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On Fuzzy ρ-Door Spaces

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Abstract

The aim of this paper is to introduce and study the concept of fuzzy ρ-door spaces as well as some relations between different well-known fuzzy topological spaces and fuzzy ρ-door spaces are investigated. Also, some interesting properties of them are established.

Keywords: Fuzzy ρ-door space, fuzzy ρ-submaximal space, fuzzy ρ-irreducible space, fuzzy ρ-irresolvable space.

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1. INTRODUCTION

In order to deal with uncertainties, the idea of fuzzy sets, fuzzy set operations was introduced by Zadeh [1] in his classical paper in the year 1965, describing fuzziness mathematically for the first time. Thereafter Chang [2] in 1968 paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. The concept of ρ-closed set was introduced by Devamanoharan et al [3]. The concept of fuzzy door spaces was introduced and studied by Anjalmose and Thangaraj [4]. The aim of this paper is to introduce and study the concept of fuzzy ρ-door spaces as well as some relations between different well-known fuzzy topological spaces and fuzzy ρ-door spaces are investigated. Also, some interesting properties of them are established.

2. PRELIMINARIES

Definition 2.1. [1] Let X be a non-empty set and I be the unit interval [0, 1]. A fuzzy set in X is an element of the set I^X of all functions from X to I.

Definition 2.2. [2] A fuzzy topology is a family τ of fuzzy sets in X which satisfies the following conditions :

(i) 0_X , $1_X \in \tau$,

(ii) if $\lambda, \mu \in \tau$, then $\lambda \wedge \mu \in \tau$,

(iii) if $\lambda_i \in \tau$ for each $i \in \mathcal{J}$ then $\vee \lambda_i \in \tau$.

τ is called a fuzzy topology on X and the ordered pair (X, τ) is called a fuzzy topological space (in short, FTS). Every member of τ is called a fuzzy open set. The complement of a fuzzy open set is called a fuzzy closed set. As (ordinary) topologies, the indiscrete fuzzy topology contains only 0_X and 1_X, while the discrete fuzzy topology contains all fuzzy sets.

Definition 2.3. [1] Let A be the subset of X. A characteristic function of A, χ_A : $X \to [0, 1]$ is defined as $\chi_A(x) = \begin{cases} 1, & \text{if } x \in A \\ 0, & \text{otherwise} \end{cases}$

Definition 2.4. [5] Let (X, τ) be a fuzzy topological space and Y be an ordinary subset of X. Then $\tau_Y = \{ \lambda/Y \mid \lambda \in \tau \}$ is a fuzzy topology on Y and is called the induced or relative fuzzy topology. The pair (Y, τ_Y) is called a fuzzy subspace of (X, τ) : (Y, τ_Y) is called a fuzzy open / fuzzy closed / fuzzy β open / fuzzy subspace if the characteristic function of Y viz. χ_Y is fuzzy open / fuzzy closed / fuzzy β open respectively.

Definition 2.5. [6] A fuzzy set λ in a fuzzy topological space (X, τ) is called fuzzy dense if there exists no fuzzy closed set μ in (X, τ) such that $\lambda < \mu < 1_X$. That is $Cl(\lambda) = 1_X$ in (X, τ).

Definition 2.6. [6] A fuzzy set λ in a fuzzy topological space (X, τ) is called fuzzy nowhere dense if there exists no non-zero fuzzy open set μ in (X, τ) such that $\mu < Cl(\lambda)$. That is $Int(Cl(\lambda)) = 0_X$ in (X, τ).

Definition 2.7. [7] A fuzzy topological space (X, τ) is called a fuzzy resolvable space if there exists a fuzzy dense set λ in (X, τ) such that $Cl(1_X - \lambda) = 1_X$. Otherwise, (X, τ) is called a fuzzy irresolvable space.

Definition 2.8. [8] A fuzzy topological space (X, τ) is called a fuzzy submaximal space if $Cl(\lambda) = 1_X$, for any non-zero fuzzy set λ in (X, τ), then $\lambda \in \tau$.

Definition 2.9. [9] A fuzzy topological space $(X,\ \tau)$ is called a fuzzy quasi-maximal space if for every fuzzy dense set λ in $(X,\ \tau)$ with $Int(\lambda)\ \neq 0_X$, $Int(\lambda)$ is also fuzzy dense in $(X,\ \tau)$.

Definition 2.10. [10] A fuzzy topological space $(X,\ \tau)$ is called a fuzzy nodec space if every non-zero fuzzy nowhere dense set λ is fuzzy closed in $(X,\ \tau)$. That is, if λ is a fuzzy nowhere dense set in $(X,\ \tau)$, then $1_X - \lambda\ \in\ \tau$.

Definition 2.11 [4] A fuzzy topological space $(X,\ \tau)$ is said to be a fuzzy door space if every fuzzy subset of X is either fuzzy open or fuzzy closed.

Definition 2.12. [11] Let λ be a fuzzy set in a fuzzy topological space X. Then the fuzzy boundary of λ is defined as $Bd(\lambda) = Cl(\lambda) \wedge Cl(\lambda')$. Obviously, $Bd(\lambda)$ is a fuzzy closed set.

Definition 2.13. [12] Let $\{\ (LF(X_t),\ \ \delta_t)\ \}_{t\in T}$ be a family of pairwise disjoint L-fts's, i.e., $X_{t_1} \cap X_{t_2} = \emptyset$ for $t_1 \neq t_2$. Consider the set $X = \ \bigcup_{t\in T} X_t$. $\forall\ t\ \in T,\ j_t$: $X_t\ \rightarrow X$ is the usual inclusion mapping (ie., $\forall\ x\ \in X_t,\ j_t(x) = x$), it naturally induces an L-fuzzy mapping $j_t : LF(X_t) \rightarrow LF(X)$. Then the final L- fuzzy topology $T_t(\delta_t,\ j_t,\ T)$ on LF(X) for $\{\delta_t\}_{t\in T}$ and is denoted by $\sum_{t\in T}\delta_t$. L-fts $(LF(X),\ \Sigma_{t\in T})$ is called the L-fuzzy sum topological (L-ftss, for short) of $\{(LF\ (X_t,\ \delta_t))\}_{t\in T}$, and written as $\sum_{t\in T}(LF\ (X_t),\ \delta_t)$, briefly $\Sigma(LF\ (X_t),\ \delta_t)$.

3. ON FUZZY ρ-DOOR SPACES

Notation 3.1. Let $(X,\ \tau)$ be a fuzzy topological space. Let $\lambda\ \in I^X$ be any fuzzy set in $(X,\ \tau)$. Then, the complement of λ is $\lambda' = 1_X - \lambda$.

Notation 3.2. Let $(X,\ \tau)$ be a fuzzy topological space. Then the fuzzy interior, fuzzy closure and fuzzy pre-closure are denoted by $FInt$, FCl and $FPCl$ respectively.

Definition 3.1. Let $(X,\ \tau)$ be a FTS. Any fuzzy set $\lambda\ \in I^X$ is said to be a fuzzy ρ-closed set (briefly, $F\rho\text{-}Cs$) in $(X,\ \tau)$ if $FPCl(\lambda) \leq FInt(\mu)$ whenever $\lambda\ \leq\ \mu$ and $\mu \in I^X$ is a fuzzy $\tilde{g}$-open set. The complement of a fuzzy ρ-closed set is said to be a fuzzy ρ-open set.

Note 3.1. Let $(X,\ \tau)$ be a FTS. Then 0_X and 1_X is both fuzzy ρ-open and fuzzy ρ-closed.

Notation 3.3. Let $(X,\ \tau)$ be a FTS. The set of all fuzzy ρ-open set in $(X,\ \tau)$ is denoted by $F\rho O(X,\ \tau)$ and the set of all fuzzy ρ-closed set in $(X,\ \tau)$ is denoted by $F\rho C(X,\ \tau)$.

Definition 3.2. Let $(X,\ \tau)$ be a FTS. Then for any $\lambda \in I^X$, the fuzzy ρ-interior of λ is denoted and defined as

$$F\rho Int(\lambda) = \ \vee \{\mu \in I^X : \mu \leq \lambda \text{ and } \mu \text{ is fuzzy } \rho - \text{open}\}.$$

Definition 3.3. Let $(X,\ \tau)$ be a FTS. Then for any $\lambda \in I^X$, the fuzzy ρ-interior of λ is denoted and defined as

$$F\rho Cl(\lambda) = \ \wedge \{\mu \in I^X : \lambda \leq \mu \text{ and } \mu \text{ is fuzzy } \rho - \text{closed}\}.$$

Proposition 3.4. Let $(X,\ \tau)$ be a FTS and $\lambda \in I^X$ be any fuzzy set in $(X,\ \tau)$. Then the following conditions hold :

(i) $1_X - F\rho Cl(\lambda) = F\rho Int(1_X - \lambda)$.
(ii) $1_X - F\rho Int(\lambda) = F\rho Cl(1_X - \lambda)$.

Definition 3.5. Let $(X,\ \tau)$ be a FTS. Then $(X,\ \tau)$ is said to be a fuzzy ρ-door space if every fuzzy set $\lambda \in I^X$ is either fuzzy ρ-open or fuzzy ρ-closed in $(X,\ \tau)$.

Example 3.1. Let $X = \{a, b\}$ and the fuzzy set $\lambda \in I^X$ be defined as : $\lambda(a) = 0.7$, $\lambda(b) = 0.8$. Then, $\tau = \{0_X,\ 1_X,\ \lambda\}$ is a fuzzy topology on X and hence the ordered pair $(X,\ \tau)$ is a fuzzy topological space. Then any fuzzy set $\mu \in I^X$ is either fuzzy ρ-open or fuzzy ρ-closed in $(X,\ \tau)$. Therefore, $(X,\ \tau)$ is a fuzzy ρ-door space.

Example 3.2. Let $X = \{a, b\}$ and the fuzzy set $\lambda \in I^X$ be defined as : $\lambda(a) = 0.3$, $\lambda(b) = 0.4$. Then, $\tau = \{0_X,\ 1_X,\ \lambda\}$ is a fuzzy topology on X and hence the ordered pair $(X,\ \tau)$ is a fuzzy topological space. Let the fuzzy set $\mu \in I^X$ be defined as follows : $\mu(a) = 0.6$, $\mu(b) = 0.6$. Then μ is neither fuzzy ρ-open nor fuzzy ρ-closed in $(X,\ \tau)$. Therefore, $(X,\ \tau)$ is not a fuzzy ρ-door space.

Definition 3.6. Let $(X,\ \tau)$ be a FTS. A fuzzy set $\lambda \in I^X$ in $(X,\ \tau)$ is called a fuzzy ρ-dense set if there exists no fuzzy ρ-closed set $\mu \in I^X$ in $(X,\ \tau)$ such that $\lambda < \mu < 1_X$ (ie., $F\rho Cl(\lambda) = 1_X$).

Example 3.3. In Example 3.1, let the fuzzy set $\mu \in I^X$ be defined as follows : $\mu(a) = 0.75$, $\mu(b) = 0.85$. Then $F\rho Cl(\mu) = 1_X$. Therefore, μ is a fuzzy ρ-dense set in $(X,\ \tau)$.

Notation 3.3. Let $(X,\ \tau)$ be a FTS. The set of all fuzzy ρ-dense set in $(X,\ \tau)$ is denoted by $F\mathcal{D}_\rho$.

Definition 3.7. Let $(X,\ \tau)$ be a FTS. A fuzzy set $\lambda\ \in I^X$ in $(X,\ \tau)$ is called a fuzzy ρ-codense set if there exists no fuzzy ρ-open set $\mu \in I^X$ in $(X,\ \tau)$ such that $\lambda > \mu > 0_X$ (ie., $F\rho Int(\lambda) = 0_X$).

Example 3.4. In Example 3.2, let the fuzzy set $\mu \in I^X$ be defined as follows : $\mu(a) = 0.3,\ \ \mu(b) = 0.4$. Then $F\rho Int(\mu) = 0_X$. Therefore, μ is a fuzzy ρ-codense set in $(X,\ \tau)$.

Definition 3.8. Let $(X,\ \tau)$ be a FTS. A fuzzy set $\lambda\ \in I^X$ in $(X,\ \tau)$ is called a fuzzy ρ-nowhere dense set if there exists no fuzzy ρ-open set $0_X \neq \mu \in I^X$ in $(X,\ \tau)$ such that $\mu < F\rho Cl(\lambda)$. That is $F\rho Int(F\rho Cl(\lambda)) = 0_X$ in $(X,\ \tau)$.

Example 3.5. In Example 3.2, let the fuzzy set $\mu \in I^X$ be defined as follows : $\mu(a) = 0.3,\ \ \mu(b) = 0.29$. Then $F\rho Int(F\rho Cl(\mu)) = 0_X$. Therefore, μ is a fuzzy ρ-nowhere dense set in $(X,\ \tau)$.

Definition 3.9. Let $(X,\ \tau)$ be a FTS. Then $(X,\ \tau\ )$ is said to be a fuzzy ρ-submaximal space if and only if every fuzzy ρ-dense set $\lambda\ \in I^X$ is fuzzy ρ-open in $(X,\ \tau)$.

Example 3.6. In Example 3.1, every fuzzy ρ-dense set $\mu\ \in I^X$ is fuzzy ρ-open in $(X,\ \tau)$. Hence $(X,\ \tau)$ is a fuzzy ρ-submaximal space.

Proposition 3.10. Let $(X,\ \tau)$ be a FTS. Then the following statements are equivalent :

(i) $(X,\ \tau)$ is a fuzzy ρ-submaximal space.
(ii) $F\rho Cl(\lambda) \wedge \lambda'$ is fuzzy ρ-closed for each $\lambda \in I^X$.
(iii) For each $\lambda \in I^X$, if $F\rho Int(\lambda) = 0_X$, then λ is fuzzy ρ-closed.
(iv) Every fuzzy ρ-codense set $\lambda \in I^X$ is fuzzy ρ-closed.
(v) Every fuzzy ρ-dense set $\lambda \in I^X$ is fuzzy ρ-open.

Proposition 3.11. Every fuzzy ρ-door space $(X,\ \tau)$ is fuzzy ρ-submaximal.

Definition 3.12. Let $(X,\ \tau)$ be a FTS. Then $(X,\ \tau\ )$ is said to be a fuzzy ρ-irreducible space if every fuzzy ρ-open set $0_X \neq \lambda\ \in I^X$ is a fuzzy ρ-dense set in $(X,\ \tau)$.

Example 3.7. Let $X = \{a, b\}$ and the fuzzy set $\lambda\ \in I^X$ be defined as : $\lambda\,(a) = 0.4$, $\lambda\,(b) = 0.5$. Then, $\tau = \{\ 0_X,\ 1_X,\ \lambda\ \}$ is a fuzzy topology on X and hence the ordered pair $(X,\ \tau)$ is a fuzzy topological space. Then any fuzzy ρ-open set $0_X \neq \mu \in I^X$ is a fuzzy ρ-dense set in $(X,\ \tau)$. Therefore, $(X,\ \tau\ )$ is a fuzzy ρ-irreducible space.

Definition 3.13. Let $(X,\ \tau)$ be a FTS. Then $(X,\ \tau)$ is said to be a fuzzy ρ-irreducible submaximal space if it is both fuzzy ρ-irreducible and fuzzy ρ-submaximal in $(X,\ \tau)$.

Proposition 3.14. Every fuzzy ρ-irreducible submaximal space is a fuzzy ρ-door space.

Definition 3.15. Let $(X,\ \tau)$ be a fuzzy topological space and Y be an ordinary subset of X. Then $\tau_Y = \{\ \lambda/Y\ = \lambda \wedge \chi_Y\ \ |\ \lambda\ \in \tau\ \}$ is a fuzzy topology on Y and is called the induced or relative fuzzy topology. The pair $(Y,\ \tau_Y)$ is called a fuzzy subspace of $(X,\ \tau)$: $(Y,\ \tau_Y)$ is called a fuzzy ρ-open subspace if the characteristic function of Y, χ_Y is fuzzy ρ-open.

Proposition 3.16. Let $(X,\ \tau)$ be a FTS and $(Y,\ \tau_Y)$ be a fuzzy ρ-open (ρ-closed) subspace of $(X,\ \tau)$. If $(X,\ \tau)$ is a fuzzy ρ-door space, then (Y, τ_Y) is a fuzzy ρ-door space.

Proposition 3.17. Let $(X_i,\ \tau_i)_{i\,\in\,\mathcal{J}}$ where $\mathcal{J}$ is an indexed set, be a family of fuzzy topological spaces and the fuzzy topological sum $(X,\ \tau) = \sum_{i\,\in\,\mathcal{J}}\ (X_i,\ \tau_i)$ the following conditions are equivalent :

(i) $(X,\ \tau)$ is a fuzzy ρ-door space.

(ii) Each $(X_i,\ \tau_i)$ is a fuzzy ρ-door space and $(X_i,\ \tau_i)$ is an indiscrete fuzzy topological space for at most one index.

Definition 3.18. Let $(X,\ \tau)$ be a FTS. Then $(X,\ \tau)$ is said to be a fuzzy ρ-resolvable space if there exists a fuzzy ρ-dense set $0_X \neq \lambda\ \in I^X$ such that $F\rho Int(\lambda) = 0_X$ ($F\rho Cl(1_X - \lambda) = 1_X$). Otherwise $(X,\ \tau)$ is called a fuzzy ρ-irresolvable space.

Example 3.8. In Example 3.7, let the fuzzy set $\mu \in I^X$ be defined as follows : $\mu(a) = 0.5$, $\mu(b) = 0.5$. Then $F\rho Cl(\mu) = 1_X$. Therefore, μ is a fuzzy ρ-dense set in $(X,\ \tau)$ and $F\rho Int(\mu) = 0_X$. Hence $(X,\ \tau)$ is a fuzzy ρ-resolvable space.

Example 3.9. In Example 3.1, for any fuzzy ρ-dense set $\mu \in I^X$, $F\rho Int(\mu) \neq 0_X$. Hence $(X,\ \tau)$ is fuzzy ρ-irresolvable space.

Proposition 3.19. Let $(X,\ \tau)$ be a FTS. Then $(X,\ \tau)$ is said to be a fuzzy ρ-resolvable space if and only if $F\rho Cl(\mu') \neq 1_X$, for each fuzzy ρ-dense set $\mu\ \in I^X$.

Definition 3.20. Let $(X,\ \tau)$ be a FTS. Then $(X,\ \tau)$ is said to be a fuzzy ρ-irreducible door space if it is both fuzzy ρ-irreducible and fuzzy ρ-door space in $(X,\ \tau)$.

Definition 3.21. Let $(X,\ \tau)$ and $(Y,\ \sigma)$ be any two $FTSs$. A fuzzy function $f:(X,\ \tau)\to(Y,\ \sigma)$ is said to be a fuzzy ρ-quasi compact function if $\lambda\in I^X$ is fuzzy ρ-open such that $f^{-1}\big(f(\lambda)\big)=\lambda$, then $f(\lambda)$ is fuzzy ρ-open in $(Y,\ \sigma)$.

Proposition 3.22. Let $(X,\ \tau)$ and $(Y,\ \sigma)$ be any two $FTSs$. If the function $f:(X,\ \tau)\to(Y,\ \sigma)$ is bijective fuzzy ρ-quasi compact function and $(X,\ \tau)$is a fuzzy ρ-door space, then $(Y,\ \sigma)$ is a fuzzy ρ-door space.

Proposition 3.23. Let $(X,\ \tau)$ and $(Y,\ \sigma)$ be any two $FTSs$. If the function $f:(X,\ \tau)\to(Y,\ \sigma)$ is a fuzzy ρ-open function, fuzzy ρ-closed surjective function and if $(X,\ \tau)$ is a fuzzy ρ-door space, then $(Y,\ \sigma)$ is a fuzzy ρ-door space.

Proposition 3.24. Let $(X,\ \tau)$ be a FTS. Then the following statements are equivalent :

(i) $(X,\ \tau)$ is a fuzzy ρ-irreducible submaximal space.
(ii) $F\rho O(X,\ \tau)=F\mathcal{D}_\rho\cup\{\emptyset\}$.
(iii) $(X,\ \tau)$ is a fuzzy ρ-irreducible door space.

Definition 3.25. Let $(X,\ \tau)$ be a FTS. Then $(X,\ \tau)$ is said to be a fuzzy ρ-quasi – maximal space if for every fuzzy ρ-dense set $\lambda\in I^X$ in $(X,\ \tau)$ with $F\rho Int(\lambda)\neq 0_X$, $F\rho Int(\lambda)$ is also fuzzy ρ-dense in $(X,\ \tau)$.

Definition 3.26. Let $(X,\ \tau)$ be a FTS. Then $(X,\ \tau)$ is said to be a fuzzy quasi – submaximal space if for every fuzzy dense set $\lambda\in I^X$, $FBd(\lambda)$ is fuzzy nowhere dense in $(X,\ \tau)$.

Definition 3.27. Let $(X,\ \tau)$ be a FTS and $\lambda\in I^X$ be any fuzzy set in $(X,\ \tau)$. Then the fuzzy ρ-boundary of λ is defined as $FBd_\rho(\lambda)=F\rho Cl(\lambda)\wedge F\rho Cl(\lambda')$.

Definition 3.28. Let $(X,\ \tau)$ be a FTS. Then $(X,\ \tau)$ is said to be a fuzzy ρ-quasi – submaximal space if for every fuzzy ρ-dense set $\lambda\in I^X$, $FBd_\rho(\lambda)$ is fuzzy ρ-nowhere dense in $(X,\ \tau)$.

Example 3.10. In Example 3.1, every fuzzy ρ-dense set $\mu\in I^X$, $FBd_\rho(\lambda)$ is fuzzy ρ-nowhere dense in $(X,\ \tau)$. Hence $(X,\ \tau)$ is a fuzzy ρ-quasi-submaximal space.

Definition 3.29. A fuzzy topological space $(X,\ \tau)$ is called a fuzzy ρ-nodec space if every fuzzy ρ-nowhere dense set $0_X\neq\lambda\in I^X$ is fuzzy ρ-closed in $(X,\ \tau)$. That is, if λ is a fuzzy ρ-nowhere dense set in $(X,\ \tau)$, then $1_X-\lambda\in F\rho O(X,\ \tau)$.

Example 3.10. In Example 3.1, every fuzzy ρ-nowhere dense set $0_X \neq \mu \in I^X$ is fuzzy ρ-closed in $(X,\ \tau)$. Hence $(X,\ \tau)$ is a fuzzy ρ-nodec space.

Proposition 3.30. Every fuzzy ρ-door space $(X,\ \tau)$ is fuzzy ρ-nodec.

Proposition 3.31. Let $(X,\ \tau)$ be a FTS. If $(X,\ \tau)$ is a fuzzy ρ-quasi–submaximal space, then $(X,\ \tau)$ is a fuzzy ρ-quasi-maximal space.

Proposition 3.32. Let $(X,\ \tau)$ be a FTS. If $(X,\ \tau)$ is a fuzzy ρ-quasi–submaximal and fuzzy ρ-nodec space, then $(X,\ \tau)$ is a fuzzy ρ-submaximal space.

Proposition 3.33. Let $(X,\ \tau)$ be a FTS. Then the following statements are equivalent :

(i) $(X,\ \tau)$ is a fuzzy ρ-door space.
(ii) $(X,\ \tau)$ is a fuzzy ρ-quasi submaximal and fuzzy ρ-nodec space.
(iii) $(X,\ \tau)$ is a fuzzy ρ-quasi maximal and fuzzy ρ-nodec space.
(iv) $(X,\ \tau)$ is a fuzzy ρ-irreducible submaximal space.

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