

Mobile Robot Path Navigation in Static Indoor Environment via AOR 9-Point Laplacian Iteration Numerical Technique

W.K. Ling^{1*}, A. A. Dahalan², and A. Saudi³

^{1,2} *Department of Mathematics, Centre for Defence Foundation Studies, National Defence University of Malaysia, 57000, Kuala Lumpur, Malaysia.*

³ *Research & Innovation Management Centre, Universiti Malaysia Sabah, Kota Kinabalu, 88999, Sabah, Malaysia.*

Abstract

Mobile robot path navigation is a crucial subject in robotics research and development. The navigation efficiency of a robot correlates directly with its overall performance. With persistent technological growth, the improvement potentials for autonomous robotic remains vast. The objective of this paper is to investigate the performance of the iteration technique called Accelerated Overrelaxation 9-Point (AOR-9P) Laplacian in enabling path navigation for mobile robots. This technique is a derivation from Laplace's equation which is used to calculate the potential fields in the 2-dimensional configuration space representation of an environment. The robotic path navigations are performed in a simulation called Robot 2D Simulator, written in Delphi Project software. After obtaining the solutions generated through AOR-9P iterative technique, the Gradient Descent Search (GDS) technique is employed to determine the best path for the mobile robot to traverse on. The performance of AOR-9P is examined by comparing the number of iterations needed to complete the navigation process. Results shown that AOR-9P enabled path navigation requires the least number of iterations to complete, thus having better performance than its predecessor techniques. In the same time, the paths produced are generally smooth and unobstructed all the way towards the goal point. For future improvements, it is recommended that the Half-Sweep (HS) and Quarter-Sweep (QS) approach to be introduced on AOR-9P iteration technique to improve its performance in solving the mobile robot path navigation problem.

Keywords: Autonomous path planning, Laplace's equation, Collision free, Optimal path, Nine-point Laplace operator.

*Corresponding author

1. INTRODUCTION

Since prehistoric times, humans have learned to make use of surrounding resources to develop tools for daily use. From sticks and stones, to incredible speed vehicles and even high soaring airplanes, countless inventions have come alight from the creative minds among humans. One class of invention that came along was the mechanical companions, called “robots”. In the early years, they were created to assist humans in performing relatively simple tasks. Naturally, humans would conduct constant research and development to increase robot performances to serve the ultimate goal, that is to push the human living standard to the highest end. At this 21st century, even though there are many high performing robots existing, they are far from being a perfect class of creation. Numerous improvements are still in order as technologies grow. One of the crucial improvements that robots are still undergoing is its ability to navigate its path through an environment. For humans ourselves, to travel from a starting point to a destination, say from one room to another, is but a simple task of taking a few steps and turns while avoiding any obstacles in between. Every measure in human mobility were performed subconsciously by the human brain, thus, making this task seem so trivial on a conscious level. On the other hand, for a robot to get to its destination, vast calculations are required to get the robot to move for even an inch, let alone travelling while avoiding any existing obstacles in between. To move, a precise number of mechanical parts in the robot must operate in harmony to get the robot from stationary to movement, and to make turns, the mechanical parts will have to respond to external commands and make changes in their operations accordingly. To move and to make turns autonomously, robots will have to undergo multiple computations at a time, including environmental analysing and best path selection for reaching goal point. These computations are called the path planning problem of the robot. In this paper, a global numerical approach is considered in order to solve the said path navigation problem. This numerical approach is based on the theory of steady-state heat transfer. Thus, the configuration spaces in which the robot operates in are modelled after Laplace’s equation for heat transfer, and the numerical functions that solves Laplace's equation here are the harmonic functions. The values obtained from the solutions represent the temperature values, and are used in the later stage to generate path for the mobile robot via a technique known as GDS technique. The numerical approach applied in this study is known as the Accelerated Overrelaxation 9-point (AOR-9P) Laplacian technique, and its performance is investigated in computing path for mobile robot in assorted 2-dimensional environment layouts.

2. BRIEF HISTORY OF RELATED WORKS

The application of potential field in determining path navigation in robotics was pioneered by Khatib [1] where a robot arm was formulated into an operational space with the existence of artificial potential field. Through this concept, the obstacles in the operational space exerted repelling force while the goal point exerted attracting force, enabling the robot arm to reach its goal point while successfully avoiding obstacles in between. Later on, Koditschek [2] would find out that at least in some geometry condition, the application of potential function method would successfully guide the end effectors from almost any starting point in the operational

space to its goal point, and that the occurrence of local minima generated through the potential functions would cause premature basin of attraction that would hinder the robot from reaching its goal point.

The occurrence of local minima would certainly pose complications for robotic path planning using potential functions, therefore Connolly et. al. [3] and Akishita et. al. [4] would independently develop global approaches in solving the issue. In their works, they would introduce the application of Laplace's equation into the computation. Harmonic functions would be applied to solve Laplace's equation, and the values obtained from the solutions would be the temperature values, employed as potential values. Dirichlet boundary condition is applied, and obstacles are treated as heat source and are assigned high potential values, while the goal point being the heat sink, possessing the lowest potential value. The path would be generated by using the GDS technique on the potential fields. The result was that smooth path would be generated by following the downward slope of the search result reaching the goal point. Application of harmonic functions were thus proven successful when the computation encountered no local minima while successfully avoiding all obstacles in between. Connolly and Gruppen [5] would go on to show that harmonic functions possess many useful properties when applied in robotics application.

Other notable works involving the use of harmonic functions include study by Waydo and Murray [6] that applied stream function which is similar to harmonic functions in conducting motion planning for vehicles. The works done by Daily and Bevely [7] which demonstrated ways to represent complex shaped obstacles in harmonic potential fields for high speed vehicles. Szulczynski et. al. [8] had also applied harmonic functions on real-time obstacles avoidance for both stationary and moving obstacles.

Traditional techniques in constructing harmonic functions were through techniques such as Jacobi, Gauss-Seidel and Successive Overrelaxation (SOR) [9] technique. Other than for path navigation problems, the iterative scheme of overrelaxations can also be used for image processings [10]. The application of AOR in robotics was recently studied in [11] in which the standard method of 5-point Laplacian is utilized for solving path navigation problem. The experimenting of 9-point Laplacian in solving robotics path navigation was pioneered by Saudi and Sulaiman [12] in which the results shown was that by employing 9-point Laplacian into computation, the path navigation performance of mobile robot was indeed better compared to the 5-point Laplacian technique. A study by Adam et. al. [13] observed that 9-point Laplacian technique converges faster than 5-point Laplacian, plus with a more accurate discretization.

3. METHODOLOGY

This section describes the method in obtaining results for the path searching performance.

3.1. The heat transfer analogy in the configuration space

For this experiment, the heat transfer analogy will be employed. The theory of heat transfer implies that heat will flow from region of higher temperature to region of lower temperature.

Therefore, in modelling the mobile robot path planning problem as a steady-state heat transfer problem, the path generated by the mobile robot shall follow the trajectory such as the heat flow trajectory in the configuration space, which will be known as the C-space from this point onwards. To model the C-space as such, the outer and inner wall boundaries of the C-space, together with the obstacles will be assigned a constant temperature value, whilst the goal point assigned the lowest temperature value. The mobile robot is represented by a point in the C-space. This setup will create a heat conduction process in which there will be temperature distributions, or in the robotics term, potential values [14]. The outer and inner wall boundaries, together with the obstacles will act as heat sources, while the goal point will be the heat sink, pulling the heat closer until it reaches the goal point. This will create heat flux lines which the mobile robot will employ as possible paths and determine which path is the best path, all while avoiding the obstacles in between.

3.2. Harmonic functions

Harmonic functions are the solutions to the Laplace's equation, and the result of the solutions are the potential values that will be used for the mobile robot path generation. Mathematically, a harmonic function on the domain $\Omega \subseteq \mathbb{R}^n$ is a function that satisfies the Laplace's equation,

$$\nabla^2 \phi = \sum_{i=1}^n \frac{\partial^2 \phi}{\partial x_i^2} = 0 \quad (1)$$

in which:

x_i = The i-th Cartesian coordinate

n = The dimension

For the domain in this experiment, within it consists the outer and inner boundary walls, the obstacles, the start point and the goal point. The Dirichlet boundary conditions are applied to cap operating space of the mobile robot in the configuration space. Dirichlet boundary conditions are given as, $\phi|_{\partial\Omega} = c$, in which c is constant.

By applying Laplace's equation into the computation, it constrains the use of harmonic functions, and the spurious creation of local minima can be avoided as the harmonic functions satisfies the min-max principle [15]. Thus, the remaining problem would be the occurrence of stationary points which would produce saddle points in the gradient vector in the computational graph, in turn halting the mobile robot navigation towards the goal point. Therefore, to prevent such complications, the path navigation algorithm will perform search on neighborhood points around the disrupted computational region.

3.3. Formulation of Accelerated Overrelaxation 9-point Laplacian (AOR-9P)

In this paper, a numerical technique called AOR-9P will be tested in its performance to enable a global path navigation for mobile robot. To understand the construction of AOR-9P numerical technique, we first visualize the C-space. The C-space, which is a rectangular 2D

planar space, is divided into small nodes that forms matrix layout for the computation to take place, as shown in Figure 1.

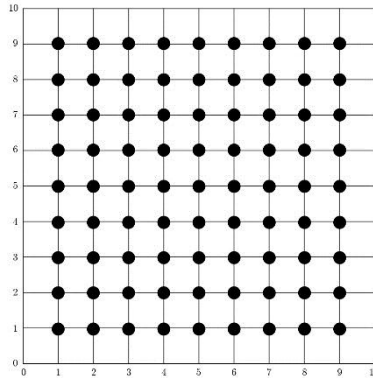


Figure 1. Nodal points arrangement in a C-space portion

In previous study [11], the nodal points computational structure was the standard 5-points structure, as shown in Figure 2(a). By considering more neighboring nodal points into calculation, in this case, 9 nodal points, the iteration performance can be increased. This is because if nodal points computational structure can be expanded, the convergence rate becomes faster because more data yields more accurate computation [13]. The nodal points structure of 9-point Laplacian method is shown in Figure 2(b).

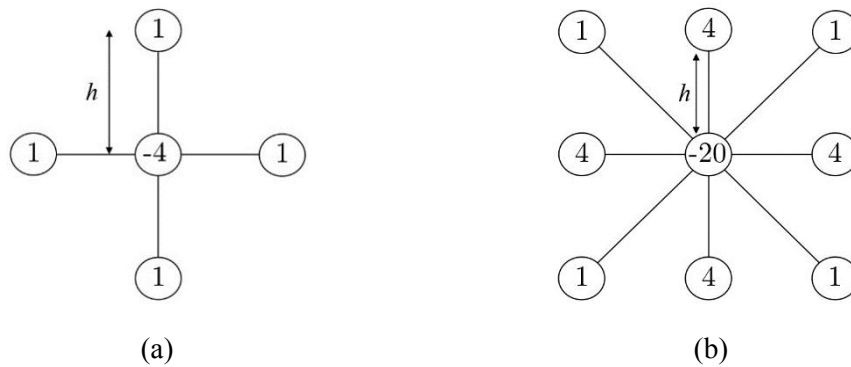


Figure 2. The nodal points set computational structure of (a) 5-point, (b) 9-point, respectively

When placed in the C-space, the mobile robot will be fed information regarding the environment it will traverse in. This information will be used to perform iterations on the nodal points to determine possible paths available. Note that only unoccupied nodal points are considered into computations, this means the nodal points occupied by obstacles will be ignored, making the iterations faster should the obstacles have taken more space in the C-space. The computational structure for both 5-point Laplacian and 9-point Laplacian in the C-space grid are as shown in Figure 3(a) and Figure 3(b), respectively.

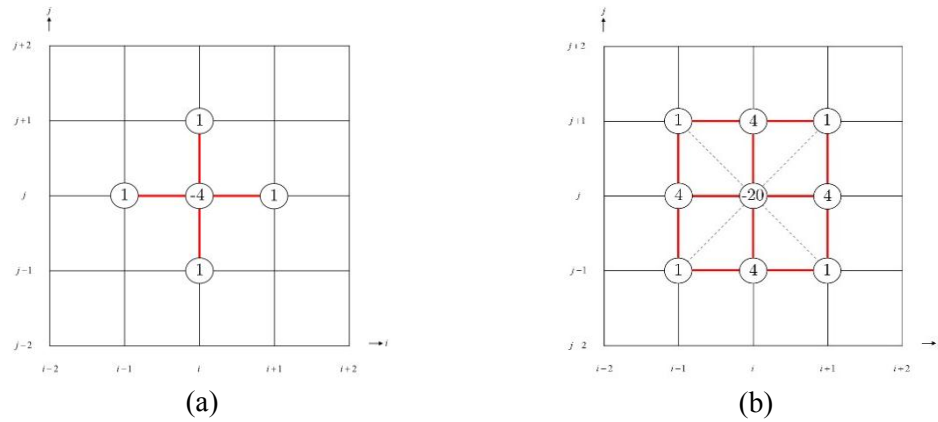


Figure 3. Computational structure of (a) 5-point, (b) 9-point, respectively, in portion C-Space

Now that the computational structure of the nodal points in C-space has been visualized, the next step is to formulate the AOR-9P numerical technique. First, consider the formula of the 2D Laplace's equation given by:

$$\nabla^2 U = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 0 \quad (2)$$

Then, apply second-order central difference scheme to form 9-point standard finite difference approximation equation, thus simplifying (2) to the following:

$$4(U_{i+1,j-1} + U_{i+1,j+1} + U_{i-1,j-1} + U_{i-1,j+1}) + U_{i+1,j} + U_{i,j+1} + U_{i-1,j} + U_{i,j-1} - 20U_{i,j} = 0 \quad (3)$$

A weighted parameter, ω , is extrapolated from (3) to get the formulation of SOR-9P, as following:

$$U_{i,j}^{(k+1)} = \frac{\omega}{5}(U_{i+1,j}^{(k)} + U_{i-1,j}^{(k+1)} + U_{i,j+1}^{(k)} + U_{i,j-1}^{(k+1)}) + \frac{\omega}{20}(U_{i-1,j-1}^{(k+1)} + U_{i+1,j+1}^{(k)} + U_{i-1,j+1}^{(k)} + U_{i+1,j-1}^{(k+1)}) + (1-\omega)U_{i,j}^{(k)} \quad (4)$$

The SOR-9P iteration technique can be further enhanced into an accelerated version of the standard overrelaxation iteration in (4), called AOR-9P, an iterative technique which this paper will investigate. To construct this, another weighted parameter can be extrapolated out of (4), that is the weighted parameter, r . Extrapolation of parameter r from (4) is via replacing $U_{i-1,j}^{(k+1)}$, $U_{i,j-1}^{(k+1)}$, $U_{i-1,j-1}^{(k+1)}$, and $U_{i+1,j-1}^{(k+1)}$, with $U_{i-1,j}^{(k)}$, $U_{i,j-1}^{(k)}$, $U_{i-1,j-1}^{(k)}$, and $U_{i+1,j-1}^{(k)}$ respectively,

while simultaneously adding the terms $\frac{r(U_{i-1,j}^{(k+1)} - U_{i-1,j}^{(k)})}{5}$, $\frac{r(U_{i,j-1}^{(k+1)} - U_{i,j-1}^{(k)})}{5}$, $\frac{r(U_{i-1,j-1}^{(k+1)} - U_{i-1,j-1}^{(k)})}{20}$, and $\frac{r(U_{i+1,j-1}^{(k+1)} - U_{i+1,j-1}^{(k)})}{20}$. Thus, the formula of AOR-9P is obtained as:

$$U_{i,j}^{(k+1)} = \frac{r}{5}(U_{i-1,j}^{(k+1)} + U_{i-1,j}^{(k)} + U_{i,j-1}^{(k+1)} + U_{i,j-1}^{(k)}) + \frac{r}{20}(U_{i-1,j-1}^{(k+1)} + U_{i-1,j-1}^{(k)} + U_{i+1,j-1}^{(k+1)} + U_{i+1,j-1}^{(k)}) + \frac{\omega}{5}(U_{i-1,j}^{(k)} + U_{i+1,j}^{(k)} + U_{i,j-1}^{(k)} + U_{i,j+1}^{(k)}) + \frac{\omega}{20}(U_{i-1,j-1}^{(k)} + U_{i+1,j+1}^{(k)} + U_{i-1,j+1}^{(k)} + U_{i+1,j-1}^{(k)}) + (1-\omega)U_{i,j}^{(k)} \quad (5)$$

Upon formulating equation (5), the linear system solution process commences. Iterations are performed on the linear system until the maximum error of the calculated solutions reach a specified tolerance error region, signaling an indication for the iterations to stop. Note that the optimal values of the weighted parameters ω and r are determined via sensitivity analysis, where the range of their values are $1 \leq \omega, r \leq 2$ [16]. It should also be noted that the maximum error value was kept at minimum, i.e. $9.992 \text{ E-}16$, to avoid flat regions on the calculations from occurring. When the iterations stop, and the potential values are obtained, the mobile robot then performs the GDS technique to form an unobstructed path from the starting point until the goal point. For comparison, performance of AOR-9P will be tested against predecessor techniques, i.e. SOR-5P, SOR-9P, and AOR-5P.

4. EXPERIMENTS AND RESULTS

The experiment to test the performance of AOR-9P technique on mobile robot path navigation is conducted on 4 types of C-Space that contain within them different shapes and numbers of obstacles. The 4 types of C-space are of 3 different pixel sizes, that are 300×300 , 600×600 and 900×900 pixels, as to differentiate the performance of path navigation in different environment sizes. The C-space contains outer and inner boundary walls, obstacles, a starting point and a goal point, where the 2 latter points are chosen arbitrarily within the C-Space. Constant temperature values are assigned to the outer and inner boundary walls, together with the obstacles, while the lowest temperature value is assigned to the goal point. The highest temperature value is set to the starting point. Computations are performed on a computer with Intel Core i5-5200U CPU running at 2.2 GHz, and has 4 GB of RAM specifications. A simulator, called Robot 2D Simulator written in Delphi Project software [17], is used to test the performance of the AOR-9P embedded navigation algorithm. The results produced are shown in Table 1 and Table 2.

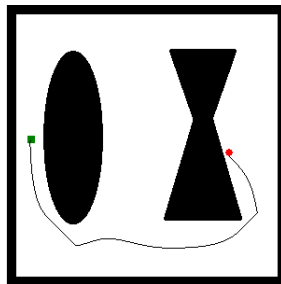
Table 1. Performance of the examined numerical method in terms of number of iterations

Methods		N × N		
		300 × 300	600 × 600	900 × 900
Environment 1	SOR-5P	1312	4185	7398
	SOR-9P	1231	3612	6541
	AOR-5P	1151	3105	5417
	AOR-9P	1113	2805	4771
Environment 2	SOR-5P	2231	4868	10916
	SOR-9P	1878	4272	9667
	AOR-5P	1726	3467	8110
	AOR-9P	1549	3055	7159
Environment 3	SOR-5P	1544	5541	9034
	SOR-9P	1284	4662	7722
	AOR-5P	1093	4081	7560
	AOR-9P	908	3448	6457
Environment 4	SOR-5P	766	2662	4962
	SOR-9P	756	2253	4264
	AOR-5P	748	2129	3884
	AOR-9P	736	1798	3322

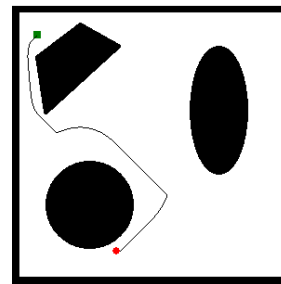
Table 2. Execution results of the examined numerical methods in terms of CPU time (in seconds)

Methods		N × N		
		300 × 300	600 × 600	900 × 900
Environment 1	SOR-5P	1.86	35.00	168.77
	SOR-9P	1.99	32.99	168.01
	AOR-5P	1.86	29.05	137.24
	AOR-9P	2.28	31.47	146.46
Environment 2	SOR-5P	3.23	42.14	247.08
	SOR-9P	3.13	41.08	249.36
	AOR-5P	2.88	33.56	208.82
	AOR-9P	3.23	34.44	222.62
Environment 3	SOR-5P	2.11	47.33	197.27
	SOR-9P	1.98	41.28	190.95
	AOR-5P	1.70	36.49	185.62
	AOR-9P	1.76	36.08	188.60
Environment 4	SOR-5P	1.05	21.19	103.46
	SOR-9P	1.17	19.75	100.79
	AOR-5P	1.17	18.77	91.91
	AOR-9P	1.44	18.74	94.55

As observed in Table 1 and Table 2, iterations performed by the AOR-9P technique has shown better potential in outperforming its predecessors in terms of number of iterations. Meanwhile in CPU time comparison, minor differences are observed between 5-point Laplacian and 9-point Laplacian approach. In the path simulator, AOR-9P also enabled a smooth and unobstructed path from the starting point until the goal point, as shown in Figure 4, with green coloured point as starting point and red coloured point as goal point. The C-space layouts in Figure 4 are based on [18].



(a)



(b)

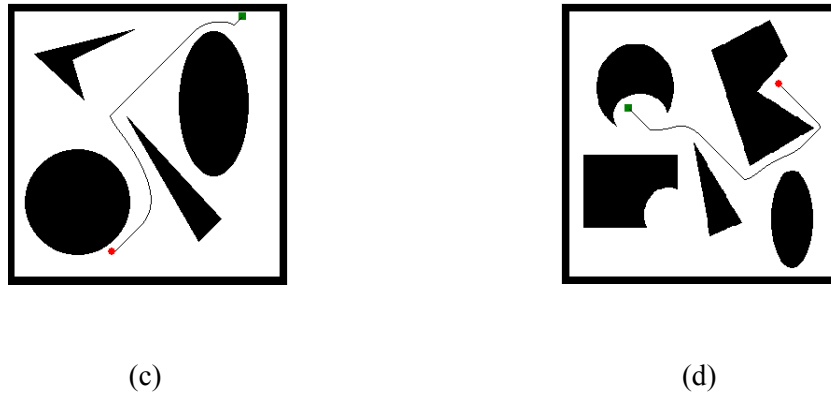
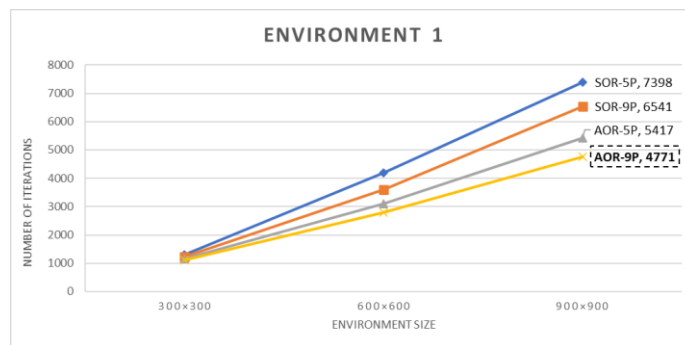
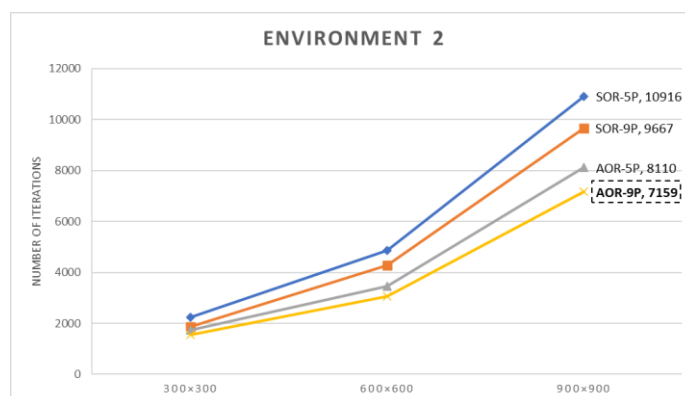


Figure 4. Path lines generated through AOR-9P for (a) Environment 1, (b) Environment 2, (c) Environment 3, and (d) Environment 4

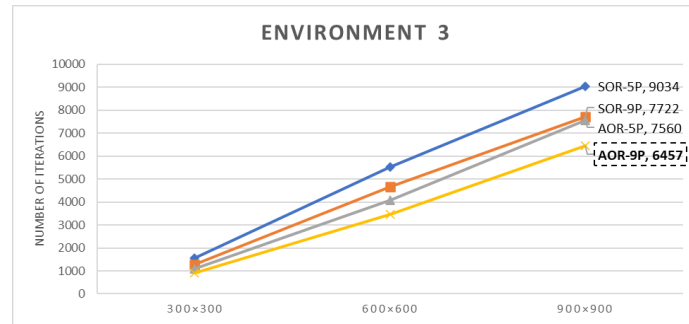
Figure 5 present further visualizations for comparison of the iteration numbers between the iteration techniques. As observed, the performance of the AOR-9P iteration technique is superior to its predecessors in all of the C-space.



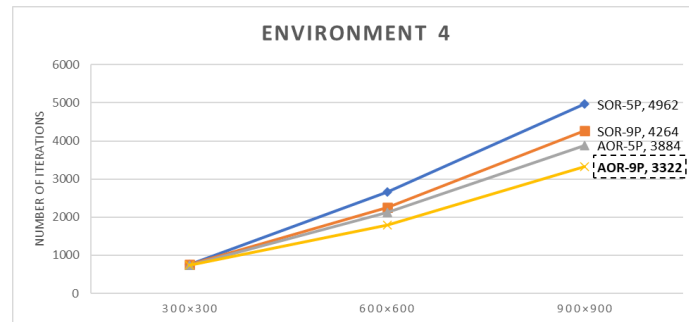
(a)



(b)



(c)



(d)

Figure 5. Comparison of path computation performance of the iteration techniques via number of iterations for (a) Environment 1, (b) Environment 2, (c) Environment 3, and (d) Environment 4.

5. SUMMARY AND FUTURE WORKS

A numerical technique based on Laplace's equation called AOR-9P is tested in solving mobile robot navigation problem. At the end of the experiment, it is shown that AOR-9P has outperformed its predecessors i.e. SOR-5P, SOR-9P and AOR-5P. Assuming optimal operating condition from the mechanical counterparts, if AOR-9P technique is applied in robot navigation, a smooth and unobstructed path is able to be generated and used by the robot to follow through swiftly until it reaches the intended destination within the C-space. It is also noted that if more obstacles are present in the C-space, the iteration process actually speeds up. This is due to the fact that nodal points being occupied by obstacles are ignored in calculation, thus, cost of computations are reduced. For future studies, it is possible to improve the numerical technique by introducing Half-Sweep (HS) [19] and Quarter-Sweep (QS) [20] approach in providing solutions to the Laplace's equation in a more efficient manner.

ACKNOWLEDGEMENT

The authors would like to express gratitude for the support by Ministry of Higher Education (Malaysia) (MOHE) through Fundamental Research Grant Scheme (FRGS/1/2018/ICT02/UPNM/03/1) on this research. The researchers declare that there is no conflict of interest regarding the publication of this study

REFERENCES

- [1] Khatib, O., 1985, "Real-Time Obstacles Avoidance for Manipulators and Mobile Robot," Proc. IEEE Int. Conf. Rob. Autom., 1, pp. 500-505.
- [2] Koditschek, D. E., 1987, "Exact Robot Navigation by Means of Potential Functions: Some Topological Considerations," Proc. IEEE Int. Conf. Rob. Autom., pp. 1-6.
- [3] Connolly, C. I., Burns, J., and Weiss, R., 1990, "Path Searching Using Laplace's Equation," Proc. IEEE Int. Conf. Rob. Autom., 3, pp. 2102-2106.
- [4] Akishita, S., Kawamura, S., and Hayashi, K., 1990, "Laplace Potential for Moving Obstacle Avoidance and Approach of a Mobile Robot," Proc. 1990 Japan-USA Symp. Flexible Autom., 1, pp. 139-142.
- [5] Connolly, C. I., and Gruppen, R. A., 1993, "The Applications of Harmonic Functions to Robotics." J. Rob. Syst., 10(7), pp. 931-946.
- [6] Waydo, S., and Murray, R. M., 2003, "Vehicle Motion Planning Using Stream Functions," Proc. IEEE Int. Conf. Rob. Autom., 2, pp. 2484-2491.
- [7] Daily, R., and Bevly, D. M., 2008, "Harmonic Potential Field Path Searching for High Speed Vehicles," Proc. IEEE Am. Control Conf., pp. 4609-4614.
- [8] Szulczynski, P., Pazderski, D., and Kozlowski, K., 2011, "Real-Time Obstacle Avoidance Using Harmonic Potential Functions," J. Autom. Mobile Rob. Intell. Syst., 5, pp. 59-66.
- [9] Sasaki, S., 1998, "A Practical Computational Technique for Mobile Robot Navigation," Proc. IEEE Int. Conf. Control Appl., 2, pp. 1323-1327.
- [10] Eng, J. H., Saudi, A., and Sulaiman, J., 2018, "Numerical Solutions of Poisson Image Blending Problem using 4-EDGSOR Iteration," J. Comput. Sci. Comput. Math., 8(4), pp. 83-86.
- [11] Dahalan, A. A., Saudi, A., Sulaiman, J., and Din, W. R. W., 2018, "Robot Navigation in Static Indoor Environment via Accelerated Iterative Method," Adv. Sci. Lett., 24, pp. 986-989.
- [12] Saudi, A., and Sulaiman, J., 2010, "Block Iterative Method Using Nine-Point Laplacian for Robot Path Planning," Eur. J. Sci. Res., 43, pp. 204-211.
- [13] Adams, L., Leveque, R., and Young, D., 1988, "Analysis of the SOR Iteration for the 9-Point Laplacian," SIAM J. Numer. Anal., 25(5), pp. 1156-1180.
- [14] Saudi, A., and Sulaiman, J., 2012, "Path Planning for Indoor Mobile Robot Using Half-Sweep SOR via Nine-Point Laplacian (HSSOR9L)," IOSR J. Math., 3, pp. 1-7, 2012.
- [15] Zachmanoglou, E. C., and Thoe, D. W., 1976, "Introduction to Partial Differential Equations with Applications," Dover Publications, New York.

- [16] Hadjidimos, A., 1978, "Accelerated Overrelaxation Method," *Math. Comp.*, 32(141), pp. 149-157.
- [17] Saudi, A., 2015, "Robot path planning using family of SOR Iterative Methods with Laplacian Behaviour-Based Control," Ph.D. thesis, Universiti Malaysia Sabah, Kota Kinabalu, Sabah.
- [18] Zelinsky, A., 1991, "Environment Exploration and Path Searching Algorithms for Mobile Robot Navigation Using Sonar," Ph.D. thesis, University of Wollongong, Australia, pp. 264.
- [19] Dahalan, A. A., Ling, W. K., Saudi, A., and Sulaiman, J., 2019, "Application of Half-Sweep MAOR Iterative Method on Autonomous Robot Path Planning in Static Indoor Environment," *J. Eng. Appl. Sci.*, 14(23), pp. 8855-8861.
- [20] Dahalan, A. A., Saudi, A., and Sulaiman, J., 2018, "Numerical Technique for Autonomous Robot Path Searching Based on QSAOR Iterative Method Using Indoor Environment," *J. Eng. Appl. Sci.*, 13(20), pp. 8414-8418.