International Journal of Computational and Applied Mathematics.
ISSN 1819-4966 Volume 12, Number 3 (2017), pp. 729-736
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A Note on Fuzzy Vertex Graceful labeling on Double Fan Graph and Double Wheel Graph

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Abstract

A labeling on a Graph G which can be gracefully numbered is called a graceful labeling. If a fuzzy graph G admits a graceful labeling and if all the vertex labelings are distinct then G is called a fuzzy vertex graceful graph. In this paper, we discussed fuzzy vertex graceful labeling on double Fan Graph and Double Wheel Graph.

Keywords: Fuzzy labeling, Graceful labeling, Fuzzy graceful labeling, Fuzzy Double Fan graph, Fuzzy Double Wheel graph.

AMS Mathematics Subject Classification: 05C, 94D

1. INTRODUCTION

Fuzzy is a newly emerging mathematical framework to exempt the phenomenon of uncertainty in real life tribulations. It was introduced by Zadeh in 1965, and the concepts were pioneered by various independent researchers, namely Rosenfeld, Kauffmann, etc.

A Fuzzy set is defined mathematically by assigning a value to each possible individual in the universe of discourse, representing its grade of membership which corresponds to the degree, to which that individual is similar or compatible with the concept represented the fuzzy set. Based on Zadeh's Fuzzy relation the first definition of a fuzzy graph was introduced by Kauffmann in 1973. The concept of a graceful labeling has been introduced by Rosa in 1967.This note is a further contribution on fuzzy graceful labeling.

Fuzzy labeling on Fan graph and Wheel graph are called a Fuzzy Fan graph and Fuzzy Wheel graph .[2,6]

2. PRELIMINARIES AND OBSERVATIONS

Definition 2.1[2]

A Fan graph $F_{m,n}$ is defined as the join of two graphs $\overline{K}_m + P_n$ where $\overline{K}_m$ is the empty set on m vertices and P_n is the path graph on n vertices.

The case m=1 Corresponds to the usual Fan graph while m=2 corresponds to the double Fan graph, etc.

A Double Fan graph with fuzzy labeling is called a fuzzy Double Fan graph.

Definition 2.2[2]

A Wheel graph W_n is a graph with n vertices ($n \geq 4$), formed by connecting a single vertex to all the vertices of an (n-1) cycle.

A Double Wheel graph DW_n of order n can be composed to $2C_N+K_1$. (ie)., it consists of order N, where the vertices of the two cycles are all connected to a common center.

A Double wheel graph with fuzzy labeling is called a fuzzy Double wheel graph.

Definition 2.3

In a fuzzy Double Fan graph($\tilde{F}_{2,n}$) if all the vertices are distinct and graceful labeling exists, then it is called a fuzzy vertex graceful labeling Double Fan graph.

A fuzzy graceful Double Fan graph $\tilde{F}_{2,n}$ consists of two vertex sets (F,F^*) and F_n with $|F| = 1$, $|F^*|$=1 and $|F_n|$>1 such that $\mu(F, F_i) > 0$ and $\mu(F^*, F_i) > 0$ where i=1 to n.

Proposition 2.4

In a fuzzy Double Fan graph ($\tilde{F}_{2,n}$), the vertex labeling $\sigma : F \rightarrow [0,1]$ and $\sigma : F^* \rightarrow [0,1]$ which satisfy the conditions that if the values of F only starts from

$\frac{n-1}{10}$ and F^* starts from $\frac{n}{10}$ then the Double Fan graph is a fuzzy vertex graceful labeling Double Fan graph.

Definition 2.5

In a fuzzy Double Wheel graph $\tilde{D}W_n$ if all the vertices are distinct and graceful labeling exists, then it is called a fuzzy vertex graceful labeling Double Wheel graph.

Proposition 2.6

A Double wheel in a fuzzy graph consists of two vertex sets V and U with $|V| = 1$, and $|U| > 1$, such that $\mu(v,u_i) > 0$ where i=1 to n and $\mu(u_i,u_{i+1}) > 0$ where i=1 to n-1.

3. MAIN RESULTS:

Theorem 3.1

Every Fuzzy Double Fan graph $\tilde{F}_{2,n}$ is a fuzzy vertex graceful Double Fan graph.

Proof:

Consider the Double Fan graph $F_{2,n}$ when m=2. Here F and F^* are the central vertices (m=2) and $F_1, F_2, F_3, \ldots, F_n$ are other vertices in the graph.

A Fuzzy Double Fan graph $\tilde{F}_{2,n}$ consists of two vertex sets(F,F^*) and F_n with $|F| = 1$, $|F^*|$=1 and $|F_n|$>1 such that $\mu(F,F_i) > 0$ and $\mu(F^*,F_i) > 0$ where i=1 to n and $\mu(F_i,F_{i+1}) > 0$ where i=1 to n-1.

If the fuzzy Double Fan graph is vertex graceful, then

$\mu(F,F_{n+1}) - \mu(F,F_n) = \mu(F_n,F_{n+1})$ Where n=1,2,…,etc. ------------------(1)

$\mu(F^*,F_{n-1}) = \mu(F^*,F_n) - \mu(F_{n-1},F_n)$

(ie)., $\mu(F^*,F_n) - \mu(F^*,F_{n-1}) = \mu(F_{n-1},F_n)$ Where n=1,2,…,etc. ------------------(2)

Also $\sigma(F_n) = \sigma(F) - \mu(F,F_n)$ **(or)**

$\sigma(F_n) = \sigma(F^*) - \mu(F^*,F_n)$.

Example 3.2

Case(i)

The value of F starts from $\frac{n-1}{10}$

Here,

$\mu(F, F_1) = 0.01,$

$\mu(F, F_2) = 0.03 = \mu(F, F_1) + \mu(F_1, F_2)$

$\mu(F, F_3) = 0.06 = \mu(F, F_2) + \mu(F_2, F_3)$ $\mu(F, F_4) = 0.10 = \mu(F, F_3) + \mu(F_3, F_4)$

Therefore,

$\mu(F, F_{n+1}) - \mu(F, F_n) = (0.01)(n+1)$ Where n=1,2,…,etc ------------(1)

Similarly,

$\mu(F_1, F_2) = 0.02$ $\mu(F_2, F_3) = 0.03$ $\mu(F_3, F_4) = 0.04$ and etc

$\mu(F_n, F_{n+1}) = (0.01)(n+1)$ Where n=1,2,…,etc. --------------------------(2)

Using (2) in (1), we get

$\mu(F, F_{n+1}) - \mu(F, F_n) = \mu(F_n, F_{n+1})$ Where n=1,2,…,etc.

Also $\sigma(F_n) = \sigma(F) - \mu(F, F_n)$

The value of F* starts from $\frac{n}{10}$

$\mu(F^*, F_4) = 0.20 = \sigma(F^*) - \sigma(F_4)$

$\mu(F^*, F_3) = 0.16 = \mu(F^*, F_4) - \mu(F_3, F_4)$

$\mu(F^*, F_2) = 0.13 = \mu(F^*, F_3) - \mu(F_2, F_3)$

and so on. Therefore

$\mu(F^*, F_{n-1}) - \mu(F^*, F_n) = (0.01)n$ ----------------- (1)

$\mu(F_1, F_2) = 0.02,$ $\mu(F_2, F_3) = 0.03,$ $\mu(F_3, F_4) = 0.04$ and etc

$\mu(F_{n-1}, F_n) = (0.01)n$ Where n=1,2,…,etc. ------ (2)

Using (2) in (1), we get

(ie). $\mu(F^*, F_n) - \mu(F^*, F_{n-1}) = \mu(F_{n-1}, F_n)$

$\sigma(F_n) = \sigma(F^*) - \mu(F^*, F_n)$

Case (ii)

The value of F starts from $\frac{n-1}{100}$

Here,

$\mu(F, F_1) = 0.001,$ $\mu(F, F_2) = 0.003 = \mu(F, F_1) + \mu(F_1, F_2)$ and so on.

Therefore, $\mu(F, F_{n+1}) - \mu(F, F_n) = (0.001)(n+1)$ Where n=1,2,…,etc

Similarly, $\mu(F_1, F_2) = 0.002$ $\mu(F_2, F_3) = 0.003$ and etc

$\mu(F_n, F_{n+1}) = (0.001)(n+1)$ Where n=1,2,…,etc.

$\mu(F, F_{n+1}) - \mu(F, F_n) = \mu(F_n, F_{n+1})$ Where n=1,2,…,etc.

The value of F* starts from $\frac{n}{100}$

$\mu(F^*, F_4) = 0.20 = \sigma(F^*) - \sigma(F_4)$

$\mu(F^*, F_3) = 0.016 = \mu(F^*, F_4) - \mu(F_3, F_4)$

and so on. Therefore

$\mu(F^*, F_n) - \mu(F^*, F_{n-1}) = (0.001)n$

$\mu(F_1, F_2) = 0.002$ $\mu(F_2, F_3) = 0.003$ and etc

$\mu(F_{n-1}, F_n) = (0.001)n$ Where n=1,2,…,etc.

Since all the membership value of edges $\mu(F, F_i) > 0$ and $\mu(F^*, F_i) > 0$ where i=1 to n and $\mu(F_i, F_{i+1}) > 0$ where i=1 to n-1 and $\mu(u,v) < \sigma(u) \wedge \sigma(v)$.

Since all the membership values of the vertices are distinct, this graph is a fuzzy vertex graceful labeling Double Fan graph.

Theorem 3.3

For some n $\geq$ 4, the Double Wheel graph DW_n is a fuzzy vertex graceful wheel graph, where the central vertex $\sigma : V \to [0,1]$ and the vertices from the inner and outer cycles are $\sigma : w_i \to [0,1]$ and $\mu(v, w_i) = 0.001 \times i$ for inner cycle and $\mu(v, w_i) = 0.001 \times (i + n)$ for outer cycle where i=1 to n.

Proof:

A Double Wheel graph DW_n is a graph with n vertices exists only if n $\geq$ 4.

A Wheel in a fuzzy graph consists of two node sets V and W with $|V| = 1$ and $|W| > 1$ such that $\mu(v, w_i) > 0$ where i=1 to n and $\mu(w_i, w_{i+1}) > 0$ where i=1 to n-1.

In the Double Wheel graph, V is the central vertex, w_i denotes the vertices in the inner and outer cycles.

Here $\sigma : V \to [0,1]$ and $\sigma : w_i \to [0,1]$.

$\sigma(w_i) = \sigma(v) - \mu(v, w_i)$ where $\mu(v, w_i) = 0.001 \times i$ for inner cycle and

$\sigma(w_i) = \sigma(v) - \mu(v, w_i)$ where $\mu(v, w_i) = 0.001 \times (i + n)$ for outer cycle where i=1 to n and $\mu(w_i, w_{i+1}) = \mu(vw_i) - \mu(vw_{i+1})$ where i=1 to n-1 (or)

$\mu(w_{n-1}, w_n) = \mu(vw_{n-1}) - \mu(vw_n)$ while $\mu(w_n, w_1) = \mu(vw_n) - \mu(vw_1) - (0.001)(n - 2)$

Example: 3.4

Case (i):

When $\sigma(v)$ starts from $\dfrac{n-1}{100}$

Here, when $\sigma : V \to [0,1]$ and $\sigma : w_i \to [0,1]$.

For Inner cycle

$\sigma(w_1) = \sigma(v) - 0.001$ $\sigma(w_2) = \sigma(v) - 0.002$

$\sigma(w_3) = \sigma(v) - 0.003$ $\sigma(w_4) = \sigma(v) - 0.004$ and etc

(ie)., $\sigma(w_i) = \sigma(v) - \mu(v, w_i)$ where $\mu(v, w_i) = 0.001 \times i$, i=1 to n

Also

$\mu(w_1, w_2) = \mu(vw_1) - \mu(vw_2) = 0.001,$
$\mu(w_2, w_3) = \mu(vw_2) - \mu(vw_3) = 0.001,$

$\mu(w_3, w_4) = \mu(vw_3) - \mu(vw_4) = 0.001$ and etc

$\mu(w_i, w_{i+1}) = \mu(vw_i) - \mu(vw_{i+1})$ where i=1 to n-1 (or)

$\mu(w_{n-1}, w_n) = \mu(vw_{n-1}) - \mu(vw_n)$ while $\mu(w_n, w_1) = \mu(vw_n) - \mu(vw_1) - (0.001)(n-2)$

For outer cycle

$\sigma(w_i) = \sigma(v) - (0.001)(i+n)$

$\sigma(w_i) = \sigma(v) - \mu(v, w_i)$ where $\mu(v, w_i) = 0.001 \times (i+n)$ where i=1 to n

(ie)., $\sigma(w_1) = \sigma(v) - 0.005,$ $\sigma(w_2) = \sigma(v) - 0.006,$ $\sigma(w_3) = \sigma(v) - 0.007$ and etc

Case (ii)

When $\sigma(v)$ starts from $\frac{n-1}{1000}$

Here, when $\sigma : V \to [0,1]$ and $\sigma : w_i \to [0,1]$.

For Inner cycle

$\sigma(w_1) = \sigma(v) - 0.0001,$ $\sigma(w_2) = \sigma(v) - 0.0002$ and etc

(ie)., $\sigma(w_i) = \sigma(v) - \mu(v, w_i)$ where $\mu(v, w_i) = 0.0001 \times i$, i=1 to n

Also $\mu(w_i, w_{i+1}) = \mu(vw_i) - \mu(vw_{i+1})$ where i=1 to n-1 (or)

$\mu(w_{n-1}, w_n) = \mu(vw_{n-1}) - \mu(vw_n)$ While
$\mu(w_n, w_1) = \mu(vw_n) - \mu(vw_1) - (0.0001)(n-2)$

For outer cycle

$\sigma(w_i) = \sigma(v) - (0.0001)(i+n)$ $\sigma(w_i) = \sigma(v) - \mu(v, w_i)$

where $\mu(v, w_i) = 0.0001 \times (i+n)$ where i=1 to n

In the Similar way, for any value of $\sigma : V \to [0,1]$ the labeling of all vertices in the inner and outer cycles w_i are distinct where i=1 to n.

Therefore by using the above , the Double wheel graph DW_n admits a fuzzy vertex graceful labeling.

4. CONCLUSION

In this paper, the concepts of fuzzy vertex graceful labeling Double Fan graph and fuzzy vertex graceful labeling Double Wheel graph have been discussed. We further extend this study on some more special classes of graphs.

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