A Note on Changing and Unchanging of Domination number of a zero-Divisor graph

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Abstract

In this paper we evaluated the changing and unchanging of domination number of a Zero-divisor graph when a graph modified by deleting a vertex (or) deleting (or) adding an edge. Let R be a commutative ring and let $\Gamma(Z_n)$ be the Zero-divisor graph of a commutative ring R, whose vertices are non-zero Zero-divisors of Z_n, such that two vertices x and y are adjacent if n divides xy. For any graph $\Gamma(Z_{2p})$, where p is any odd prime number, then there exists a non-pendant vertex $v \in V$ such that $\gamma\ (\Gamma(Z_{2p})-v) > \gamma\ (\Gamma(Z_{2p}))$, the deletion of vertex v from a graph G results in a graph (G-v) such that $\gamma\ (G-v) \geq \gamma\ (G)$.

Keywords: Commutative ring, Zero-divisor graph, Domination number, changing and unchanging domination number of Zero-divisor graph.

1. INTRODUCTION:

 All graphs are considered in this paper are finite, undirected, Simple and connected.we followed the Hedetniemi et al terminologies.The domination Concept is applied on a commutative ring and with this basic concepts, we discussed about the changing and unchanging of domination number of a Zero-divisor graph.

Let R be a commutative ring and let Z(R) be its set of Zero-divisors. We relate a graph $\Gamma(R)$ to R with vertices $Z(R)^* = Z(R)-\{0\}$, the set of non-zero zero divisors of

R. Throughout this paper, we consider the commutative ring R by Z_n where Z_n is a non-zero Zero-divisor and Zero-divisor graph $\Gamma(R)$ is denoted by $\Gamma(Z_n)$.

Definition 1.1:

If R is a commutative ring, then a $\neq 0$ in R is said to be a Zero-divisor if there exists an element b in R with b$\neq$0 such that a . b = 0.

Definition 1.2:

Let R be a commutative ring and let $\Gamma(Z_n)$ be the Zero-divisor graph of a commutative ring R, whose vertices are non-zero Zero-divisors of Z_n, such that two vertices x and y are adjacent if n divides xy.

The idea of a Zero-divisor graph of a commutative ring was introduced by I. Beek in (3), where he mainly focused on colourings.

In general, we followed [1,8,9,11,12] for graph theory terminology and the notations for commutative ring from [10]. We checked the effects on domination number when $\Gamma(Z_n)$ is modified by deleting a vertex (or) deleting (or) adding an edge. Harary and Haynes [5] surveyed the problem of characterizing the graph G in the following cases.

(i) $\gamma\ (\Gamma(Z_{2p})-v)\ >\gamma\ (\Gamma(Z_{2p}))$

(ii) $\gamma\ (G-v) >\gamma\ (G)$

(iii) $\gamma\ (G-v) = \gamma\ (G)$ when G is a zero divisor graph

- ❖ d(u,v) = Length of the shortest path between the vertices u and v.
- ❖ Eccentricity : The eccentricity e(v) of a vertex v in a connected graph G is maximum of d (u,v) for all u in G.
- ❖ Girth: The girth of a graph G is the length of the shortest cycle in G.
- ❖ Radius: The radius r(G) is the minimum eccentricity for the vertices.
- ❖ Diameter: The maximum eccentricity is called the diameter.
- ❖ Central vertex: A vertex v is a central vertex if e(v) = r(G).
- ❖ Centre: The centre of G is the set of all central vertices.
- ❖ Centroid vertex: A vertex v is a centroid vertex of a tree T, if c has minimum weight and the centroid of T consists of all such vertices.

2. DOMINATION NUMBER:

Let $G = (V,E)$ be a graph of order n. A non-empty subset $D \subseteq V$ is a dominating set if every vertex in V-D is adjacent to at least one vertex in D. The Domination number of a graph G is denoted by γ (G) and it is the minimum cardinality of the minimal dominating set of G.

A dominating set D is a minimal dominating set if no proper subset of 'D' is a dominating set of G.

2.1 Unchanging of domination number due to vertex removal:

Carrington, Harary and Haynes characterized graphs for which γ (G -v) $=\gamma$ (G).

Theorem 2.1:

A graph G has γ (G-v) $= \gamma$ (G) for any vertex v $\in$ V if and only if G has no isolated vertex and for each vertex v, either

(i) There is a γ -set in V-D such that for every vertex v$\in$V-D and for each γ -set D with vertex u in D, $p_n[v;D]$ contains at least one vertex from V-D, or

(ii) For every v$\in$V-D ,there is a subset of γ (G) of vertices in G-N[v] that dominates G-v.

Theorem 2.2:

An edge e = uv is in every p_n γ -set of G if and only if for every γ -set D of G, (u, v)$\subseteq$ D or {u, v}$\subseteq$V –D.

2.2 Changing of domination due to edge addition

The domination number is changed when any single edge is added.

(i.e) γ (G + e) $= \gamma$ (G) -1

The addition of an edge can decrease the domination number by at most one. Sunner and Blitch characterised these graphs only in the cases for which γ (G) is 1 or 2.

Theorem 2.3:

A graph G with γ (G) = 1 is in Changing domination number in edge removal if and only if G is complete.

2.3 Unchanging of Domination due to Edge addition:

The domination number is unchanged when any single edge is added.

The changing and unchanging terminology was first suggested by Harary. It is useful to partition the vertices of G into three sets according to how their removal affects γ (G) .

Let $V = V^0 \cup V^+ \cup V^-$ for,

$V^0 = \{v \in V : \gamma (G - v) = \gamma (G)\}$

$V^+ = \{v \in V : \gamma (G - v) > \gamma (G)\}$

$V^- = \{v \in V : \gamma (G - v) < \gamma (G)\}$

Similarly, the edge set can be partitioned into,

$E^0 = \{uv \in E : \gamma (G - uv) = \gamma (G)\}$

$E + = \{uv \in E : \gamma (G - uv) > \gamma (G)\}$

EXAMPLE 2.4:

Consider the zero divisor graph $\Gamma(Z_{14}) = \{0,1,2,3,4,5,6,7,8,9,10,11,12,13\}$

The vertex set $V(Z_{14}) = \{2,4,6,7,8,10,12\}$ and the edge set

$E(Z_{14}) = \{(2,7),(4,7),(6,7),(7,8),(7,10),(7,12)\}$
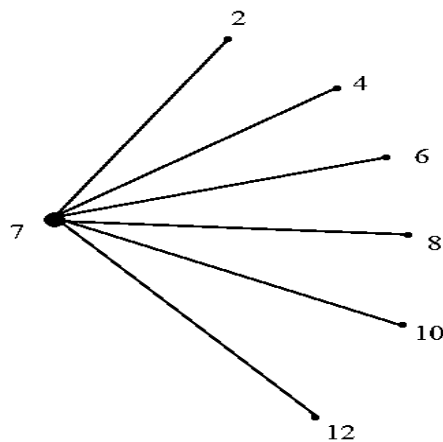
Here $G = \Gamma(Z_{14})$ is a Zero divisor graph and the dominating set $D = \{7\}$. Therefore the domination number γ (G) =1.

3. CHANGING AND UNCHANGING OF DOMINATION NUMBER OF $\Gamma(Z_N)$ DUE TO VERTEX DELETION:

Let G-v denote the graph formed by deleting the vertex v and G-e is the graph

formed by deleting an edge e from G.

Here, C represents Changing

U represents Unchanging

V represents Vertex set

E represents Edge set

D represents deletion.

Then we have the following cases

(i) γ (G -v) $\neq \gamma$ (G) for all v $\in$ V, for changing of domination number due to vertex deletion.

(ii) γ (G -e) $\neq \gamma$ (G) for all e$\in$ E, for changing of domination number due to edge deletion.

(iii) γ (G -v) = γ (G) for all v $\in$ V, for Unchanging of domination number due to vertex deletion.

(iv) γ (G -e) = γ (G) for all e$\in$ E, for Unchanging of domination number due to edge deletion.

Theorem: 3.1

For any graph $\Gamma(Z_{2p})$, where p is any odd prime, then there exists a non-pendant vertex

v $\in$ V such that γ ($\Gamma(Z_{2p})$−v) $> \gamma$ ($\Gamma(Z_{2p})$).

Proof:

Let $\Gamma(Z_{2p})$ be a Zero-divisor graph when p is any odd prime number.

To Prove γ ($\Gamma(Z_{2p})$−v) $> \gamma$ (Γ (Z_{2p})),for a non-pendant vertex v $\in$ V.

Since $\Gamma(Z_{2p})$ has atleast one vertex v with deg(v)$\geq$2 which is adjacent to atleast two end vertices,

If v is adjacent to two or more end vertices i.e if x and y are any two end vertices of v. Clearly v is in every γ - set of $\Gamma(Z_{2p})$ and γ ($\Gamma(Z_{2p})$−x)= γ ($\Gamma(Z_{2p})$).

Let ($\Gamma'(Z_{2p})$−x)= $\Gamma(Z_{2p})$,for any graph $\Gamma(Z_{2p})$.if degree of x is 1 then

γ ($\Gamma(Z_{2p})$−x)= γ ($\Gamma(Z_{2p})$). if $\Gamma'(Z_{2p})$=$\Gamma(Z_{2p})$ −v then we get

γ ($\Gamma(Z_{2p})$−v) >γ ($\Gamma(Z_{2p})$), where deg (v)=2.

Corollary: 3.2

If p is any odd prime number then there exists a pendant vertex u∈V such that
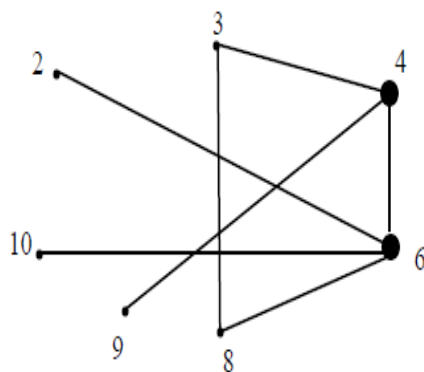
γ ($\Gamma(Z_{2p})$−u)= γ ($\Gamma(Z_{2p})$).

Example: 3.3

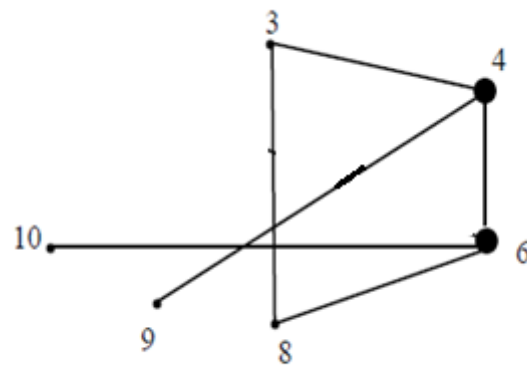Let $\Gamma(Z_{12})$ be the zero divisor graph and is denoted by G,

    $\Gamma(Z_{12})$ ={0,1,2,3,4,5,6,7,8,9,10,11}

    V(Z_{12})={2,3,4,6,8,9,10}

    E(Z_{12})={(2,6),(3,4),(3,8),(4,6),(4,9),(6,18),(6,10)}

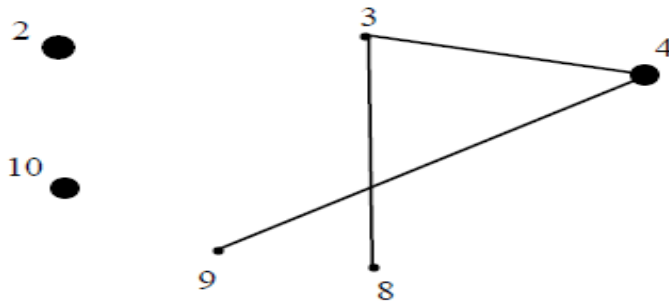   (i)   Graph G   (ii)   Graph $G_{1=}$ (G-2)

    Here D = {4, 6}, γ (G) = 2 and γ (G_1) = 2

    From (i) & (ii), γ (G-v) = γ (G)

Now take the graph $G_2 = (G-6)$:

Here D={2,4,8,10} γ (G-6)=4

(i.e) changing of domination number γ (G-6)>γ (G)

Example: 3.4

We take the graph $G=\Gamma(z_{10})$

$\Gamma(Z_{10})$ ={0,1,2,3,4,5,6,7,8,9}

where $V(Z_{10})$={2,4,5,6,8,} and

$E(Z_{10})$={(2,5),(4,5),(5,6),(5,8)}

Here the dominating set D= {5} and the domination number γ (G) =1.

Now the graph G_1= (G-4)

In this graph the dominating set D_1={5} and the domination number γ (G-4)=1.

Hence γ (G)=γ (G-4) which is the unchanging domination number due to vertex deletion.

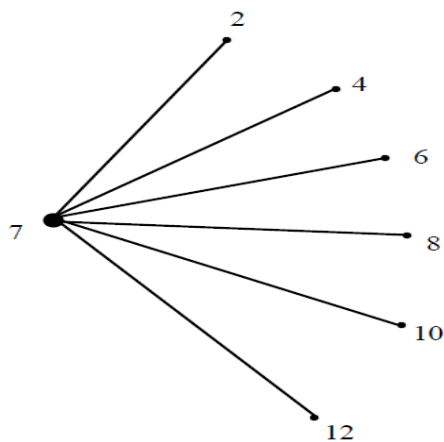**4. CHANGING OF DOMINATION NUMBER DUE TO EDGE DELETION OF A ZERO DIVISOR GRAPH:**

The removal of an edge from a graph G can increase the domination number by atmost one and it cannot decrease the domination number (i.e) γ (G-e)=γ (G)+1

Example:4.1

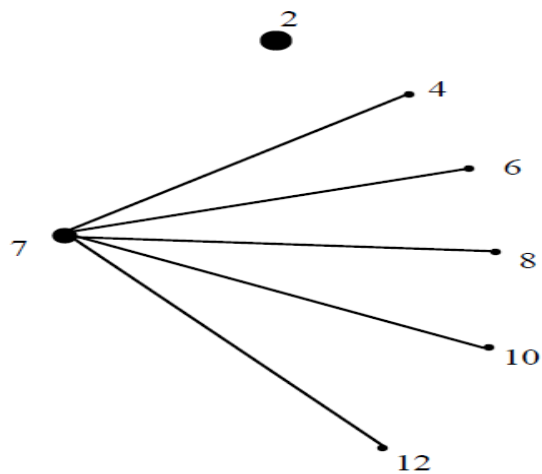Let G= $\Gamma(Z_{14})$ ={0,1,2,3,4,5,6,7,8,9,10,11,12,13}

$V(Z_{14})$={2,4,6,7,8,10,12} and

$E(Z_{14})$={(2,7),(4,7),(6,7),(7,8),(7,10),(7,12)}

Here D={7} and $\gamma(G)$=1

when we removed an edge (2,7) from the graph G, we have

(i.e) G_1=(G-e)=(G-(2,7))

Here D = {2,7} and γ (G_1)=2

From the above diagram we have γ (G-e) = γ (G) +1

Note:

To find the changing domination number in a zero divisor graph, we should take zero divisor graph which has a domination number greater than one.

5. APPLICATIONS:

We selected our state Tamil Nadu, let 4 and 6 be the distributors of certain product to the whole state. It is distributed to 4 major villages denoted by 3,8, 9,10 then the domination number of the graph is 2 . In an emergency case, let the distributor 6 is dead and the remaining distributor could not manage to supply the product for the whole state then the domination number gets changed.

This is an example for Changing domination number due to vertex deletion.

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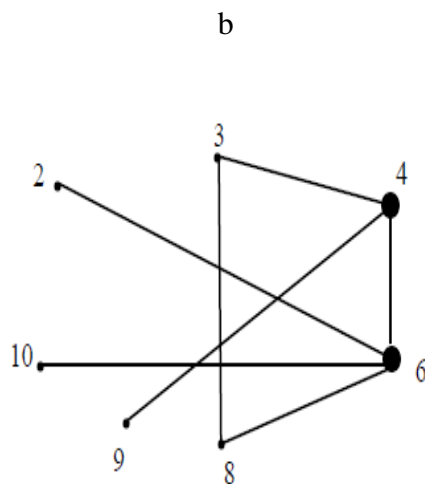
Here in this graph G, the dominating set D={4,6} and the dominating number γ (G)=2

When G_1=G-6 then D_1= {2, 4, 10} and γ (G-6)=3.
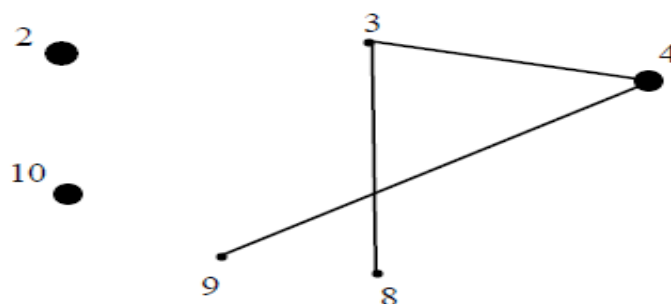
$G_1=(G-6)$

6. CONCLUSION:

In this paper we discussed "Changing and Unchanging Domination number of a zero-Divisor graph", by deleting a vertex or deleting or adding an edge. Related theorems and results were proved. The important concept of finding the domination number of zero divisor graph is very much applicable and useful to find the algebraic structures and the properties of rings.

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