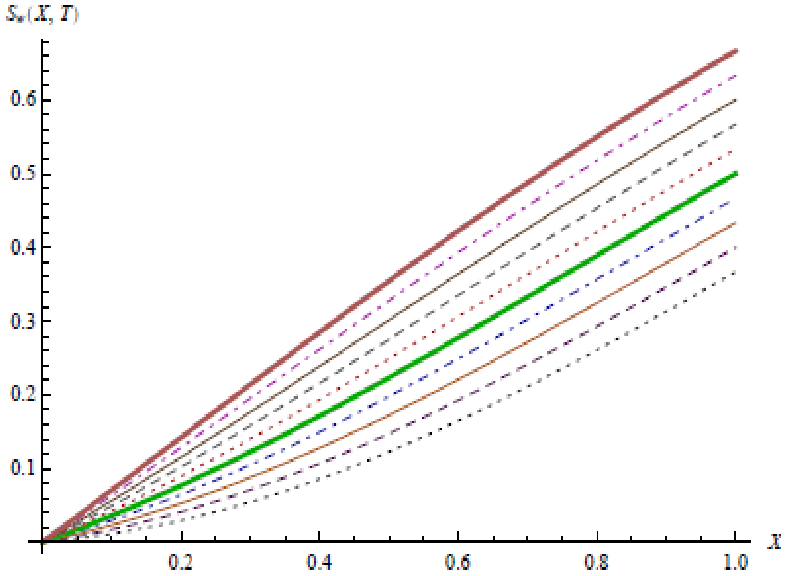

Figure 6: Saturation of water v/s distance for fixed time $T = 0.1, 0.2, \ldots, 1$ for $\theta = 10^\circ$.

5. CONCLUSIONS

The mathematical model of cocurrent imbibition phenomenon in inclined heterogeneous porous medium is developed. The nonlinear partial differential equation is derived for cocurrent imbibition. Homotopy analysis solution is obtained for cocurrent imbibition phenomenon with appropriate boundary conditions. The solution satisfies both the boundary conditions. We have discussed the numerical interpretation and graphical interpretation of solution for different angle of inclination. As angle of inclination with porous matrix increases, the saturation of injected water increases. We conclude that the saturation of injected water increases when the distance increases for given time T .

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