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Properties of Intuitionistic Fuzzy Graphs of Second Type

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Abstract

In this paper, we define the path, bridge and cut-vertex on intuitionistic fuzzy graphs of second type and establish some of their properties.

Key Words : Intuitionistic Fuzzy Graphs, Intuitionistic Fuzzy Graphs of Second Type, strength, path, bridge, cut-vertex.
2010 AMS Subject Classification : 05C72, 03E72

1. INTRODUCTION :

Fuzzy sets were introduced by Lotfi. A. Zadeh [10] in 1965 as a generalisation of classical (crisp) sets. Further the fuzzy sets are generalised by Krassimir.T. Atanassov [1] in which he has taken non-membership values also into consideration and introduced Intuitionistic Fuzzy Set [IFS] and its extension namely Intuitionistic Fuzzy Sets of Second Type [IFSST]. Also he introduced the concept of Intuitionistic fuzzy relations. R. Parvathi and M. G. Karunambigai [6] introduced Intuitionistic Fuzzy Graphs [IFG] elaborately and also they introduced the path, bridge and cut-vertex on IFG. The authors introduced [9] Intuitionistic Fuzzy Graphs of Second Type [IFGST] and studied some of their properties. In section 2, we give some basic definitions and in section 3, further the authors propose the path, bridge and cut-vertex on intuitionistic fuzzy graphs of second type and establish some more properties. The paper is concluded in section 4 .

2. PRELIMINARIES:

In this section, we give some basic definitions.

Definition 2.1 [1] **:** An Intuitionistic Fuzzy set [IFS] A in a universal set E is defined as an object of the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in E\},$$

Where $\mu_A: E \to [0,1]$ and $\nu_A: E \to [0,1]$ denote the degree of membership and the degree of non-membership of the element x in E respectively, satisfying $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Definition 2.2 [1] **:** An Intuitionistic Fuzzy sets of second type [IFSST] A in a universal set E is defined as an object of the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in E\},$$

Where $\mu_A: E \to [0,1]$ and $\nu_A: E \to [0,1]$ denote the degree of membership and the degree of non-membership of the element x in E respectively, satisfying $0 \leq \mu_A(x)^2 + \nu_A(x)^2 \leq 1$.

Definition 2.3 [6] **:** An Intuitionistic Fuzzy Graph [IFG] is of the form $G = [V, E]$ where

(i) $V = \{v_1, v_2, \dots v_n\}$ such that $\mu_1: V \to [0,1]$ and $\nu_1: V \to [0,1]$ denote the degree of membership and non-membership of the element $v_i \in V$, respectively, and

$0 \leq \mu_1(v_i) + \nu_1(v_i) \leq 1$ for every $v_i \in V,\ (i = 1,2, \dots n)$

(ii) $E \subseteq V \times V$ where $\mu_2: V \times V \to [0,1]$ and $\nu_2: V \times V \to [0,1]$ are such that

$\mu_2(v_i, v_j) \leq \min[\mu_1(v_i), \mu_1(v_j)]$,

$\nu_2(v_i, v_j) \leq \max[\nu_1(v_i), \nu_1(v_j)]$

and $0 \leq \mu_2(v_i, v_j) + \nu_2(v_i, v_j) \leq 1$ for every $(v_i, v_j) \in E,\ (i, j = 1,2, \dots n)$

Definition 2.4 [6] **:** A path P of an IFG is a sequence of distinct vertices $v_1, v_2, v_3 \dots v_n$ such that either one of the following conditions are satisfied

(a) $\mu_{2ij} > 0$ and $\nu_{2ij} > 0$

(b) $\mu_{2ij} = 0$ and $\nu_{2ij} > 0$

(c) $\mu_{2ij} > 0$ and $\nu_{2ij} = 0$ for some i and j $(i,j = 1,2,3..n)$

Definition 2.5 [6] **:** In an IFG, $G = [V, E]$ the μ-strength of a path $P = v_1 v_2 \dots v_n$ connecting any two vertices is defined as, $\max\{\mu_{2ij}\}$ and is denoted by $(\mu_{2ij})^{\infty}$ and the ν-strength of a path $P = v_1 v_2 \dots v_n$ connecting any two vertices is defined as, $\min\{\nu_{2ij}\}$ and is denoted by $(\nu_{2ij})^{\infty}$ for all $i, j = 1,2,3 \dots n$

Definition 2.6 [6] **:** Let $G = [V, E]$ be an IFG. Let v_i, v_j be any two distinct vertices and

$H = [V', E']$ be an intuitionistic fuzzy subgraph of G obtained by deleting an edge (v_i, v_j) that is $H = [V', E']$ where $\mu'_{2ij} = 0$ and $\nu'_{2ij} = 0$ and $\mu'_2 = \mu_2$ and $\nu'_2 = \nu_2$ for all other edges.

Now (v_i, v_j) is said to be a bridge in G, if either

$(\mu'_{2xy})^{\infty} < (\mu_{2xy})^{\infty}$ and $(\nu'_{2xy})^{\infty} \geq (\nu_{2xy})^{\infty}$ (or)

$(\mu'_{2xy})^{\infty} \leq (\mu_{2xy})^{\infty}$ and $(\nu'_{2xy})^{\infty} > (\nu_{2xy})^{\infty}$ for some $v_x, v_y \in V$

In other words, deleting an edge (v_i, v_j) reduces the strength of connectedness between some pair of vertices (or) (v_i, v_j) is a bridge if, there exists v_x, v_y such that, $(v_i, v_j)$is an edge of every strongest path from v_x to v_y

Definition 2.7 [6] **:** A vertex v_i is said to be a cut-vertex in IFG, G if deleting a vertex v_i reduces the strength of connectedness between some pair of vertices (or) v_i is a cut-vertex if and only if there exists v_x, v_y such that v_i is a vertex of every strongest path from v_x to v_y .

In other words,

$(\mu'_{2xy})^{\infty} \leq \mu_{2xy}$ and $(\nu'_{2xy})^{\infty} < \nu_{2xy}$ (or)

$(\mu'_{2xy})^{\infty} < \mu_{2xy}$ and $(\nu'_{2xy})^{\infty} \leq \nu_{2xy}$ for some $v_x, v_y \in V$

Definition 2.8 [9] **:** An Intuitionistic Fuzzy Graphs of Second Type [IFGST] is of the form $G = [V, E]$ where

(i) $V = \{v_1, v_2, \dots v_n\}$ such that $\mu_1 : V \to [0,1]$ and $\nu_1 : V \to [0,1]$ denote the degree of membership and non-membership of the element $v_i \in V$, respectively, and

$0 \leq \mu_1(v_i)^2 + \nu_1(v_i)^2 \leq 1$ for every $v_i \in V$, $(i = 1,2, \dots n)$

(ii) $E \subseteq V \times V$ where $\mu_2 : V \times V \to [0,1]$ and $\nu_2 : V \times V \to [0,1]$ are such that

$\mu_2(v_i, v_j) \leq \min[\mu_1(v_i)^2, \mu_1(v_j)^2]$,

$\nu_2(v_i, v_j) \le \max\left[\nu_1(v_i)^2, \nu_1(v_j)^2\right]$

and $0 \le \mu_2(v_i, v_j)^2 + \nu_2(v_i, v_j)^2 \le 1$ for every $(v_i, v_j) \in E,\ (i, j = 1,2, \dots n)$

3. INTUITIONISTIC FUZZY GRAPHS OF SECOND TYPE

In this section, we define the path, bridge and cut-vertex on IFGST and prove some of their properties.

Definition 3.1: A path P of an IFGST is a sequence of distinct vertices $v_1, v_2, v_3 \dots v_n$ such that either one of the following conditions are satisfied

(a) $\mu_{2ij} > 0$ and $\nu_{2ij} > 0$

(b) $\mu_{2ij} = 0$ and $\nu_{2ij} > 0$

(c) $\mu_{2ij} > 0$ and $\nu_{2ij} = 0$ for some i and j $(i, j = 1,2,3..n)$

Definition 3.2: A path $P = v_1 v_2 \dots v_{n+1}$ $(n > 0)$ then the length of P is n

Definition 3.3: A path $P = v_1 v_2 \dots v_{n+1}$ is called a cycle if $v_1 = v_{n+1}$ and $n \ge 3$

Example 3.1:

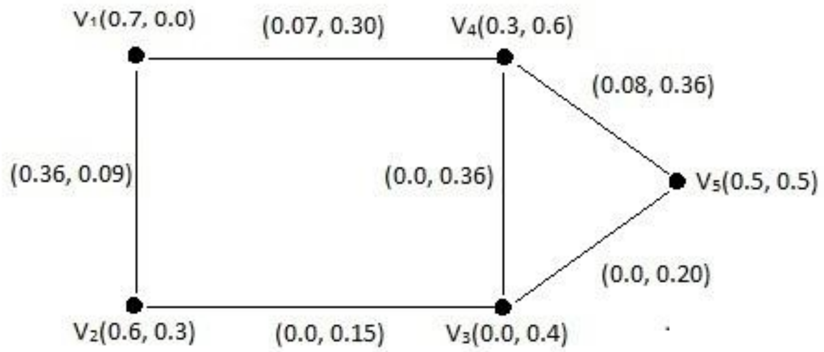

FIGURE 1.

In the above FIGURE 1. $v_1 v_2 v_3 v_5$ is a path and the length of this path is 3. Also $v_1 v_4 v_5 v_3 v_2 v_1$ is a cycle.

Definition 3.4: In an IFGST, $G = [V, E]$ the μ-strength of a path $P = v_1 v_2 \dots v_n$ connecting any two vertices is defined as, $\max\{\mu_{2ij}\}$ and is denoted by $(\mu_{2ij})^{\infty}$ and the ν-strength of a path $P = v_1 v_2 \dots v_n$ connecting any two vertices is defined as, $\min\{\nu_{2ij}\}$ and is denoted by $(\nu_{2ij})^{\infty}$ for all $i, j = 1,2,3 \dots n$

Remark 1

If an edge possesses both μ-strength and ν-strength, then it is the strength of the path P and is denoted by S_P that is $S_P = \left(\left(\mu_{2ij}\right)^{\infty}, \left(\nu_{2ij}\right)^{\infty} \right)$for all $i, j = 1,2,3 \dots n$

Definition 3.5: Let $G = [V, E]$ be an IFGST. Let v_i, v_jbe any two distinct vertices and $H = [V', E']$ be an intuitionistic fuzzy subgraph of G obtained by deleting an edge (v_i, v_j).

So, $H = [V', E']$ where $\mu'_{2ij} = 0$ and $\nu'_{2ij} = 0$ and $\mu'_{2xy} = \mu_{2xy}$ and $\nu'_{2xy} = \nu_{2xy}$ for all other edges (v_x, v_y)

We say that (v_i, v_j) is bridge in G, if either

$\left(\mu'_{2xy}\right)^{\infty} < \mu_{2ij} < \mu^{\infty}_{2ij}$ and $\left(\nu'_{2xy}\right)^{\infty} \geq \nu_{2ij} \geq \nu^{\infty}_{2ij}$ (or)

$\left(\mu'_{2xy}\right)^{\infty} \leq \mu_{2ij} \leq \mu^{\infty}_{2ij}$ and $\left(\nu'_{2xy}\right)^{\infty} > \nu_{2ij} > \nu^{\infty}_{2ij}$ for some $v_x, v_y \in V$

In other words, (v_i, v_j) is a bridge in G whose removal reduces the strength of the connectedness between some pair of vertices in G.

Example 3.2:

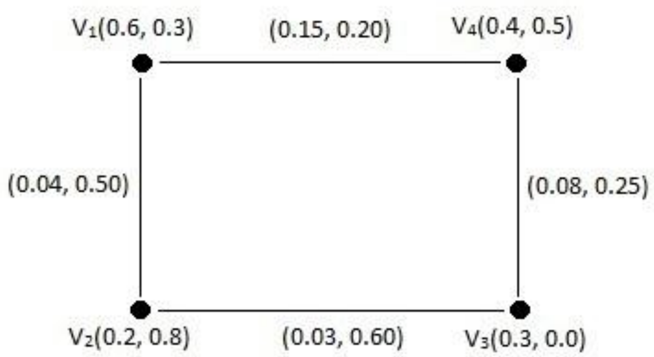

FIGURE 2.

In the above FIGURE 2.

The strength of the path(P) v_1v_4 is (0.15,0.20) and also the strength of the path(P) $v_1v_2v_3v_4$ is (0.08,0.25)

Clearly, we can see that when we remove an edge v_1v_4 then the strength of the connectedness between v_1 and v_4 in the path P is reduced.

Hence v_1v_4 is a bridge.

Definition 3.6: Let $G = [V, E]$ be an IFGST. Let v_i, v_jbe any two distinct vertices and $H = [V', E']$ be an intuitionistic fuzzy subgraph of G obtained by deleting an edge (v_i, v_j).

So, $H = [V', E']$ where $\mu'_{2ij} = 0$ and $\nu'_{2ij} = 0$ and $\mu'_{2xy} = \mu_{2xy}$ and $\nu'_{2xy} = \nu_{2xy}$ for all other edges (v_x, v_y)

A vertex v_i is said to be a cut-vertex in G whose removal reduces the strength of connectedness between some pair of vertices.

In other words,

$\left(\mu'_{2xy}\right)^{\infty} < \mu_{2ij} < \mu^{\infty}_{2ij}$ and $\left(\nu'_{2xy}\right)^{\infty} \geq \nu_{2ij} \geq \nu^{\infty}_{2ij}$ (or)

$\left(\mu'_{2xy}\right)^{\infty} \leq \mu_{2ij} \leq \mu^{\infty}_{2ij}$ and $\left(\nu'_{2xy}\right)^{\infty} > \nu_{2ij} > \nu^{\infty}_{2ij}$ for some $v_x, v_y \in V$

Example 3.3:

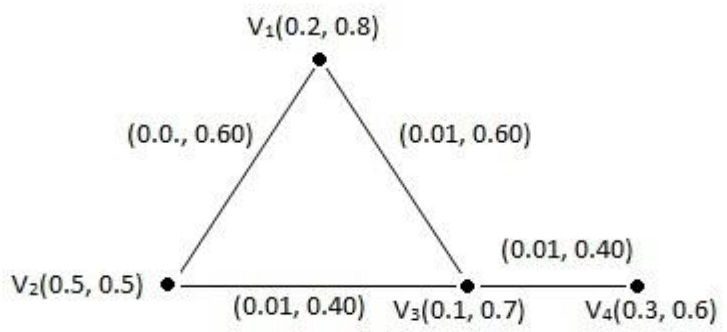

FIGURE 3.

In the above FIGURE 3.

The strength of the path(P) $v_1v_2v_3$ is (0.03,0.60) and also the strength of the path(P) v_1v_3 is (0.01,0.60)

Clearly, we can see that when we remove a vertex v_2 then the strength of the connectedness between v_1 and v_3 in the path P is reduced.

Hence v_2 is a cut-vertex.

Theorem 3.1: In an IFGST, $G = [V, E]$, after deleting an edge (v_i, v_j) we have an IFGST , $G' = [V', E']$ of vertices v_x, v_y $(x, y = 1,2, \dots n)$ then the following conditions are equivalent

(i) $\left(\mu'_{2xy}\right)^{\infty} < \mu_{2ij}$ and $\left(\nu'_{2xy}\right)^{\infty} > \nu_{2ij}$

(ii) (v_i, v_j) is a bridge

(iii) (v_i, v_j) is not an edge of any cycle.

Proof:

To prove (i) ⇒ (ii)

Assume that $\left(\mu'_{2xy}\right)^{\infty} < \mu_{2ij}$ and $\left(\nu'_{2xy}\right)^{\infty} > \nu_{2ij}$

Now we have to prove (v_i, v_j) is a bridge.

Suppose that (v_i, v_j) is not a bridge, then

$\left(\mu'_{2xy}\right)^{\infty} = \mu^{\infty}_{2ij} \geq \mu_{2ij}$ and $\left(\nu'_{2xy}\right)^{\infty} = \nu^{\infty}_{2ij} \leq \nu_{2ij}$

This is contradiction to (i)

Hence (v_i, v_j) is a bridge.

To prove (ii) ⇒ (iii)

Assume that (v_i, v_j) is a bridge.

To prove (v_i, v_j) is not an edge of any cycle.

If (v_i, v_j) is an edge of a cycle, then any path involving an edge (v_i, v_j) by using the rest of the cycle as a path from v_i to v_j

This is contradiction to our assumption.

Therefore (v_i, v_j) is not an edge of any cycle.

To prove (iii) ⇒ (i)

Assume that (v_i, v_j) is not an edge of any cycle.

To prove $\left(\mu'_{2xy}\right)^{\infty} < \mu_{2ij}$ and $\left(\nu'_{2xy}\right)^{\infty} > \nu_{2ij}$

Suppose that $\left(\mu'_{2xy}\right)^{\infty} \geq \mu_{2ij}$ and $\left(\nu'_{2xy}\right)^{\infty} \leq \nu_{2ij}$ then there is a path from v_i to v_j

not involving (v_i, v_j) that has strength greater than or equal to μ_{2ij} and less than or equal to ν_{2ij} . Also this path together with (v_i, v_j) form a cycle. This is contradiction to our assumption.

Hence $\left(\mu'_{2xy}\right)^{\infty} < \mu_{2ij}$ and $\left(\nu'_{2xy}\right)^{\infty} > \nu_{2ij}$

Therefore (i),(ii) and (iii) are equivalent.

Theorem 3.2: Let IFGST, $G = [V, E]$ then

(i) If μ_{2ij} and ν_{2ij} are constants for all $v_i, v_j \in V$, then G has no bridge.

(ii) If μ_{2ij} and ν_{2ij} are not constants for all $v_i, v_j \in V$, then G has at least one bridge.

Proof:

(i) Let μ_{2ij} and ν_{2ij} are constants for all $v_i, v_j \in V$

Let $\mu_{2ij} = c_1$ and $\nu_{2ij} = c_2$ for all $v_i, v_j \in V$, where $0 \leq c_1 \leq 1$ and $0 \leq c_2 \leq 1$.

In this case each edge in a graph has the same degree of membership and non-membership values. Suppose we can delete any edge is does not reduce the strength of connectedness between any pair of vertices.

Hence G has no bridge.

(ii) Assume that μ_{2ij} and ν_{2ij} are not constants for all $v_i, v_j \in V$

Choose an edge $(v_x, v_y) \in E$ such that $\mu_{2xy} = \max\{\mu_{2ij}\}$ and $\nu_{2xy} = \min\{\nu_{2ij}\}$ for all

$(v_i, v_j) \in E$

Now, there exists at least one edge (v_s, v_t) distinct from (v_x, v_y) such that $\mu_{2st} < \mu_{2xy}$ and $\nu_{2st} > \nu_{2xy}$

If we delete an edge (v_x, v_y) then the strength of connectedness between v_x and v_y in G is decreased.

Therefore (v_x, v_y) is a bridge in G.

4. CONCLUSION:

In this paper, we defined the path, bridge and cut-vertex on IFGST and established some of their properties. In future we will study some more properties and applications of IFGST.

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