

Chemical Reaction and Soret effect on MHD free Convective Flow past an Infinite Vertical Porous Plate with Variable Suction

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Abstract

This paper deals with the study of chemical reaction and soret effects on unsteady MHD free convective mass transfer flow past an infinite vertical porous plate with variable suction, where the plate temperature oscillates with the same frequency as that of variable suction velocity. The governing boundary layer equations are solved analytically using two-term harmonic and non-harmonic functions. The effects of various thermal physical parameters on the velocity, temperature, concentration and skin-friction coefficient are discussed in detail through graphs.

Key Words: MHD, porous plate, chemical reaction, soret, variable suction

1. INTRODUCTION:

In recent years, the problems of free convective heat and mass transfer flows through a porous medium under the influence of a magnetic field have been attracted the attention of a number of researchers because of their possible applications in many branches of science and technology, such as its applications in transportation cooling of reentry vehicles and rocket boosters, cross-hatching on ablative surfaces and film

vaporization in combustion chambers. On the other hand, flow through a porous medium have numerous engineering and geophysical applications, for example, in chemical engineering for filtration and purification process; in agriculture engineering to study the underground water resources; in petroleum technology to study the movement of natural gas, oil and water through the oil reservoirs. In view of these applications, many researchers have studied MHD free convective heat and mass transfer flow in a porous medium; some of them are Raptis and Kafoussias [1], Sattar [2] and Kim [3]. Due to the importance of Soret (thermal-diffusion) and Dufour (diffusion-thermo) effects for the fluids with very light molecular weight as well as medium molecular weight many investigators have studied and reported results for these flows of whom the names are Eckert and Drake [4], Dursunkaya and Worek [5], Anghel et al. [6], Postelnicu [7] are worth mentioning. Alam and Rahman [8] studied the Dufour and Soret effects on steady MHD free convective heat and mass transfer flow past a semi-infinite vertical porous plate embedded in a porous medium.

Many practical diffusive operations, the molecular diffusion of a species are involved in the presence of chemical reaction within or at the boundary. There are two types of reactions i.e. homogeneous and heterogeneous reactions. A homogeneous reaction occurs uniformly throughout a given phase. In such type of reaction the species generation is analogous to internal source of heat generation. In contrast, a heterogeneous reaction takes place in a restricted region or within the boundary of a phase. It is so treated as a boundary condition similar to the constant heat flux condition in heat transfer. All industrial chemical processes are so designed that the cheaper raw materials can be transformed to high value products by chemical reaction. For a specific chemistry, the reactor performance is a complex function of the underlying transport processes. An analysis of the transport processes and their interaction with chemical reactions are quite difficult and is directly connected to the underlying fluid dynamics. Moreover, the chemical reaction, heat and mass transfer on MHD flow over a vertical stretching surface with heat source and thermal stratification have been presented by Kandaswamy et al. [9]. Al-Odat and Al-Azab [10] studied the influence of magnetic field on unsteady free convective heat and mass transfer flow along an impulsively started semi-infinite vertical plate taking into account a homogeneous chemical reaction of first order. The effect of radiation on the heat and fluid flow over an unsteady stretching surface has been analyzed by El-Aziz [11]. J Prakash et.al [12] has studied the thermo diffusion and chemical reaction effects on MHD three dimensional free convective couette flow. Singh et. al. [13] investigated MHD oblique stagnation-point flow towards a stretching sheet with heat transfer for steady and unsteady cases. Elbashbeshy et. al. [14] investigated the effects of thermal radiation and magnetic field on unsteady boundary layer mixed convection flow and heat transfer problem from a vertical porous stretching surface. The opposing buoyancy effects on simultaneous heat and mass transfer by natural convection in a fluid saturated porous medium investigated by Angirasa et.al. [15]. Ahmed [16] investigates the effects of unsteady free convective MHD flow through a porous medium bounded by an infinite vertical porous plate. Mythreye et al. [17] analyzed the effect of chemical reaction on unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption.

In this paper, an analytical study was performed to investigate the effects of chemical reaction and soret on the unsteady MHD free convective mass transfer flow past an infinite vertical porous plate with variable suction. The governing equations of motion are solved analytically by using a regular perturbation technique. The behaviour of velocity, temperature, concentration and skin-friction coefficient for different values of thermo-physical parameters has been computed and the results are presented graphically and discussed in detail.

2. FORMATION OF THE PROBLEM:

Unsteady flow of an incompressible, electrically conducting viscous fluid past an infinite vertical porous plate embedded under the influence of a uniform transverse magnetic field is considered. Here the origin of the coordinate system is taken to be at any point of the plate. Let the components of velocity along x and y axes are u, v and which are chosen in the upward direction along the plate and normal to the plate respectively. The polarization effects are assumed to be negligible. Hence the governing equations of the problem are:

$$\frac{\partial v'}{\partial y'} = 0 \quad (1)$$

$$\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} = g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) + \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\nu}{k'} u' - \frac{\sigma\beta_0^2}{\rho'} u' \quad (2)$$

$$\frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} = \frac{k_r}{\rho' c_p'} \frac{\partial^2 T'}{\partial y'^2} \quad (3)$$

$$\frac{\partial C'}{\partial t'} + v' \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2} + D_1 \frac{\partial^2 T'}{\partial y'^2} - K_r (C' - C'_\infty) \quad (4)$$

The corresponding boundary conditions are:

$$t > 0 \quad u' = 0, \quad T' = T'_w = 1 + \varepsilon e^{i\omega t}, \quad C' = C'_w \quad \text{at} \quad y' = 0 \quad (5)$$

$$u' \rightarrow 0, \quad T' \rightarrow T'_\infty, \quad C' \rightarrow C'_w \quad \text{as} \quad y' \rightarrow \infty$$

From the continuity equation, it can be seen that v' is either a constant or a function of time. So, assuming suction velocity to be oscillatory about a non-zero constant mean, one can write

$$v' = -v_0(1 + \varepsilon A e^{i\omega t}) \quad (6)$$

Where v_0 is the mean suction velocity and ε, A are small such that $\varepsilon A \ll 1$. The negative sign indicates that the suction velocity is directed towards the plate. In order to write the governing equations and the boundary conditions in dimensional following non-dimensional quantities are introduced.

$$y = \frac{v_0 y'}{v}, \quad u = \frac{u'}{v_0}, \quad t = \frac{t' v_0^2}{4v}, \quad T = \frac{(T' - T'_\infty)}{(T'_w - T'_\infty)}, \quad C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \quad Sc = \frac{v}{D}, \quad v = \frac{v'}{v_0},$$

$$Gr = \frac{g\beta v(T'_w - T'_\infty)}{v_0^3}, \quad Gc = \frac{g\beta^* v(C'_w - C'_\infty)}{v_0^3}, \quad \omega = \frac{4v\omega'}{v_0^2}, \quad M = \frac{\sigma B_0^2 v}{\rho v_0^2}, \quad K = \frac{K' v_0^2}{v^2},$$

$$So = \frac{D_1(T'_w - T'_\infty)}{v(C'_w - C'_\infty)}, \quad Pr = \frac{\mu Cp}{K_T} \quad (7)$$

Hence, using the above non - dimensional quantities, the equations (2)-(5) in the non-dimensional form can be written as

$$\frac{1}{4} \left(\frac{\partial u}{\partial t} \right) - (1 + \varepsilon A e^{i\omega t}) \frac{\partial u}{\partial y} = GrT + GcC + \frac{\partial^2 u}{\partial y^2} - \left(M + \frac{1}{K} \right) u \quad (8)$$

$$\frac{1}{4} \left(\frac{\partial T}{\partial t} \right) - (1 + \varepsilon A e^{i\omega t}) \frac{\partial T}{\partial y} = \frac{1}{Pr} \frac{\partial^2 T}{\partial y^2} \quad (9)$$

$$\frac{1}{4} \left(\frac{\partial C}{\partial t} \right) - (1 + \varepsilon A e^{i\omega t}) \frac{\partial C}{\partial y} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} + So \frac{\partial^2 T}{\partial y^2} - KrC \quad (10)$$

The corresponding boundary conditions are

$$t > 0, \quad u = 0, T = T'_w = 1 + \varepsilon e^{i\omega t}, C = 1 \quad \text{at} \quad y = 0$$

$$u \rightarrow 0, T \rightarrow 0, C \rightarrow 0 \quad \text{as} \quad y \rightarrow \infty \quad (11)$$

Here t is the time, g the acceleration due to gravity, β the coefficient of volume expansion, β^* the coefficient of thermal expansion with concentration, T temperature of the fluid, T'_∞ the temperature of the fluid far away from the plate, C the species concentration, C'_∞ the species concentration of the fluid far away from the plate, T'_w the plate temperature, C'_w the species concentration near the plate, v the kinematic viscosity, ρ the density, C_p the specific heat at constant pressure, K_T the thermal conductivity, D the chemical molecular diffusivity, μ the coefficient of viscosity, M the magnetic field parameter, So the soret number, Sc the Schmidt number and chemical reaction parameter Kr , all the physical quantities have their usual meaning.

3. SOLUTION OF THE PROBLEM:

In order to reduce the above system of partial differential equation to a system of ordinary differential equations, the velocity, temperature and concentration in the neighbourhood of the porous plate are taken as

$$\begin{aligned} u(y,t) &= u_0(y) + \varepsilon e^{i\omega t} u_1(y) + O(\varepsilon^2) \\ T(y,t) &= T_0(y) + \varepsilon e^{i\omega t} T_1(y) + O(\varepsilon^2) \end{aligned} \quad (12)$$

$$C(y,t) = C_0(y) + \varepsilon e^{i\omega t} C_1(y) + O(\varepsilon^2)$$

Substituting equation (12) in equations (8) to (10) and equating the harmonic and non-harmonic terms, and neglecting the higher order terms of $O(\varepsilon^2)$, we obtain

$$u_0'' + Lu_0' - \left(M + \frac{1}{K} \right) u_0 = -GrT_0 - GcC_0 \quad (13)$$

$$u_1'' + Lu_1' - \left(\frac{i\omega}{4} + M + \frac{1}{K} \right) u_1 = -GrT_1 - GcC_1 \quad (14)$$

$$T_0'' + LPrT_0' = 0 \quad (15)$$

$$T_1'' + LPrT_1' - \left(\frac{i\omega Pr}{4} \right) T_1 = 0 \quad (16)$$

$$C_0'' + LScC_0' - ScKrC_0 = -SoSc(L^2 Pr^2 e^{-LPr y}) \quad (17)$$

$$C_1'' + LScC_1' - \left(\frac{Sci\omega}{4} + ScKr \right) C_1 = -SoScT_1'' \quad (18)$$

And the corresponding boundary conditions are

$$\begin{aligned} t > 0, \quad u_0 = u_1 = 0, \quad T_0 = T_1 = 1, \quad C_0 = 1, \quad C_1 = 0 \quad \text{at } y = 0 \\ u_0 = u_1 \rightarrow 0, \quad T_0 = T_1 \rightarrow 0, \quad C_0 = C_1 \rightarrow 0 \quad \text{as } y \rightarrow \infty \end{aligned} \quad (19)$$

Solving the above equations (13) to (18) and using boundary conditions (19), the solution of equations are expressed as

$$\begin{aligned} u(y,t) &= -(A_3 + A_4 + A_5)e^{-m_8 y} + (A_3 + A_5)e^{-LPr y} + A_4 e^{-m_4 y} + \varepsilon e^{i\omega t} (-(A_6 + A_7 + A_8)e^{-m_{10} y} \\ &+ (A_6 + A_8)e^{-m_2 y} + A_7 e^{-m_6 y}) \end{aligned} \quad (20)$$

$$T(y,t) = e^{-LPr y} + \varepsilon e^{i\omega t} e^{-m_2 y} \quad (21)$$

$$C(y,t) = (1 - A_1)e^{-m_4 y} + A_1 e^{-LPr y} + \varepsilon e^{i\omega t} A_2 (e^{-m_2 y} - e^{-m_6 y}) \quad (22)$$

Skin-friction:

$$\left(\frac{\partial u}{\partial y}\right)_{y=0} = m_8(A_3 + A_4 + A_5) - L \text{Pr}(A_3 + A_5) - m_4 A_4 + \varepsilon(m_{10}(A_6 + A_7 + A_8) - m_2(A_6 + A_8) - m_6 A_7) e^{i\omega t} \quad (23)$$

Where

$$L = 1 + \varepsilon A e^{i\omega t}, \quad m_2 = \frac{\text{Pr} L + \sqrt{\text{Pr}^2 L^2 + i\omega \text{Pr}}}{2}, \quad m_4 = \frac{L \text{Sc} + \sqrt{L^2 \text{Sc}^2 + 4 \text{Sc} K r}}{2},$$

$$N = M + \frac{1}{\left(K + \frac{i\omega}{4}\right)}, \quad A_1 = \frac{\text{SoSc} L^2 \text{Pr}^2}{-L^2 \text{Pr}^2 + L^2 \text{Sc} \text{Pr} + \text{Sc} K r},$$

$$m_6 = \frac{L \text{Sc} + \sqrt{L^2 \text{Sc}^2 + 4 \text{Sc} \left(\frac{i\omega}{4} + K r\right)}}{2},$$

$$A_2 = \frac{\text{SoSc} m_2^2}{-m_2^2 + L \text{Sc} m_2 + \left(\frac{\text{Sc} i\omega}{2} + \text{Sc} K r\right)}, \quad A_3 = \frac{\text{Gr}}{-L^2 \text{Pr}^2 + L^2 \text{Pr} + \left(M + \frac{1}{K}\right)},$$

$$A_4 = \frac{\text{Gc}(1 - A_1)}{-m_4^2 + L m_4 + \left(M + \frac{1}{K}\right)}, \quad m_{10} = \frac{L + \sqrt{L^2 + 4N}}{2}, \quad A_6 = \frac{\text{Gr}}{-m_2^2 + m_2 L + N},$$

$$A_7 = \frac{\text{Gc} A_2}{m_6^2 - L m_6 - N}, \quad A_8 = \frac{A_2}{-m_2^2 + L m_2 + N}, \quad m_8 = \frac{L + \sqrt{L^2 + 4\left(M + \frac{1}{K}\right)}}{2}$$

4. RESULT AND DISCUSSION:

To assess the physical depth of the problem, the effects of various parameters like Schmidt number Sc, thermal Grashof number Gr, magnetic parameter M, chemical reaction parameter Kr, mass Grashof number Gc, solet effect number So and prandtl number Pr on velocity, temperature and concentration distribution are studied in figures 1-12, the numerical values of Prandtl number Pr are chosen as Pr = 0.71, and Pr = 7, which corresponds to air and water at 20°C respectively. The numerical values of the remaining parameters are chosen arbitrarily.

Figure 1 presents typical velocity profiles in the boundary layer for various values of the thermal Grashof number. The thermal Grashof number defines the ratio of the species buoyancy force to the viscous hydrodynamic force. As expected, the fluid

velocity increases and the peak value is more distinctive due to increase in the species buoyancy force. The influence of the Solutal Grashof number G_c on the velocity is presented in Figure 2. The Solutal Grashof number signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. As expected, it is observed that there is a rise in the velocity due to the enhancement of thermal buoyancy force. Here, the positive values of G_c correspond to cooling of the plate. Also, as G_c increases, the peak values of the velocity increases rapidly near the porous plate and then decays smoothly to the free stream velocity. Figure 3 displays the influence of thermodiffusion (soret) parameter over the dimensionless velocity. It is evident that the thermodiffusion parameter accelerates the velocity of the flow field. Figures 4 and 5 displays the effects of the Schmidt number (Sc) on the velocity and concentration profiles, respectively. As the Schmidt number increases, the concentration decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. The reduction in the velocity and concentration profiles are accompanied by simultaneous reductions in the momentum and concentration boundary layers thickens. These behaviors are clearly shown in Figures 4 and 5.

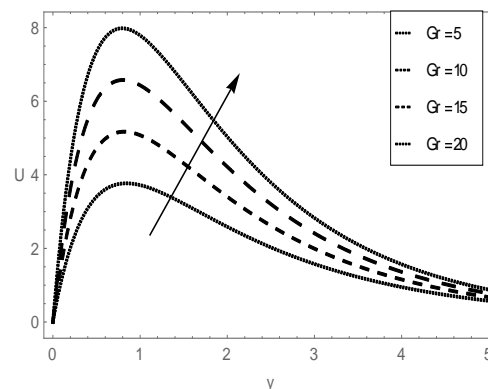


Fig.1: Effects of thermal Grashof number Gr on velocity

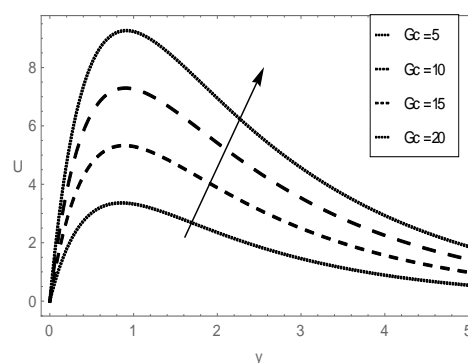


Fig.2: Effects of mass Grashof number G_c on velocity

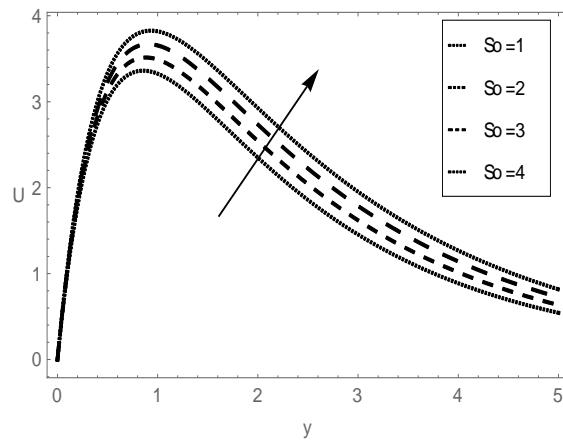


Fig.3: Effects of Soret number So on velocity

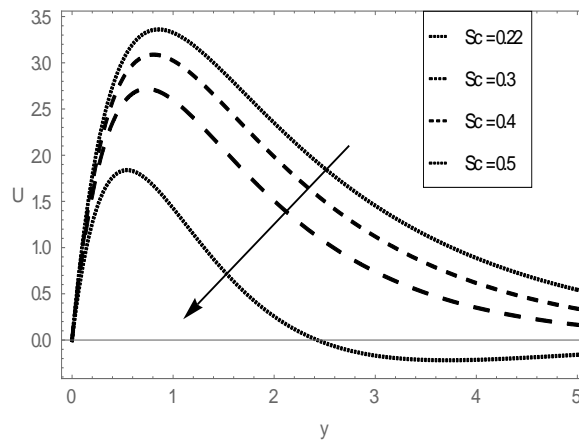


Fig.4: Effects of Schmidt number Sc on velocity

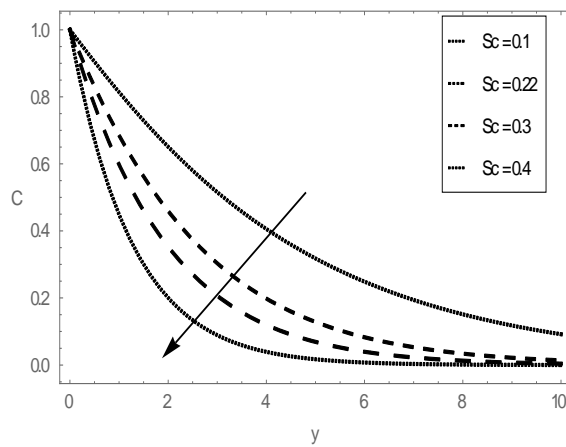


Fig.5: Effects of Schmidt number Sc on Concentration

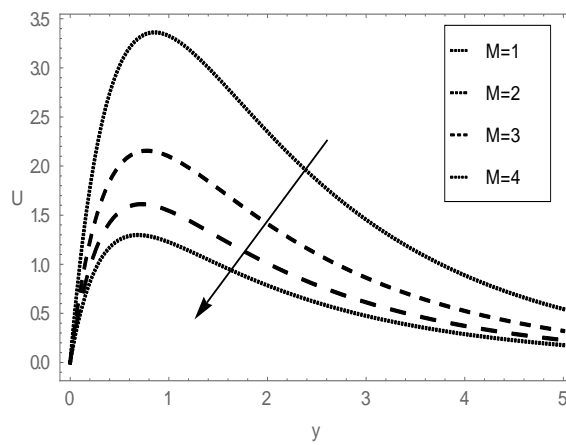


Fig.6: Effects of magnetic parameter M on velocity

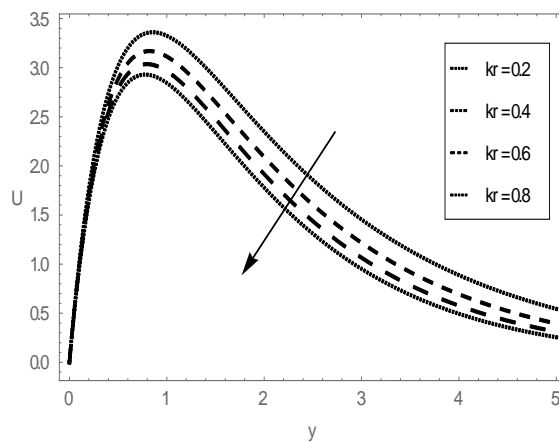


Fig.7: Effects of chemical reaction parameter Kr on velocity

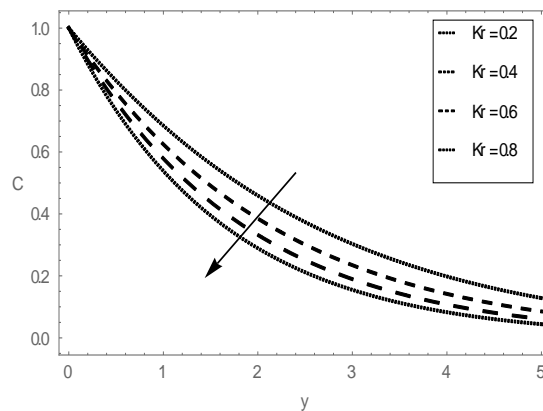


Fig.8: Effects of Chemical reaction parameter Kr on Concentration

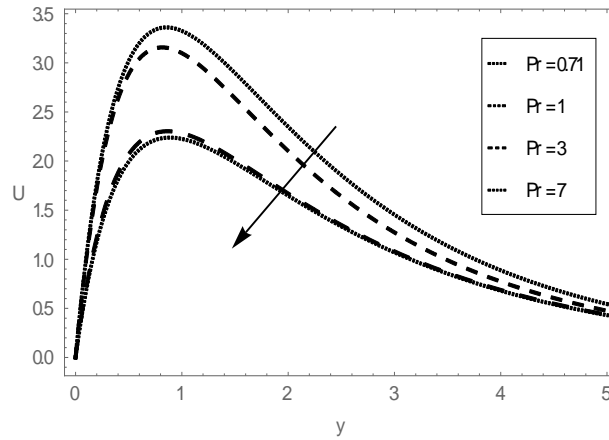


Fig.9: Effects of Prandtl number Pr on velocity

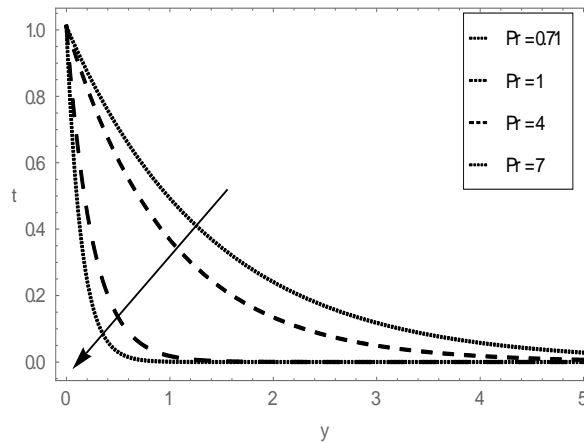


Fig.10: Effects of Prandtl number Pr on temperature

For different values of the magnetic field parameter M , the velocity profiles are plotted in Figure. 6. It is obvious that the effect of increasing values of the magnetic field parameter results in a decreasing velocity distribution across the boundary layer. Figures 7 and 8, display the effects of the chemical reaction parameter (Kr) on the velocity and concentration distributions respectively. It is seen, that the velocity and concentration decreases with increasing the chemical reaction parameter. Figures 9 and 10 illustrate the velocity and temperature profiles for different values of the Prandtl number Pr . The Prandtl number defines the ratio of momentum diffusivity to thermal diffusivity. The numerical results show that the effect of increasing values of Prandtl number results in a decreasing velocity (Figure 9). From Figure 10, it is observed that an increase in the Prandtl number results a decrease of the thermal boundary layer thickness. The reason is that smaller values of Pr are equivalent to increasing the thermal conductivities, and therefore heat is able to diffuse away from

the heated plate more rapidly than for higher values of Pr. Hence in the case of smaller Prandtl numbers as the boundary layer is thicker and the rate of heat transfer is reduced.

5. CONCLUSIONS:

The governing equations for unsteady MHD convective mass transfer past an infinite vertical porous plate with variable suction were formulated. The plate temperature oscillates with the same frequency as that of variable suction velocity with the chemical reaction effects. The computed values obtained from analytical solutions for the velocity, temperature, concentration fields and skin-friction coefficient are presented through graphs. In the absence of chemical reaction ($Kr=0$), these results are in good agreement with the results of Seethamahalakshmi et al. [18].

We conclude the following after analyzing the graphs:

1. The velocity decreases with increasing the Prandtl number, magnetic field parameter, chemical reaction parameter and Schmidt number whereas reverse trend is seen with increasing the Soret number, thermal and solutal Grashof numbers.
2. The temperature decreases as the values of Prandtl number increase.
3. The concentration decreases as the values of the chemical reaction parameter and Schmidt number increases.

REFERENCES:

- [1] Raptis, N.G. Kafoussias (1982), Magneto hydrodynamic free convection flow and mass transfer through porous medium bounded by an infinite vertical porous plate with constant heat flux, *Can. J. Phys.*, **60**(12), 1725–1729.
- [2] Sattar M.A. (1993), Unsteady hydromagnetic free convection flow with Hall current mass transfer and variable suction through a porous medium near an infinite vertical porous plate with constant heat flux, *Int. Energy Research*, **17**, 1-5.
- [3] Kim (2004), Heat and mass transfer in MHD micropolar flow over a vertical moving porous plate in a porous medium, *Transport in Porous Media*, **56**(1), 17–37, 2004.
- [4] E.R.G. Eckert, R.M. Drake, *Analysis of Heat and Mass Transfer*, McGraw-Hill, New York, 1972.
- [5] Dursunkaya, Z W.M. (1992) Worek, Diffusion-thermo and thermal-diffusion effects in transient and steady natural convection from vertical surface, *Int. J. Heat Mass Transfer*, **35**(8), 2060–2065.
- [6] M. Anghel, H. S. Takhar, I. Pop,(200). Dufour and Soret effects on free-convection boundary layer over a vertical surface embedded in a porous medium, *Studia Universities Babes-Bolyai, Mathematica*, **XLV**(4),11–21.

- [7] Postelnicu, A (2004), Influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects, *Int. J. Heat Mass Transfer*, **47**(6–7), 1467–1472.
- [8] Alam, M. S. and M.M. Rahman (2005). Dufour and Soret effects on MHD free convective heat and mass transfer flow past a vertical flat plate embedded in a porous medium, *J. Naval Architecture and Marine Engng.*, **2**(1), 55–65.
- [9] Kandasamy, R., Periasamy, K. and Sivagnana Prabhu, K.K (2005). Chemical reaction, heat and mass transfer on MHD flow over a vertical stretching surface with heat source and thermal stratification effects, *Int. J. Heat and Mass Transfer*, **48** (21-22), 4557-4561.
- [10] Al-Odat, M. Q. and Al-Azab, (2007). Influence of chemical reaction on transient MHD free convection over a moving vertical plate, *Emirates J. Engg. Res.*, **12** (3), 15-21.
- [11] El-Aziz M.A. (2009), Radiation effect on the flow and heat transfer over an unsteady stretching sheet, *International Communications in Heat and Mass Transfer*, **36**, 521-524.
- [12] Prakash J, K.S. Balamurugan and S.V.K. Varma (2015). Thermodiffusion and Chemical reaction effects on MHD three dimensional free convective couette flow, *Walailak Journal of Science and Technology*, **12**(9) 805-830.
- [13] Singh, P, N.S. Tomer, S. Kumar and D. Sinha, (2010). MHD oblique stagnation point flow towards a stretching sheet with heat transfer, *International Journal of Applied Mathematics and Mechanics*, **6**(13), 94-111.
- [14] Elbashbeshy, E. M. A. and D. M. Yassmin and A. A. Dalia, (2010). Heat transfer over an unsteady porous stretching surface embedded in a porous medium with variable heat flux in the presence of heat source or sink, *African Journal of Mathematics and Computer Science Research*, **3**(5), 68-73
- [15] Angirasa, D., Peterson, G. P. and Pop, (1997). Combined heat and mass transfer by natural convection with opposing buoyancy effects in a fluid saturated porous medium, *Int. J. Heat Mass Trans.*, **40**(12), 2755-2773.
- [16] Ahmed, S (2007). Effects of unsteady free convective MHD flow through a porous medium bounded by an infinite vertical porous plate, *Bull. Cal. Math. Soc.*, **99** (5), 511-522.
- [17] Mythreye, A, J.P. Pramod and K.S. Balamurugan, (2015), Chemical Reaction on unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption, *Procedia Engineering* **127**, 613-620.
- [18] Seethamahalakshmi, B.D.C.N. Prasad and G.V. Ramana Reddy (2012). MHD free convective mass transfer flow past an infinite vertical porous plate with variable suction and soret effect, *Asian J. of Current Engg.* **1** (2), 49-55.