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Edge – Odd Gracefulness of Different Types of Shell Graphs by Removing Two Chords

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INTRODUCTION

Choudum and Kishore [1999] found graceful labeling of the union of paths and cycles, Bhat-Nayak and Selvam [2003] got graceful labeling for n-cone $Cm \vee K_n$. Barrientos [2005] obtained the graceful labeling for unions of cycles and complete bipartite graphs. Cheng et. al. [2008] analyzed graceful labeling for generalized spiders and caterpillars. Guo [1994, 1995] investigated graceful labelings for bipartite graph B(m, n) and B(m, n, p). Liu [995] proved that the star graph with top sides is graceful. Seoud and Youssef [2000] showed that some classes of families in terms of disconnected from paths and cycles are graceful. Xu et.al. [2008] verified that the graphs $C_{13}{}^{(t)}$ are graceful where $t \equiv 0, 1 \pmod 4$. Sethuraman and Jesintha developed a new class of graceful lobsters.

SECTION 2: Edge-odd gracefulness of few semi-shell graphs

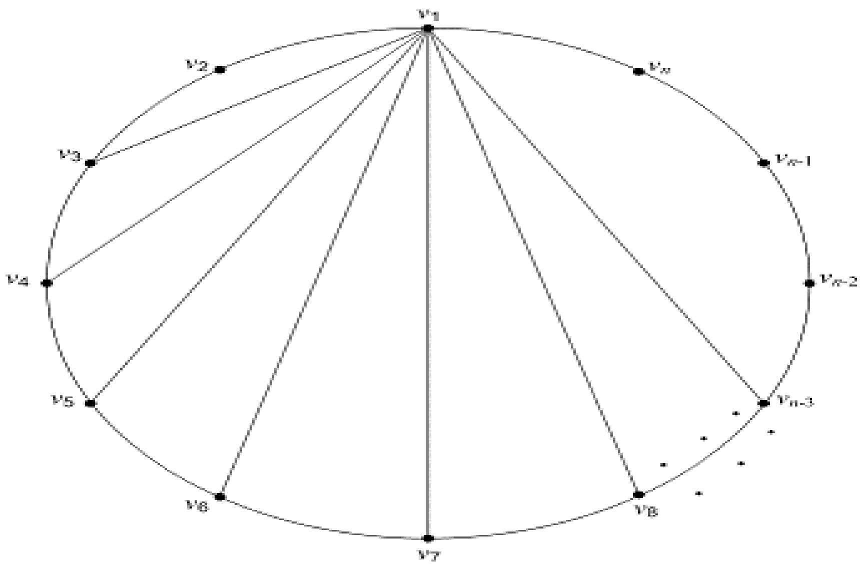
Definition 2.1: Graceful Graph: A function f of a graph G is called a graceful labeling with m edges, if f is an injection from the vertex set of G to the set {0, 1, 2, …, m} such that when each edge uv is assigned the label $|f(u) - f(v)|$ and the resulting edge labels are distinct. Then the graph G is graceful.

Definition 2.2: Edge-odd graceful graph: A (p, q) connected graph is edge-odd graceful graph if there exists an injective map f: $E(G) \rightarrow \{1, 3, \ldots, 2q-1\}$ so that induced map f_+: V(G) → {0, 1, 2, …,(2k-1)} defined by $f_+(x) \equiv \Sigma f(x, y) \pmod{2k}$, where the vertex x is incident with other vertex y and k = max {p, q} makes all the edges distinct and odd. Hence the graph G is edge- odd graceful.

Definition 2.3: C(n , n-4) is a connected (p, q)-graph whose vertex set is $\{v_1, v_2, v_3, \ldots, v_n\}$ and edge set is $\{v_i v_{i+1}: i = 1 \text{ to } (n-1)\} \cup \{v_n v_1\} \cup \{v_1 v_j: j = 3, 4, 5, \ldots, (n-3)\}$. Here p = n ; q = 2n-5. It is obtained from a shell graph by removing two additional chords at v_1. It is called a weak shell graph.

Theorem 2.4: The weak shell graph C (n, n-4) is edge-odd graceful if n ≥ 9.

Proof: One of the arbitrary labeling for vertices is as follows:



The given graph C (n, n-1) is connected whose vertex set is $\{v_1, v_2, v_3, \ldots, v_n\}$ and

edge set is $\{v_i v_{i+1}: i=1 \text{ to } (n-1)\} \cup \{v_n v_1\} \cup \{v_1 v_j: j=3,4,5,\ldots,(n-3)\}$. Here p = n ; q = 2n-5

Define a map f: E(C(n , n-4))→{1,2,3,…,q} by

$f(v_i v_{i+1}) = 2i-1, \quad i = 1 \text{ to } (n-1)$

$f(v_n v_1) = 2n-1$

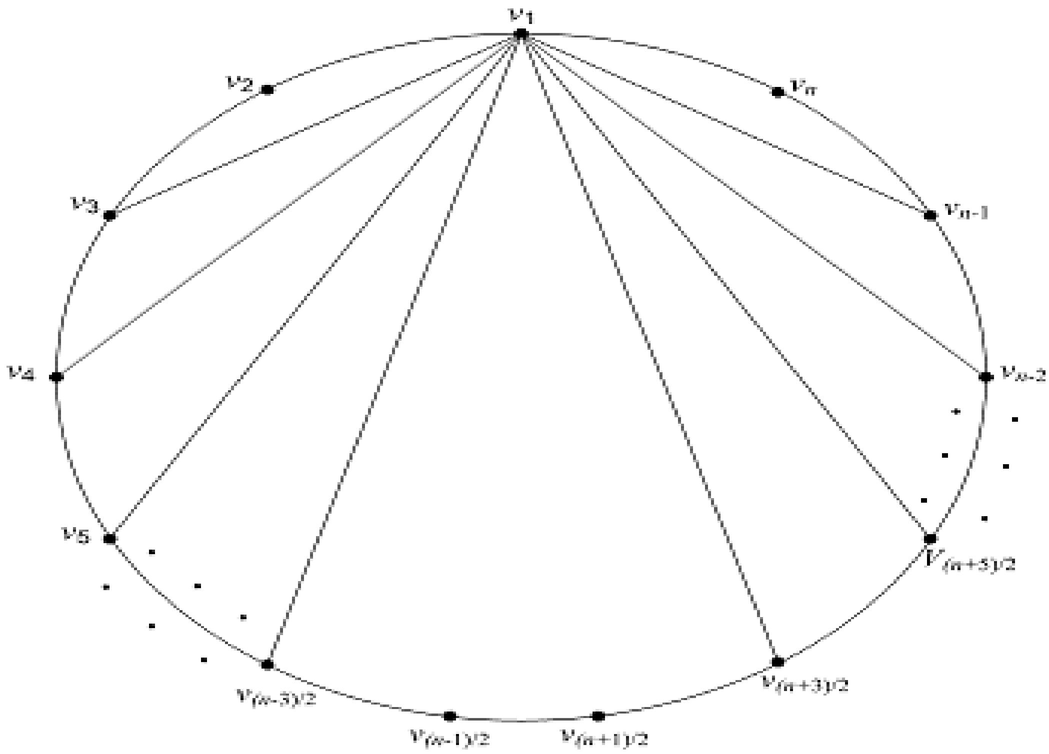
$f(v_i v_j) = (2n-1) + 2(j-2), \quad j = 3, 4, 5, \ldots, (n-3).$

Define $f^{+}: V(C(n , n\text{-}4)) \rightarrow \{0, 1, 2,....,q\}$ by $f^{+}(U)= \sum_{V\in G} f(UV)$ (mod 2q) where this sum run over all edges through the vertex U.

Hence the map f and the induced map f_{+} provide labels as distinct odd numbers for edges and also the labelings for vertex set have distinct values in $\{0, 1, 2,\ldots, (2k\text{-}1)\}$. Hence the graph C(n , n-4) is edge-odd graceful.

Definition 2.5: $C_n{*}S_{n\text{-}4}$ is a connected graph whose vertex set = $\{v_1 ,v_2,\ldots\ldots,v_n\}$ and edge set is $\{v_iv_{i+1}$:i=1 to (n-1)$\} \cup\{v_nv_1\}\cup\{v_1v_j : j=3,4,5,\ldots,\frac{n-3}{2}, \frac{n+3}{2},\frac{n+5}{2},\ldots,(n\text{-}1)\}$. Here it is obtained from a shell graph by removing two chords at the middle vertex. It is called also weak shell graph.

Theorem 2.6: The weak shell graph $C_n{*}S_{n\text{-}4}$ is edge-odd graceful where n is an odd integer.



Proof: Let n be an odd integer. The given graph has the vertex set = $\{v_1 ,v_2,\ldots\ldots,v_n\}$ and edge set is $\{v_iv_{i+1}$:i=1 to (n-1)$\} \cup\{v_nv_1\} \cup \{v_1v_j : j=3,4,5,\ldots,\frac{n-3}{2}$,

$\frac{n+3}{2}, \frac{n+5}{2}$,…,(n-1)}.

One of the arbitrary labeling for vertices is as above:

Define a map f: E (C_n*S_{n-4}) → (1, 3, …, (2q-1)}

$f(v_iv_{i+1}) = 2i-1$, i=1 to (n-1), $i\neq, \frac{n-3}{2}, \frac{n-1}{2}, \frac{n+1}{2}, n, (n-2)$

$f(v_{(n-3)/2}v_{(n-1)/2}) = n$

$f(v_{(n-1)/2}v_{(n+1)/2}) = n+2$

$f(v_{(n+1)/2}v_{(n+3)/2}) = n+4$

$f(v_nv_1) = 2n-1$

$f(v_{n-2}v_{n-1}) = 2n-5$

$f(v_1v_j) = (4n-5)-2(j-3)$, j=3,4,5,…, $\frac{n-3}{2}$

$f(v_1v_j) = (3n-6)-2[j-(\frac{n+3}{2})]$, j=$\frac{n+3}{2}, \frac{n+5}{2}$,….(n-1).

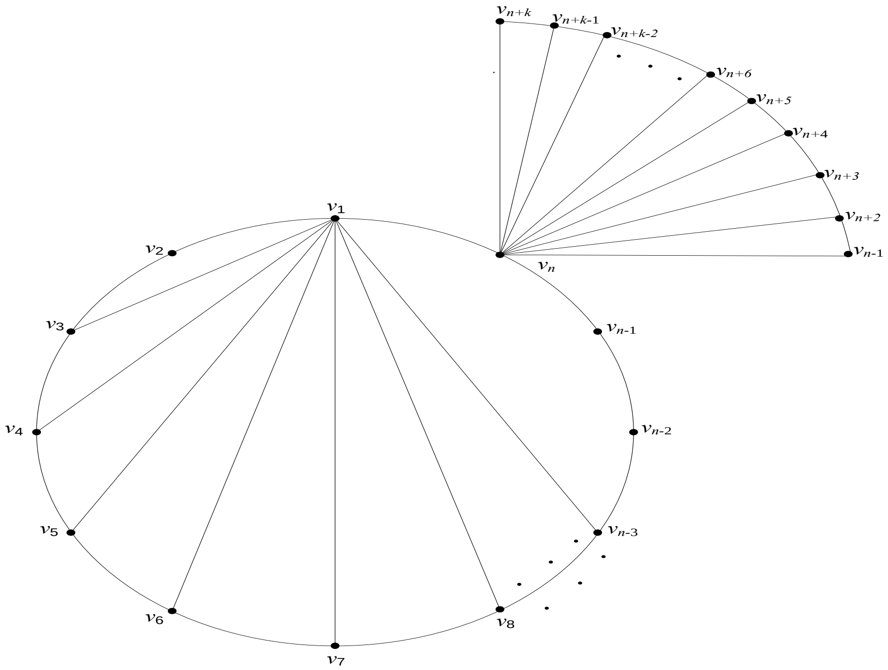
Define f^+: V(C_n*S_{n-4}) → {0, 1, 2,....,q} by $f^+(U)=\sum_{V\in G} f(UV)$ (mod 2q) where this sum run over all edges through the vertex U.

Hence the map f and the induced map f_+ provide labels as distinct odd numbers for edges and also the labelings for vertex set have distinct values in {0, 1, 2,…., (2k-1)}. Hence the graph C_n*S_{n-4} is edge-odd graceful.

Definition 2.7: C(n, n-4) ∗ F_k is a connected graph whose vertex set is {v_1 ,v_2,……,v_{n+k}} and edge set is {v_iv_{i+1}: i = 1 to (n-1) } ∪{v_nv_1} ∪ {v_1v_j: j = 3, 4, 5,…,(n-3)} ∪ {$v_{n+1}v_{n+i+1}$, i = 1, 2,…, (k-1)} ∪ {v_nv_{n+j} , j = 1, 2,.., k}.

Theorem 2.8: The connected graph C(n, n-4) ∗ F_k is edge-odd graceful if n is an odd integer.

Proof: One of the arbitrary labeling for vertices is as follows:



Its vertex set is $\{v_1, v_2, \ldots, v_{n+k}\}$.

Its edge set is $\{v_iv_{i+1}: i = 1 \text{ to } (n-1)\} \cup \{v_nv_1\} \cup \{v_1v_j: j = 3, 4, 5, \ldots, (n-3)\}$

$\cup \{v_{n+1}v_{n+i+1}, i = 1, 2, \ldots, (k-1)\} \cup \{v_nv_{n+j}, j = 1, 2, \ldots, k\}$.

Define a map $f: E(C(n, n-4) * F_k) \rightarrow \{1,2,3,\ldots,q\}$ by

$f(v_iv_{i+1}) = 2i-1, \; i = 1,2,\ldots,(n-1), \; i \neq (n-3),(n-2)$

$f(v_{n-3}v_{n-2}) = 2n-5$

$f(v_{n-2}v_{n-1}) = 2n-7$

$f(v_nv_{n+j}) = (2n-1) + 4(j-1), \; j=1,2,\ldots,(k-1)$

$f(v_{n+i}v_{n+i+1}) = (2n+1)+4(j-1), \; i=1,2,\ldots,(k-2)$

$f(v_{n+k-1}v_{n+k}) = (2n-1)+4(k-1)$

$f(v_nv_{n+k}) = (2n+1)+4(k-2)$

$f(v_nv_1) = (2n+3)+4(k-2)$

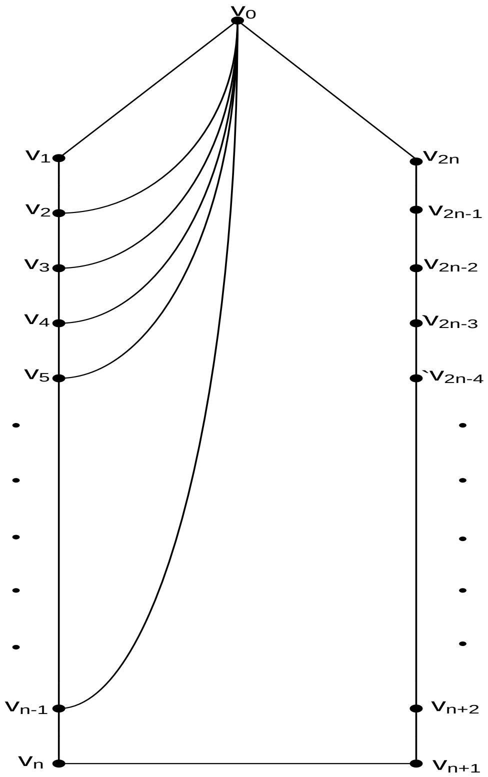
$f(v_1v_i) = (2n+3)+4(k-2)+2(j-2)$, $j=3,4,\ldots,(n-3)$

Define f^+: $V(C(n, n\text{-}4) * F_k) \rightarrow \{0, 1, 2,....,q\}$ by $f^+(U)= \sum_{V\in G} f(UV)$ (mod 2q) where this sum run over all edges through the vertex U.

Hence the map f and the induced map f_+ provide labels as distinct odd numbers for edges and also the labelings for vertex set have distinct values in {0, 1, 2,…., (2k-1)}. Hence the graph C(n, n-4) ∗ F_k is edge-odd graceful.

Definition 2.9: The semi-shell graph $C(n, \frac{n-5}{2})$ is connected whose vertex set : $\{v_0, v_1,\ldots,v_{2n}\}$ and edge set is $\{v_iv_{i+1};\ i = 0 \text{ to } (2n-1)\} \cup \{v_{2n}v_0\} \cup \{v_0v_j: j = 2 \text{ to } (n-1)\}$

Theorem 2.10: The semi-shell graph $C(n, \frac{n-5}{2})$ is edge-odd graceful where n is an odd integer.



Its vertex set is $\{v_0, v_1, \ldots\ldots, v_{2n}\}$, and edge set is $\{v_iv_{i+1};$ i varies from 0 to (2n-1)$\}$ $\cup \{v_{2n}v_0\} \cup \{v_0v_j:$ j =2 to (n-1)$\}$.

Define a map f: $E(C(n, \frac{n-5}{2})) \rightarrow \{0,1,2,\ldots,q\}$ by

$f(v_iv_{i+1}) = 2i-1$; $i = 0, 1, 2,\ldots,\frac{3n+1}{4}$ if $n \equiv 1 \pmod 3$.

$= 2i-1$; $i = 0, 1, 2,\ldots,\frac{3n-1}{4}$ if $n \equiv 2 \pmod 3$..

$= 2i-1$; $i = 0, 1, 2,\ldots,\frac{3n}{4}$ if $n \equiv 0 \pmod 3$..

$f(v_{(3n+1)/4}\, v_{(3n+5)/4}) = f(v_{(3n+1)/4}\, v_{(3n+1)/4}) + 4$.

$f(v_iv_{i+1}) = f(v_{(3n+1)/4}\, v_{(3n+5)/4}) + 4\,(i - \frac{3n+1}{4})$, $\quad i = \frac{3n+5}{4}, \frac{3n+13}{4}, \ldots (2n-1)$.

$= 2i-1$ if i = 2 to (n-1)

$F(v_{2n}v_0) = \frac{3n-1}{2} + 1 = \frac{3n+1}{4}$.

Define f^+: $V(C(n, \frac{n-5}{2})) \rightarrow \{0, 1, 2,\ldots,q\}$ by $f^+(U)= \sum_{V\in G} f(UV) \pmod{2q}$ where this sum run over all edges through the vertex U.

Hence the map f and the induced map f_+ provide labels as distinct odd numbers for edges and also the labeling for vertex set have distinct values in $\{0, 1, 2,\ldots, (2k-1)\}$. Hence the graph $C(n, \frac{n-5}{2})$ is edge-odd graceful.

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