International Journal of Computational and Applied Mathematics.
ISSN 1819-4966 Volume 12, Number 3 (2017), pp. 811-813

http://www.ripublication.com

A Note on the Structure of Finite Cyclic Groups

S.K. Pandey

Dept. of Mathematics, Sardar Patel University of Police, Security and Criminal Justice, Daijar-342304, Jodhpur, India.

Abstract

This note deals with finite cyclic groups. It exhibits that if G_1 is a cyclic group of order $q-1$ associated with a finite field of order q then G_1 does have additional structure. We explore this idea by means of a simple but suitable example.

Key-words: Cyclic group, finite group, finite field, even square group.

MSCS 2010: 11E57, 20B05.

1. INTRODUCTION

The concept of a group is very old and well established [1-2]. It is well known that any two cyclic groups of the same order are algebraically equivalent. This note describes that cyclic groups associated with a finite field have additional structure.

We have recently introduced the notion of even square rings [3]. This led to introduce the notion of even square semigroups [4]. Recall that a multiplicative semigroup S together with a unary operation $f : S \to S$ defined by $f(a) = 2a, \forall a \in S$ is called a unary semigroup. A unary semigroup S is called an even semigroup if $a \in 2S, \forall a \in S$. Similarly a unary semigroup S is called an even square semigroup if $a^2 \in 2S, \forall a \in S$. Analogously we define even square groups [5].

In this note we mainly describe that a finite cyclic group G_1 associated with a field does have additional structure. However another cyclic group having the same order does not necessarily have the same structure.

RESULTS

Proposition. Let R be a finite filed of order q and G be the associated multiplicative cyclic group of order $q-1$ then G is equipped with a self-map defined by $f(a) = na, \forall a \in R$. Here $n \neq mp \in Z^+$, p is the characteristic of R and m is a positive integer.

Corollary. Let R be a finite filed of characteristic $\neq 2$ and G be the associated cyclic group then G is equipped with a self-map defined by $f(a) = 2a, \forall a \in R$.

Remark 1. One may find that above result holds for a field of infinite order also. Clearly R is closed under the above unary operation.

Example. Let $G_1 = \{2,4,6,8\}$. One can easily see that G_1 is a cyclic group under multiplication modulo ten. G_1 is a finite cyclic group of order four associated with a finite field of order five. It can be seen that G_1 is equipped with a self-map as described in the above proposition. On the other hand let $G_2 = \{1,-1,i,-i\}$. Here $i^2 = -1$. Clearly G_2 is a multiplicative cyclic group of order four. But G_2 is not equipped with a self-map as like G_1. In the light of the above corollary and the notion of even square groups we may also say that G_1 is an even square group but G_2 is not an even square group however both are cyclic groups of the same order.

Thus two cyclic groups of same order may differ on account of a self-map described by the above proposition.

Remark 2. Though we have given just one example however one may see that the cyclic group associated with every finite field does have additional structure as described above.

REFERENCES

[1] M. Artin, Algebra, Prentice Hall of India Pvt. Ltd., New Delhi, 2000.

[2] T. W. Hungerford, Algebra, Springer-India, New Delhi, 2005.

[3] S. K. Pandey, Nil Elements and Even Square Rings, International Journal of Algebra, Vol. 11, no. 1, 1-7, 2017.

[4] S. K. Pandey, Even Square Semigroups, International Journal of Mathematics and its Applications, Vol. 5 , issue 1-A, 76-78, 2017.

[5] S.K. Pandey, Even Square Algebraic Structures, International Journal of Pure Algebra, Vol. 7 (1), 476-478, 2017.